Dynamic Traffic Assignment Models for System Optimal Future Mobility Analysis

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Dynamic Traffic Assignment Models for System Optimal Future Mobility Analysis

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Thesis/Dissertation Sheet

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System optimum dynamic traffic assignment (SODTA) models predict a time-dependent traffic state with optimal network performance, providing a benchmark for controlling and managing dynamic traffic networks. This thesis explores the applications of these models for congestion mitigation based on a novel optimisation framework for system-level future mobility analysis.

This thesis has three aims: (1) to provide a mathematical foundation for developing a framework for network-level analysis of traffic flow, (2) to explore the usefulness of the proposed model for various network-level design problems with advanced congestion mitigation strategies, and (3) to explore the practicality of the proposed model for futuristic transport scenarios in an automation heavy network. These aims are achieved and presented in the three core chapters of this thesis.
The first core chapter develops the base model of SODTA embedding the link transmission model (LTM) for dynamic network loading and traffic flow propagation and implements it on single-OD and multi-OD networks.

The second core chapter explores three strategies for congestion mitigation with system-level mobility analysis based on further development of the base model. The three strategies involves a classical example of network design problem with potential capacity enhancements, a departure time incentive scheme to encourage commuters to shift their departure times to maintain an optimal system performance and a shared mobility service to cater to the travel demand of a network where commuters are incentivised to share their rides to reduce overall congestion in the network.

Finally, the third core chapter develops the base model even further to analyse a network which includes both legacy vehicles (LVs) and vehicles with automation features such as cooperative adaptive cruise control, speed harmonisation and cooperative merging. Here, an integrated mixed-integer programming framework is proposed for optimal exclusive lane design for these automated vehicles (AVs) on a freeway network which accounts for commuters' demand split among AVs and LVs via a logit model incorporating class-based utilities.

Overall, this thesis exploits the potentialities of SODTA models to evaluate futuristic transport scenarios. The recent technological advancements in transport industry indicate an exponential rise in cooperation and coordination between transportation system and its stakeholders rendering these models essential tools for future mobility analysis.

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This thesis is dedicated to my beloved sister

Didibhai (Antara Chakraborty)

for her constant support throughout my entire academic career

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Jai Gurudev!

Abstract

System optimum dynamic traffic assignment (SODTA) models predict a time-dependent traffic state with optimal network performance, providing a benchmark for controlling and managing dynamic traffic networks. These models possess great potential for various applications such as network design, emergency evacuation of traffic management, signal control and congestion pricing. This thesis explores the applications of these models even further for congestion mitigation based on a novel optimisation framework for system-level future mobility analysis.

This thesis has three aims: (1) to provide a mathematical foundation for developing a framework for network-level analysis of traffic flow, (2) to explore the usefulness of the proposed model for various network-level design problems with advanced congestion mitigation strategies, and (3) to explore the practicality of the proposed model for futuristic transport scenarios in an automation heavy network. These aims are achieved and presented in the three core chapters of this thesis. The first core chapter develops the base model of SODTA embedding the link transmission model (LTM) for dynamic network loading and traffic flow propagation and implements it on single-OD and multi-OD networks. The second core chapter explores three strategies for congestion mitigation with system-level mobility analysis based on further development of the base model. The first strategy involves a classical example of network design problem which is solved with potential capacity enhancements in crucial locations in the network leading to optimal network performance respecting budget constraints. In the second strategy, a departure time incentive scheme is developed encouraging commuters to shift their departure times to maintain an optimal system performance. Whereas, in the third strategy a shared mobility service is developed catering to the travel demand of a network where commuters are incentivised to share their rides to reduce overall congestion in the network. Here, the novelty lies in designing these incentive schemes based on the impact on endogenously computed arrival times of commuters due to departure time shift or ride-sharing. Both of these formulations are implemented on multi-OD test networks showing significant improvements in overall system performance by managing the travel demand effectively.

Finally, the third core chapter develops the base model even further to analyse a network which includes both legacy vehicles (LVs) and vehicles with automation features such as cooperative adaptive cruise control, speed harmonisation and cooperative merging. Here, an integrated mixed-integer programming framework is proposed for optimal exclusive lane design for these automated vehicles (AVs) on a freeway network which accounts for commuters' demand split among AVs and LVs via a logit model incorporating class-based utilities. The LTM is modified to integrate two vehicle classes namely, LVs and AVs with a lane-based approach. The presence of binary variables to represent lane design and the logit model for endogenous demand estimation results in a non-convex mixed-integer non-linear program (MINLP) formulation. To tackle this challenging optimization problem, a Benders' decomposition approach is adopted. The proposed approach iteratively explores possible lane designs in the Benders' master problem and, at each iteration, solves a sequence of SODTA problems which is shown to converge to fixed-points representative of logit-compatible demand splits. Further, it is proven that the proposed solution method converges to a local optima of the nonconvex problem and conditions are identified under which this local optima is a global solution. The proposed approach is implemented on two hypothetical freeway networks with single and multiple origins and destinations where route choice modelling is obviated. The numerical results reveal that the optimal lane design of freeway network is non-trivial and can inform on the value of accounting for endogenous demand in the proposed freeway network design.

Overall, this thesis exploits the potentialities of SODTA models to evaluate futuristic transport scenarios. The recent technological advancements in transport industry indicate an exponential rise in cooperation and coordination between transportation system and its stakeholders rendering these models essential tools for future mobility analysis.

List of Publications

Journal articles (published)

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

Transportation has been one of the bare necessities for humankind from time immemorial. It has been the engine of growth in human history facilitating trades of goods and services provided by different modes which evolved over time. The accessibility and mobility provided by transport networks have been the most important decisive factor in determining locations of cities and industries leading to economic and social progress of nations. Understanding the importance of mobility and transport with regards to the prosperity of a nation, countries around the world established transport itself as an industry to drive the well-being of national economies.

With the perpetual growth in transport, sustainability took the back-seat which brought in traffic congestion, air pollution and accidents along with freedom in movement. The vicious cycle of reappearing congestion due to more motorways to match rising travel demand stalled transportation infrastructure investments with a broad agreement that the current trends in transport is not sustainable. The growing awareness about finite fossil fuel resources and increase in greenhouse gas emissions along with the experience of recurring congestion and traffic accidents ignited a fundamental change in technology, design, operation and financing of transport system towards a sustainable approach.

A panoramic approach encompassing individual needs is absolutely necessary to have a sustainable development plan in transport. A system-level approach coupled with state-of-the-art optimisation methods is essential to identify the crucial areas in a network where tax-payers' money should be invested for transport infrastructure improvement. However, our self-centred travel behaviour has always been one of the main obstacles in implementing such system-level approaches. In this scenario, a macroscopic objective can still be achieved if we, as selfish commuters, are lured to co-operate for a bigger cause which benefits all the stakeholders in a transport network.

This thesis is motivated to develop a single narrative embedding such a sustainable system-level approach for improving network-wide traffic performance with co-operation from commuters with incentives, whenever needed.

1.2 Background

As mentioned earlier, a sustainable transport planning process is an indispensable component for the economic growth of a nation. Traditionally, this transport planning process involves four steps: trip generation, trip distribution, mode choice and traffic assignment. The trip generation constitutes of two components: trip production and trip attraction. Trip production (attraction) estimate the number of trips originated from (destined to) a traffic analysis zone based on collected data regarding travel attributes in a traffic analysis zone. Subsequently, an origin-destination matrix is created in the trip distribution step, indicating the overall inter and intra-zonal flow of traffic. Depending on the availability, accessibility and affordability, commuters choose different modes of transport to make these inter and intra-zonal trips which is modelled in the mode choice step. The output of the mode choice step estimates the number of vehicles a network has to deal with. Finally, the traffic assignment models come in to allocate these vehicles in different routes in the network completing the four-step transport planning process.

Traffic assignment models typically describe the interaction between travel demand and available transport infrastructure. These models have been crucial throughout the years for strategic transport planning as well as for predicting future transport flows. These models generally consist of two fundamental components: route choice model and network loading model. Based on the spatial, temporal and behavioural assumptions of these components, traffic assignment models could be categorised accordingly (Bliemer et al., 2017). The most popular distinction in the literature is based on temporal assumptions where these models are mainly categorised into two categories: static and dynamic.

Static models consider fixed travel demand and a single time period for both route choice and network loading. The single time period generally represents a stand-alone peak period, unperturbed by traffic flows outside this period. The network loading component assures that all vehicles reach their respective destinations within this time period. Static models are simple and efficient. Hence, they are still in practice and research and most commonly implemented throughout the world for strategic transport planning purposes. However, temporal assumptions in static models are quite strong and may not represent the ground traffic conditions correctly. The most crucial drawback of traditional static models is the absence of explicit capacity constraints which produces erroneous results in modelling bottlenecks and estimating travel times and delays. The modified version of the traditional static models does consider these capacity constraints with a provision of accumulating excess traffic at bottlenecks in residual queues.

A sequence of such static models may capture the fluctuation in travel demand

and traffic flow propagation over time within a network with a fixed route choice proportion for each model. Such sequence of models is called a semi-dynamic model which includes features such as connectivity between two time periods for passing residual vehicles.

Dynamic Traffic Assignment (DTA) models aim to develop a time-dependent traffic flow pattern from a time-varying travel demand on a transport network. Two types of traffic flow patterns are commonly examined: selfish routing (user equilibrium) and social welfare (system optimum). The former aims to represent an equilibrium in users' utilities whereas the latter is beneficial for network-wide system performance—often measured by the total system travel time (TSTT).

When the agents in a traffic network choose their routes to minimize their own expected travel time, the system progresses towards a state of user equilibrium (UE) where no further reduction in travel time is possible between each origin-destination pair (Wardrop, 1952). The link flow rates corresponding to this state can be obtained by solving a non-linear convex program minimizing the Beckmann function (Beckmann et al., 1956). The constraints of this optimization problem include the flow conservation and non-negativity constraints. As convex programming has appealing mathematical properties, this method is widely studied for traffic assignment in the literature. However, UE flow patterns may lead to an underutilization of the network infrastructure compared to system optimum (SO) flow patterns.

System optimum dynamic traffic assignment (SODTA) allocates traffic on a network in such a way that the TSTT is minimized. In this case, all agents in the traffic network do not experience equal and minimal travel time which might incentivise drivers to switch routes in pursuit of a path with lesser travel time. Hence, the SODTA flow pattern may not be sustainable by itself unless the commuters are cooperative in maximum social welfare. However, SODTA provides a measure of the optimal usage of network infrastructure and has attracted substantial attention over the years due to its application in various traffic management strategies such as congestion pricing, incentive-based travel demand management, traffic control measures, emergency evacuations, etc. It also allows a transport modeller to internalise the externalities of congestion, accidents and environmental impacts for a better assessment of network level traffic conditions. Furthermore, with the advent of connected and autonomous vehicles, system-level traffic control measures might be much more attainable compared to the current era of greedy driving behaviour.

1.3 Research questions

SODTA models attracted considerable attention over the years for its application on day-to-day traffic operation and management to disaster emergency evacuations to network planning and transport policy evaluation. This thesis is focussed on developing an SODTA framework to address various such network-level design problems. The SODTA framework embeds the link transmission model (LTM) for dynamic network loading and traffic flow propagation due to its simpler traffic flow modelling approach and reduced computational complexity compared to the other solution methods.

This thesis attempts to achieve the following three research aims.

Aim 1. Develop a framework for network-level traffic flow analysis

First of all, this thesis aims to provide a mathematical foundation for developing a framework for network-level analysis of traffic flow.

Aim 2. Explore the application of the proposed framework to analyse advanced congestion mitigation strategies

The second aim of this thesis is to explore the usefulness of the proposed model for various network-level design problems in existing traffic scenarios such as capacity enhancement, travel demand management with departure time incentives and shared mobility service.

Aim 3. Explore the application of the proposed framework for evaluating futuristic traffic scenarios

Finally, this thesis aims to explore the practicality of the proposed model for futuristic transport scenarios in an automation heavy network.

1.4 Thesis overview

The three aims of this thesis are fulfilled over the following chapters.

Chapter 2. Literature review

This chapter documents a review of literature relevant to the research questions.

Chapter 3. Link Transmission Model-based System Optimum Dynamic Traffic Assignment

This chapter presents a linear programming formulation for system optimum dynamic traffic assignment using the link transmission model (LTM) as the underlying traffic flow model. The original LTM was adapted by incorporating link-sending and receiving flows using linear inequalities. Furthermore, route choice was relaxed, and transfer flow variables were used to model vehicles' routing decisions within the network. The objective function of the linear program aims to minimize the total difference between the cumulative inflows and outflows of each link subject to flow-conservation, link capacity and jam density constraints. These cumulative inflows and outflows are represented using transfer flows from connected links. The resulting formulation is a linear program that represents a dynamic system optimum traffic flow pattern, embedding the LTM's network loading procedure for single-OD and multi-OD networks. The performance of the proposed formulation is evaluated on two test networks.

Chapter 4. System-level Mobility Analysis for Congestion Mitigation

In this chapter, the LTM-SODTA model proposed in the previous chapter is implemented for system-level mobility analysis for congestion mitigation with three strategies: network design with capacity enhancement and travel demand management with departure-time incentives and shared mobility service. The LTM-SODTA model is modified accordingly while implementing these three strategies with a limited budget. Network design with capacity enhancement is modelled by updating the sending and receiving flow constraints in the base model with a scope for improvement in capacity of selected links that will have the maximum effect on system-level performance. In this case, a comparison is made with the popular cell transmission model (CTM) while solving the network design problem (NDP). CTM is found to involve more number of variables compared to LTM due to its cell-based structure. From network design perspective, it is inferred that the cell representation of a network may not necessarily provide additional valuable degrees of freedom for NDP as improving only a segment of a link may not increase the throughput of vehicles in the network. This chapter also includes evaluating two travel demand management strategies where a novel method of endogenous estimation of travel time is designed and adopted for incentivising commuters to shift their departure-time and share their rides in two separate formulations.

Chapter 5. Freeway Network Design under Endogenous Autonomous Vehicles Demand

In this chapter, an integrated mixed-integer programming framework is proposed for optimal AV-exclusive lane design on a freeway network which accounts for commuters' demand split among AVs and LVs via a logit model incorporating class-based utilities. The link transmission model (LTM) is incorporated as the underlying traffic flow model due to its computational efficiency for system optimum dynamic traffic assignment. The LTM is modified to integrate two vehicle classes namely, LVs and AVs with a lane-based approach. The presence of binary variables to represent lane design and the logit model for endogenous demand estimation results in a nonconvex mixed-integer nonlinear program (MINLP) formulation. To tackle this challenging optimization problem, a Benders' decomposition approach is adopted. The proposed approach iteratively explores possible lane designs in the Benders' master problem and, at each iteration, solves a sequence of system-optimum dynamic traffic assignment (SODTA) problems which is shown to converge to fixed-points representative of logit-compatible demand splits. Further, it is proven that the proposed solution method converges to a local optima of the nonconvex problem and conditions are identified under which this local optima is a global solution. The proposed approach is implemented on two hypothetical freeway networks with single and multiple origins and destinations. The numerical results reveal that the optimal lane design of freeway network is non-trivial and can inform on the value of accounting for endogenous demand in the proposed freeway network design.

Chapter 6. Conclusion

This chapter concludes the thesis with summarising the findings and contribution of the study along with presenting future research directions.

Chapter 2

Literature Review

2.1 Introduction

In 1952, John Glen Wardrop stated two principles to formalise different notions of equilibrium leading to the emergence of user-equilibrium (UE) and system optimal (SO) traffic assignment. Under UE conditions, each traveller seeks to minimise his/her travel time/cost with selfish driving behaviour which represents a realistic travel behaviour with equal travel time/cost on each used path between any origin-destination pair. Although realistic, the system performance takes a toll under UE conditions which might be one of the main reasons for traffic congestion in any urban transport network around the world. On the other hand, under SO traffic conditions, a panoramic approach is developed where the objective is to reach optimal system performance with full cooperation from all the users in the network. Obtaining this overall cooperative driving behaviour is near impossible in the world of selfish drivers. However, with the advent of automation technologies in transport sectors along with well-designed incentive schemes towards congestion mitigation strategies such as departure-time shifts, rides-sharing etc., a significant shift in driving behaviour could be obtained. Following this shift, SO traffic assignment might become one of the most desirable solutions to most traffic related issues in urban transport networks.

System optimum dynamic traffic assignment (SODTA) originated from the dynamic extension of Wardrop's second principle (Wardrop, 1952) which is used to predict a time-dependent traffic state with optimal network performance and provided a benchmark for controlling and managing dynamic traffic networks. SODTA is also implemented extensively for long-term technology development in evaluating transportation policies such as congestion pricing (Yang and Hai-Jun, 1997), network access control (Zhang and Shen, 2010), traffic management under evacuation (Hsu and Peeta, 2014), dynamic parking (Yang et al., 2013; Ma and Zhang, 2017) etc. Despite extensive exploration, SODTA remains an active research area since modelling and optimizing dynamic traffic networks is highly complex (Papageorgiou, 1990). Further, the time-dependent traffic flow pattern solution of SODTA involves complex spatial and temporal interactions of traffic with non-convex constraints. To account for this non-convexity issue, various theoretical studies introduced a number of simplifications, such as networks with fixed route choice (Gazis, 1974; D'Ans and Gazis, 1976), networks with a single bottleneck (Vickrey, 1969; Hendrickson and Kocur, 1981), networks with only parallel routes (Arnott et al., 1990) or the adoption of a path-based approach to eliminate the non-convex constraints (Ghali and Smith, 1995; Shen et al., 2007; Qian et al., 2012). Another simplified version of the SODTA problem involves analysing a network with multiple origins and a single-destination (Merchant and Nemhauser, 1978; Carey, 1987; Wie, 1998; Ziliaskopoulos, 2000). This restrictive formulation attempts to avoid the basic non-convexity which occurs due to first-in-first-out requirement of traffic agents in a dynamic flow problem (Carey, 1992). Although, single-destination SODTA deals with a single commodity and may not overcome the drawbacks of existing analytical models, it can provide valuable insights to solve multi-destination network problem. Even without this further development to multiple destinations, SODTA with single-destination can be applicable to scenarios such as emergency evacuation towards a common assembly point (Dixit and

Wolshon, 2014; Liu et al., 2006; Chiu et al., 2007) or modelling the morning peak traffic commute towards a high-demand centroid (e.g. central business districts) (Shen and Zhang, 2014). Other successful implementation of SODTA models include congestion pricing (Yang and Meng, 1998; Carey and Watling, 2012), signal control (Lo, 2001; Lin and Wang, 2004) and network design (Waller and Ziliaskopoulos, 2001; Waller et al., 2006)

2.2 Network loading and traffic flow propagation

SODTA models consist of two components: dynamic network loading (DNL) and route choice. DNL models represent the propagation of traffic flow over time in road networks based on the kinematic wave theory, introduced by Lighthill and Whitham (1955); Richards (1956). Typically, DNL models consist of two components: a link model and a node model. Link models determine sending flows from upstream links and receiving capacities of downstream links at each time step based on traffic flow interactions within the links. Node models determine the actual transfer flow from the upstream link to the downstream link which is the minimum of sending and receiving flows and the capacities of the corresponding links.

The computational complexity of SODTA for practical application purposes strongly depends on the underlying traffic flow model and its specific formulation involving these link and node models. Various traffic flow models have been adopted in the formulation of the SODTA optimization problem such as link exit flow function (Merchant and Nemhauser, 1978; Carey, 1987; Wie, 1998), deterministic queuing models with no link interactions (Yang and Meng, 1998; Chang et al., 1989) and cell transmission model (CTM) (Ziliaskopoulos, 2000). In this thesis, an SODTA framework embedding the link transmission model (LTM), as developed by Yperman (Yperman et al., 2005), is proposed for exploring various network-level design applications. A brief discussion about the application of CTM and LTM for SODTA is provided in the following subsections.

2.2.1 Cell transmission model

Ziliaskopoulos (2000) proposed a linear programming (LP) formulation for SODTA problem based on the cell transmission model (CTM), introduced by Daganzo (Daganzo, 1994, 1995). This model uses cells to represent link flow which captures traffic flow variability inside the link in contrast to models relying on link exit functions. The motivation behind this LP approach was to make use of the vast existing literature on LP for a better understanding and resolution of SODTA. Furthermore, the dual variables associated with the flow conservation constraint of this LP represented the marginal contribution of an additional unit of demand at a given cell and in a given time interval, thereby providing means for developing marginal cost pricing strategies. This CTM-based LP approach has been adopted later on to solve single-level continuous network design problem (NDP) (Waller et al., 2006; Ukkusuri, 2002).

The objective function of an NDP typically minimises the TSTT subject to flow conservation as well as budget constraints. However, the CTM-based NDP formulation involves a large number of variables and constraints due to the presence of large number of cells, even for small-size test networks. As single-destination NDP formulation represent traffic flow propagations explicitly by link-based variables, the number of variables and constraints of the optimization problem is directly proportional to the scale of the network and length of the study period (Shen and Zhang, 2014). Hence, CTM based NDP approaches may become computationally intractable for real-sized traffic networks.

2.2.2 Link transmission model

The Link Transmission Model (LTM), proposed by Yperman (Yperman et al., 2005), improves on the CTM space discretization to a large extent by analysing traffic flow at link boundaries only. This reduction in space discretization in turn reduces the computational complexity by n times for the same level of accuracy, where n is the average number of cells in a homogeneous network link (Yperman et al., 2005). This reduction in computational complexity is expected to enable the proposed LP to scale to real-size traffic networks. In the literature, LTM-based SODTA model has been developed with environmental objectives (Long et al., 2018), to study strict implementation of first-in-first-out (FIFO) constraints (Long and Szeto, 2019) and with dial-a-ride service constraints for shared autonomous vehicle routing (Levin, 2017). Considering the importance of developing a solution approach for various network design problems, an LTM based LP formulation for SODTA is proposed in this study and substantially developed further for four distinct network design applications involving automation in transport sector.

2.3 SODTA for modelling automation in transport sector

As mobility demand increases, transport infrastructures remain under-utilized majorly due to various human factors e.g., slow reflexes, poor and heterogeneous driving behaviour, safety concerns etc. Automation in transport sector is gaining more attention than ever to minimize these human inputs to exploit transport resources in an efficient and sustainable way. This growing attention in automation translates to a projected growth of US$173B in global autonomous driving market by 2030 (Sullivan, 2018). Potential benefits of vehicles with automation (AV) include improved throughput (Vander Laan and Sadabadi, 2017; Levin and Boyles, 2016a; Shladover et al., 2012), traffic safety (Fernandes and Nunes, 2012), travel speed and energy consumption (Mersky and Samaras, 2016). Along with these benefits, AVs have the potential to provide cost-effective mobility options with overall system-level benefits in terms of congestion and vehicular emissions (Greenblatt and Saxena, 2015) as well. However, in arterial networks with heterogeneous traffic, fewer AVs have been found to have a negative impact on the average travel speed and string stability of the traffic flow (Van Arem et al., 2006; Talebpour and Mahmassani, 2016; Monteil et al., 2018). Additional resource allocation at the network level, such as AV-exclusive lanes, may handle such issues fostering the usage of AVs further rendering this mode of travel more attractive than legacy vehicles (LV). Along with this, it is also necessary to find the crucial locations in the network where providing these dedicated lanes would reap the maximum benefits.

2.3.1 Effect of exclusive lanes

The safety concerns due to mixed vehicular interactions could be minimised with AV-exclusive lanes segregating AVs and LVs in a network, providing a smooth transition of the current transport system to an automation-heavy transport system. These dedicated lanes would be advantageous in the current situation of imperfect AV technologies, which pose critical problems in traffic safety during lane changing in a congested network with mixed traffic. At network intersections, reservation-based models have been shown to have the potential to increase intersection capacity (Dresner and Stone, 2004; Fajardo et al., 2011; Qian et al., 2014; Levin and Rey, 2017). More recently, Rey and Levin (2019) proposed a hybrid network control policy in such networks with dedicated lanes that provide access to “blue phases” during which only AVs can traverse traffic intersections.

Yu et al. (2019) adopted a microscopic traffic simulation method to investigate the efficiency and safety of mixed traffic on highways with AV-exclusive lanes. Although, the safety of mixed traffic was found to be worsened with low market penetration of AVs, the simulation results from the car-following models of this study showed an increment of up to 84% in throughput of the traffic network due to presence of AV-exclusive lanes. These dedicated lanes may also facilitate cooperative adaptive cruise control (CACC) contributing to better traffic flow performance (Van Arem et al., 2006), improved highway capacity (Milanés et al., 2013), decrease in fuel consumption and emissions (Ploeg et al., 2011). On freeways, CACC was found to significantly increase capacity with a moderate to high market penetration rate (Shladover et al., 2012). However, with a naive deployment strategy of AV-exclusive lanes, CACC was found to increase the total system travel time on freeways (Melson et al., 2018). Talebpour et al. (2017) simulated traffic flow under different penetrations rates of AVs on a two-lane and a four-lane freeway segment in Chicago, Illinois with mandatory and optional usage of AV-exclusive lanes. At market penetration rates of more than 50% for the two-lane highway and 30% for the four-lane highway, a potential benefit in terms of throughput and travel time reliability was observed in this study, with optional use of the AV-exclusive lanes yielding the most benefit. In another study, this optional use of AV-exclusive lanes was found to be beneficial only with a high market share of AVs (Mahmassani, 2016).

2.3.2 Effect of automated/coordinated/cooperative mobility services on mode choice

With the advent of AV technology in the market, it is critical to gain individual motivations for choosing to own or use AVs as a service. Hence, an endogenous demand model becomes imperative in such traffic flow modelling involving multiple vehicle classes. Based on a stated preference survey from 721 individuals living across Israel and North America, 44% of the sample population were found to prefer LVs over AVs (Haboucha et al., 2017). Another stated choice survey for the adoption of shared AVs suggested service attributes including travel cost, travel time and waiting time to be critical determinants of the use of shared AVs (Krueger et al., 2016).

To account for the effect of disruptive technologies like AVs on mode choice, this thesis explores the potential benefits of introducing AV-exclusive lanes in a network where AVs are used as a service and the AV-demand is estimated endogenously. Along this line, the proposed model also investigates the problem of locating AV-exclusive lanes in a freeway network so as to reap maximum benefits of the deployed infrastructure.

2.4 Summary

This thesis is built upon the literature of system optimum dynamic traffic assignment (SODTA) formulations to represent traffic dynamics, and obtain an analytical model amenable for exact optimization. Based on the review of the literature presented in this chapter, the following gaps have been identified to be crucial.

1. It has been observed that the potentiality of SODTA models is not fully explored in the following avenues:
    (a) accounting for various aspect of network data: elastic demand, departure time and OD travel time
    (b) analysing various travel demand management strategies such as departure-time incentives and ride-sharing
    (c) multimodal traffic flow analysis especially with automated, coordinated and cooperative mobility services.
2. From the review of the existing literature, it is also realised that researchers around the world might be a bit sceptical in developing SODTA models as the system-optimum routes generated by these models may not represent the ground

traffic conditions correctly. However, this thesis would like to advocate that with the forthcoming wave of automation in transport industry with vehicle-to-vehicle communication and vehicle-to-system communication, network-level models like SODTA would become much more relevant and easily applicable on large transport networks. Without these automations, these models might be limited to networks with fixed or single paths between each OD-pair hence obviating route choice modelling.

With an attempt to bridge some of these gaps in the literature, this thesis develops an SODTA framework with LTM as the underlying traffic flow model and implements on general networks. The framework is further developed to model network design with capacity enhancements as well as travel demand management with two separate strategies: departure-time incentives and shared mobility services. A futuristic traffic scenario involving vehicles with minimal level of automation (level 1 as per SAE (2013)) is also analysed by adapting the framework to accommodate multiple vehicle classes exploring optimal dedicated lane allocation strategies for automated vehicles. In this formulation, the route choice modelling is averted with an implementation on a freeway network with single paths between each OD pair.

Chapter 3

Link Transmission Model-based System Optimum Dynamic Traffic Assignment[1]

This chapter presents a linear programming formulation for system optimum dynamic traffic assignment using the link transmission model (LTM) as the underlying traffic flow model. The original LTM is adapted by incorporating link-sending and receiving flows using linear inequalities. Furthermore, route choice is relaxed, and transfer flow variables are used to model vehicles' routing decisions within the network. The objective function of the linear program aims to minimize the total difference between the cumulative inflows and outflows of each link subject to flow-conservation, link capacity and jam density constraints. These cumulative inflows and outflows are represented using transfer flows from connected links. The resulting formulation is a linear program that represents a dynamic system optimum traffic flow pattern, embedding the LTM's network loading procedure for single-OD and multi-OD networks. The performance of the proposed formulation is evaluated on two test networks.

3.1 Introduction

Dynamic Traffic Assignment (DTA) models are useful for both short term (prediction of traffic conditions for traffic management) and long term (strategic transport plan-

[1]This Chapter is partially based on:

Chakraborty, S., Rey, D., Moylan, E., Waller, S.T., 2018. Link transmission model-based linear programming formulation for network design. Transportation Research Record 2672, 139-147.

ning processes for infrastructure investments) analysis of a transport network. These models have an edge over their static counterparts due to their ability of capturing time-varying travel demand along with describing various important traffic flow dynamics. However, these advantages come at a cost of high computational complexity depending on the intricacies of the DTA framework.

A typical DTA framework consists of two components: dynamic network loading (DNL) model and route choice model. DNL models consists of a link model and a node model describing the traffic flow interactions at link and node levels respectively. The traffic flow propagation through a link is modelled by link model which estimates the sending and receiving flows of each link whereas, the actual transfer flows between a pair of links are modelled by node models . The route choice model yields path flow for the DNL model for each origin-destination pair. In this chapter, an LP formulation is proposed using link transmission model (LTM) as the DNL model whereas, route choice at each node is governed by a system level objective. The overall formulation is a system optimum dynamic traffic assignment (SODTA) model with LTM as the underlying traffic flow model. This formulation is named as LTM-SODTA which provides the foundation of this thesis.

This chapter is structured as follows. Section 3.2 provides an overview of LTM and introduces the LP formulation for single-OD and multi-OD SODTA. The chapter concludes by summarizing the findings and discussing future research directions.

3.2 Linear program for SODTA with link transmission model

LTM, proposed by Yperman et al. (2005), is a solution method for dynamic network loading (DNL) models based on the simplified triangular fundamental diagram (Newell, 1993a). The model keeps track of the traffic flow rates at each time step in terms of cumulative inflows and outflows at link boundaries only, eliminating the minute space discretization used in Daganzo's (Daganzo, 1994) cell transmission model (CTM). Given a time-varying traffic demand and turning fractions at each node, the LTM determines the dynamic link travel times on a traffic network based on the cumulative vehicle inflows and outflows at each time step. The sending and receiving flows for each link pair are calculated based on the time-varying demand and the network and traffic flow characteristics. Consequently, the traffic flow is propagated based on transfer flows at each node of the network.

Let $G = (N, A)$ be a directed network where N is the set of nodes and A is the set of arcs. In the proposed formulation, each node of the network is represented by a set of incoming and outgoing arcs. The set of arcs is partitioned into three subsets: source centroid connectors denoted as A_r, sink centroid connectors as A_s, and physical links formed by the remaining links of the set; i.e., $A \backslash \{A_r \cup A_s\}$. The set of origin-destination pairs is represented by K. $\Gamma^-(i)$ and $\Gamma^+(i)$ represent the set of predecessor and successor links of link i. Two different sets of discretised timesteps, R and T, are proposed for dynamic network loading ($D(t)$) and traffic flow propagation respectively. Table 3.1 presents the rest of the notations of the proposed formulation.

The entire formulation involves three sets of variables: transfer flows between links ($y_{i,j}(t)$), cumulative inflows ($z_i^+(t)$) and outflows ($z_i^-(t)$) from each link at each timestep. As LTM propagates traffic flow based on updating cumulative inflows and outflows of links at each timestep, $z_i^+(t)$ and $z_i^-(t)$ are introduced as a set of decision variables in the proposed formulation to represent these cumulative numbers at the boundaries of each link. These cumulative variables get updated based on the transfer flows between each links summed over time until the previous timestep. In turn, transfer flows are estimated based on the cumulative flows from the previous timestep creating a circular reference. Practically, it is possible to construct the whole formulation using the transfer flows alone. However, this may lead to a complex set

Table 3.1: Mathematical notations for the single-OD SODTA

Sets	
A	set of all links and centroid connectors
A_r	set of source centroid connectors
A_s	set of sink centroid connectors
K	set of origin-destination pairs
$\Gamma^{-}(i)$	set of predecessor links of link $i \in A$
$\Gamma^{+}(i)$	set of successor links of link $i \in A$
T	set of discretised time steps for traffic flow propagation $(t_0, t_1, ..., t_n)$
R	set of discretised time steps for demand loading
Parameters	
$D(t)$	demand at $t \in T$
q_i	capacity of link $i \in A$
L_i	length of link $i \in A$
K_{jam}	jam density
v	free-flow speed
w	backward shockwave speed
δ	time step length for traffic flow propagation
Variables	
$y_{i,j}(t) \geq 0$	transfer flow from link $i \in A$ to link $j \in A$ at time $t \in T$
$z_i^{+}(t) \geq 0$	cumulative inflow to link $i \in A \setminus A_s$ at time $t \in T$
$z_i^{-}(t) \geq 0$	cumulative outflow from link $i \in A \setminus A_s$ at time $t \in T$

of constraints making the model quite incomprehensible. Hence, in this study, both transfer and cumulative flows are used as decision variables.

The transfer flows in LTM are obtained from the node model, considering the number of vehicles intends to travel from an upstream link, spaces available in the downstream link and their respective capacities at each timestep. In a single-origin-single-destination network, the link model in LTM estimates this sending flow ($S_i(t)$) from an upstream link i and the receiving flow ($R_j(t)$) into a downstream link j at each time step based on equations (3.1) and (3.2). The maximum sending flow from link i at time t is defined as follows.

$$S_i(t) = \min\left\{(z_i^{+}(t_s) - z_i^{-}(t)), q_i\delta\right\} \tag{3.1}$$

Here, $t_s = t + \delta - \frac{L_i}{v}$, where δ is discretised time step for traffic flow propagation, L_i and v are the length and free-flow speed of link i respectively, the ratio of which provides the free-flow travel time in that link. According to the LTM theory, if a free flow state of traffic is observed at the downstream end of a link at time t, this state must have emerged from the upstream end and travelled at free flow speed of the link (Yperman et al., 2005). Hence, the sending flow from each upstream link towards all the downstream links during each time interval $[t, t+\delta]$ must be less than or equal to the difference in cumulative inflows at upstream at $t_s = t+\delta-\frac{L_i}{v}$ and at downstream at t.

Similarly, the maximum receiving flow on link i at time t is defined as follows.

$$R_j(t) = \min\left\{\left(K_{jam}L_j - (z_j^+(t) - z_j^-(t_r))\right), q_j\delta\right\} \tag{3.2}$$

Here, $t_r = t + \delta - \frac{L_i}{w}$, where w is the backward wave speed due to congestion. As per the LTM theory, if a congested traffic state is observed at the upstream end of a link, this state must have emerged from the downstream end, travelling backwards at a speed equal to the congestion wave speed, w (Yperman et al., 2005). Based on this concept, the amount of empty spaces left in a receiving link at each time step defines the receiving flow of that link as presented in Equation (3.2). Both sending and receiving flows of a link are constrained by its capacity (q).

In the node model, these sending and receiving flows are used to determine the transfer flow ($y_{i,j}(t)$) between a pair of connected links as presented in Equation (3.3).

$$y_{i,j}(t) = \min\{S_i, R_j\} \tag{3.3}$$

$$y_{i,j}(t) = \min\left\{(z_i^+(t_s) - z_i^-(t)), q_i\delta, \left(K_{jam}L_j - (z_j^+(t) - z_j^-(t_r))\right), q_j\delta\right\} \tag{3.4}$$

The existence of the min function in Equation (3.4) brings in non-linearity in the model. Further, in the original LTM formulation, the transfer flows also depend on the route choice model which dictates the traffic flow to a downstream link in terms of turning ratios. In this study, this min function of the original LTM is relaxed with linear constraints and the route choice is optimized by the SODTA model. The linearised set of constraints which is a relaxation of Equation (3.4) is presented as follows.

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \leq \left(z_i^+(t_s) - z_i^-(t)\right) \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.5}$$

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \leq q_i \delta \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.6}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \leq \left(K_{jam} L_j - (z_j^+(t) - z_j^-(t_r))\right) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{3.7}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \leq q_j \delta \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{3.8}$$

Note that the left-hand side of Constraints (3.5)-(3.8) represents the total flow that could potentially exit link i (sending) or enter link i (receiving) while accounting for the topology of the network. In addition, it should be noted that the min functions in (3.1) and (3.2) are relaxed within Constraints (3.5)-(3.8). While this linearisation technique is commonly used (Ziliaskopoulos, 2000), it does not impose a strict minimum, thus relaxing the original LTM formulation. This relaxation will be tightened by using an objective function that penalizes the travel time of vehicles in the network, thus incentivizing vehicles to "move along" instead of waiting. Further, the numerical experiments conducted in this study show that a strict minimum is always observed when implementing the proposed LTM-SODTA formulation.

Accordingly, the cumulative inflow and outflows ($z_i^+(t)$,$z_i^-(t)$) are updated based on the transfer flows as shown in Equations (3.9) and (3.10). These constraints define the cumulative inflows and outflows of a link at each time step as the total transfer flow from the predecessor links and total transfer flow to the successor links respectively, until the previous time step.

$$z_i^+(t) = \sum_{t'<t} \sum_{h\in\Gamma^-(i)} y_{h,i}(t') \qquad \forall i \in A \setminus A_r, \forall t \in T \tag{3.9}$$

$$z_i^-(t) = \sum_{t'<t} \sum_{j\in\Gamma^+(i)} y_{i,j}(t') \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.10}$$

The time-dependent travel demand is incorporated in the formulation as the cumulative inflow of source centroid connectors ($i \in A_r$) as shown in Equation (3.11).

$$z_i^+(t) = \sum_{t'<t} D(t') \qquad \forall i \in A_r, \forall t \in T \tag{3.11}$$

To ensure all vehicles reach the destination, a trip completion constraint is added in the model as follows. This constraint (3.12) equates the cumulative inflow into the sink centroid connector at the last timestep ($\bar{t}$) to the total demand in the network.

$$z_i^+(\bar{t}) = \sum_{t\in R} D(t) \qquad \forall i \in A_s \tag{3.12}$$

The traditional objective function in SODTA formulations is to minimize the total system travel time (TSTT), which is the sum of the experienced travel times of all the users in the network summed over the entire analysis period. As the traffic density of link i at t is $(z_i^+(t) - z_i^-(t))$, at each time interval, the experienced travel time of vehicles would be $\delta(z_i^+(t) - z_i^-(t))$, where δ is the duration of a timestep. As per the Courant-Friedrichs-Lewy (CFL) condition (Courant et al., 1928) of LTM, this δ is required to be at most equal to the free flow travel time ($\delta \leq \frac{L_i}{v}$) of the shortest link in the network to prevent vehicles from moving out of a link during the time interval. The objective function of the proposed LP can be presented as follows.

$$\min TSTT = \min \delta \sum_{i\in A\setminus A_r} \sum_{t\in T} \Big(z_i^+(t) - z_i^-(t) \Big) \tag{3.13}$$

Note that, the objective function (3.13) also minimises the waiting time of the vehicles within the source centroid connector that are queued up due to congestion in the network. Hence, the combination of Constraint (3.11) and the objective function ensures that the travel demand departing from centroid $i \in A_r$ at time t is accounted for in the computation of the TSTT.

The single-origin-single-destination LTM-SODTA formulation is presented in (3.14)-(3.25). This model is named as the base model for the sake of brevity.

$$\min TSTT = \min \delta \sum_{i \in A \setminus A_r} \sum_{t \in T} \Big(z_i^+(t) - z_i^-(t) \Big) \tag{3.14}$$

subject to,

$$z_i^+(t) = \sum_{t' < t} D(t') \qquad \forall i \in A_r, \forall t \in T \tag{3.15}$$

$$z_i^+(t) = \sum_{t' < t} \sum_{h \in \Gamma^-(i)} y_{h,i}(t') \qquad \forall i \in A \setminus A_r, \forall t \in T \tag{3.16}$$

$$z_i^-(t) = \sum_{t' < t} \sum_{j \in \Gamma^+(i)} y_{i,j}(t') \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.17}$$

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \le \big(z_i^+(t_s) - z_i^-(t) \big) \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.18}$$

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \le q_i \delta \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.19}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \le \big(K_{jam} L_j - (z_j^+(t) - z_j^-(t_r)) \big) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{3.20}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \le q_j \delta \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{3.21}$$

$$z_i^+(\bar{t}) = \sum_{t \in R} D(t) \qquad \forall i \in A_s \tag{3.22}$$

$$y_{i,j}(t) \ge 0 \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.23}$$

$$z_i^+(t) \ge 0 \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.24}$$

$$z_i^-(t) \ge 0 \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.25}$$

Constraint (3.15) loads the time-varying demand into the network through source centroid connectors. Equations (3.16) and (3.17) are definitional constraints defining the cumulative inflows and outflows of a link at each time step as the total transfer flow from the predecessor links and total transfer flow to the successor links respectively until the previous time step. Constraints (3.18) and (3.19) refer to the sending flow from each incoming link summed over all outgoing links during each time interval $[t, t+\delta]$ which must be less than or equal to the difference in cumulative inflows at $t_s = t+\delta-\frac{L_i}{v_{f,i}}$ and outflows at t. As the value of t_s is likely to fall in between two time steps, the cumulative flows are obtained using a linear interpolation method between these two corresponding time steps. The constraint (3.19) presents the upper limit of this sending flow as the capacity of the incoming link.

Constraints (3.20) and (3.21) refer to the receiving flows of each link accounting for congestion effects. Constraint (3.20) describes the amount of empty spaces left in a receiving link at each time step. In this constraint, the amount of empty spaces is presented as the difference between total spaces in the link (product of jam density and length of the link) and the occupied spaces (difference in cumulative flows accounting for congestion). The number of occupied spaces is determined by subtracting the cumulative outflows at $t_r = t + \delta - \frac{L_i}{w_i}$ from cumulative inflows at t. The cumulative flow at t_r is obtained following a similar interpolation method as that of t_s. According to Constraint (3.20), the transfer flow to each outgoing link summed over all incoming links must be less than or equal to the remaining empty spaces in the outgoing link. Constraint (3.21) presents the upper limit of the receiving flow as the capacity of the receiving link. Constraint (3.22) ensures all vehicles complete their trips at the end of the analysis period. Constraints (3.23)–(3.25) state the non-negativity conditions.

It should be noted that the proposed LP does not define separate set of constraints for merge and diverge situations. Constraints (3.18)–(3.21) are formulated in such a way that they apply to both merge and diverge nodes. This combined set of constraints allows the proposed LP to have a simpler form compared to other existing formulations.

In the following section, the proposed LP is implemented on a single-OD and multi-OD test networks.

3.2.1 Application on a single-OD network

In this section, we illustrate the proposed LP formulation by solving an example network with six nodes and eight links. The network characteristics are presented in Table 3.2. Figure 3.1 presents the schematic diagram of the network.

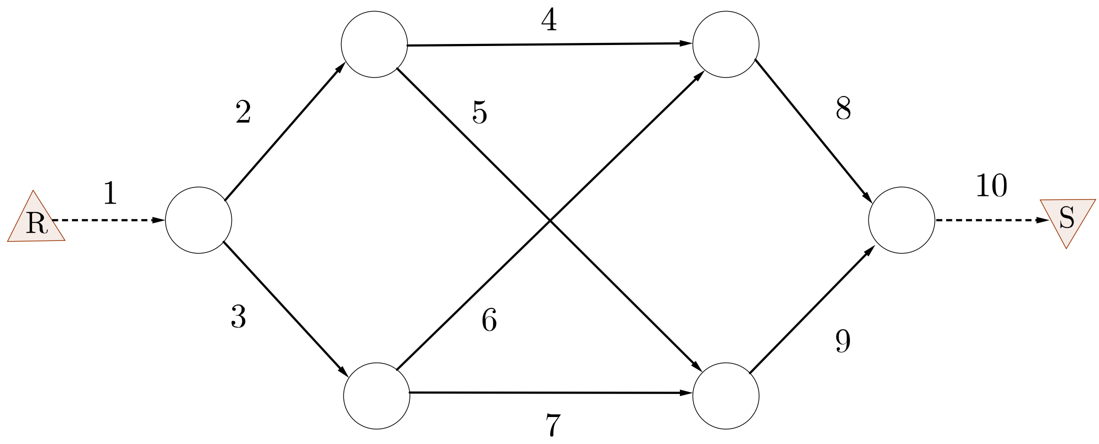

Figure 3.1: Single-OD test network for LTM-SODTA

The network consists of a single origin, R and a single destination S which are connected to the physical network by centroid connectors 1 and 10. There exists four paths between this OD pair: R-1-2-4-8-10-S, R-1-2-5-9-10-S, R-1-3-7-9-10-S and R-1-3-6-8-10-S. The length of the centroid connectors 1 and 10 are specified as zero whereas the capacity, jam density and free-flow speed are set at very high values. The physical links 2, 3, 4 and 9 have a link length of 900m and links 5, 6, 7 and 8 have a length of 1200m. The capacity and jam density of the links 4 and 7 are 1800 veh/hr (5 veh/timestep) and 150 veh/km respectively. The remaining links has a capacity of 3600 veh/hr (10 veh/timestep) and jam density of 300 veh/km. We assume the free-flow speed to be 30 m/s throughout the network.

Table 3.2: Characteristics of the test network (Figure 3.1)

Links	**Length**	**Capacity (veh/hr)**	**Jam density (veh/km)**	**Free-flow speed (m/sec)**
1	0	100000	100000	100000
2	900	3600	300	30
3	900	3600	300	30
4	900	1800	150	30
5	1200	3600	300	30
6	1200	3600	300	30
7	1200	1800	150	30
8	1200	3600	300	30
9	900	3600	300	30
10	0	100000	100000	100000

The total period of the analysis is discretised into time steps. In this network, we assume a time step of 10 seconds and an assignment period of 140 seconds. The time-varying demand of 40 vehicles is loaded into the network at each time step through connector 1. It should be noted that the demand at time step t specifies the number of vehicles loaded into the network during the time interval $[t, t + \delta]$.

The proposed LP is implemented on a Intel(R) Core(TM) i7-6700 @3.40GHz CPU with 16GB RAM and CPLEX 12.10.0.0 is used as the optimization solver. The optimal occupancies in each link at each timestep have been presented in Table 3.3. The LP formulation for this case results in a program with 465 variables, 779 constraints and 31 dual simplex iterations to converge to a TSTT value of 4700 veh-seconds. It is observed that the demand (40) loaded at the first timestep through link 1 is more than the combined capacities (20) of the downstream links 2 and 3. Hence, vehicles queue up at link 1 for one more timestep before entering into the network. The proposed model in this study considers this waiting time as well while minimising the TSTT.

The proposed formulation is further enhanced to capture multiple OD-pairs and

Table 3.3: Optimal occupancy (in number of vehicles)

Links	**Time steps**														
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	40	20	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	10	20	20	10	0	0	0	0	0	0	0	0	0
3	0	0	10	20	20	10	5	0	0	0	0	0	0	0	0
4	0	0	0	0	0	5	10	10	5	0	0	0	0	0	0
5	0	0	0	0	0	5	10	10	10	5	0	0	0	0	0
6	0	0	0	0	0	5	5	5	5	0	0	0	0	0	0
7	0	0	0	0	0	5	10	15	15	10	5	0	0	0	0
8	0	0	0	0	0	0	0	0	5	15	15	15	10	0	0
9	0	0	0	0	0	0	0	0	0	10	20	25	15	5	0

implemented on the Sioux Falls network in the following section.

3.2.2 Application on multi-OD networks

In the previous section, a minimal working example of the base model on a single-origin-single-destination network is illustrated. In this section, the model is adapted to accommodate multiple OD pairs and implemented on the popular Sioux Falls network. This multi-OD LTM-SODTA model is presented as follows (Equations (3.26)–(3.37)).

$$\min TSTT = \min \delta \sum_{(o,d)\in K} \sum_{i\in A\setminus A_r} \sum_{t\in T} \left(z_i^{o,d+}(t) - z_i^{o,d-}(t) \right) \tag{3.26}$$

subject to,

$$z_o^{o,d+}(t) = \sum_{t'<t} D^{o,d}(t') \qquad \forall (o,d) \in K, \forall t \in T \tag{3.27}$$

$$z_i^{o,d+}(t) = \sum_{t'<t} \sum_{h \in \Gamma^-(i)} y_{h,i}^{o,d}(t') \qquad \forall i \in A \setminus A_r, \forall (o,d) \in K, \forall t \in T \tag{3.28}$$

$$z_i^{o,d-}(t) = \sum_{t'<t} \sum_{j \in \Gamma^+(i)} y_{i,j}^{o,d}(t') \qquad \forall i \in A \setminus A_s, \forall (o,d) \in K, \forall t \in T \tag{3.29}$$

$$\sum_{j \in \Gamma^+(i)} y_{i,j}^{o,d}(t) \leq \left(z_i^{o,d+}(t_s) - z_i^{o,d-}(t) \right) \qquad \forall i \in A \setminus A_s, \forall (o,d) \in K, \forall t \in T \setminus \{t_n\} \tag{3.30}$$

$$\sum_{(o,d) \in K} \sum_{j \in \Gamma^+(i)} y_{i,j}^{o,d}(t) \leq \delta q_i \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{3.31}$$

$$\sum_{(o,d) \in K} \sum_{i \in \Gamma^-(j)} y_{i,j}^{o,d}(t) \leq K_{jam} L_j - \sum_{(o,d) \in K} \left(z_j^{o,d+}(t) - z_j^{o,d-}(t_r) \right) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{3.32}$$

$$\sum_{(o,d) \in K} \sum_{i \in \Gamma^-(j)} y_{i,j}^{o,d}(t) \leq \delta q_j \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{3.33}$$

$$z_d^{o,d+}(\bar{t}) = \sum_{t \in R} D^{o,d}(t) \qquad \forall (o,d) \in K \tag{3.34}$$

$$y_{i,j}^{o,d}(t) \geq 0 \qquad \forall i \in A \setminus A_r, \forall t \in T \tag{3.35}$$

$$z_i^{o,d+}(t) \geq 0 \qquad \forall i \in A \setminus A_r, \forall t \in T \tag{3.36}$$

$$z_i^{o,d-}(t) \geq 0 \qquad \forall i \in A \setminus A_r, \forall t \in T \tag{3.37}$$

In this model, the three sets of variables as presented in the base model are now indexed for each OD pair: transfer flows ($y_{i,j}^{o,d}(t)$), cumulative inflows ($z_i^{o,d+}(t)$) and outflows ($z_i^{o,d-}(t)$). The objective function minimises the TSTT for all the OD-pairs as presented in Equation (3.26). Constraint (3.27) loads the dynamic OD-based demand into the network through source centroid connectors. After these vehicles enter the network, they are tracked with OD indices until they reach their respective destinations. The modified versions of the definitional constraints of cumulative inflows and outflows (Equations (3.28) and (3.29)) as well as Equation (3.30) of sending flow remain quite similar to the base model while being applied to each OD pair. However, in cases of the capacity constraint (Equation (3.31)) for sending flow and the set of receiving flow constraints (Equations (3.32) and (3.33)), the transfer flows are summed over all the OD-pairs as the sending and receiving flow capacities of a link do not change due to more number of OD-pairs in the network. Finally, the trip completion constraint (3.34) ensures that all vehicles reach their respective destinations (d) at the end of the analysis period ($\bar{t}$) followed by non-negativity constraints of the variables (3.35)–(3.37).

This multi-OD formulation is implemented on the Sioux Falls network with 6 OD pairs. The network is presented in Figure 3.2 which consists of 24 nodes and 76 links. The network characteristics are presented in Table 3.4. The travel demand originates from nodes 1, 3 and 13 towards destination nodes 6, 18 and 20.

The timestep (δ) for traffic flow is considered to be 30 seconds which renders 120 timesteps corresponding to an one hour analysis period. The free-flow speed and backward wave speed are considered as 54 and 36 km/hr respectively. Note that, the triangular fundamental diagram of LTM may dictate a variable wave speed depending on the capacities of links. However, this may provide unrealistic wave speeds in a network like Sioux Falls which has very high capacities for a number of links. Hence, in this study a fixed value of backward wave speed is considered. The value of δ (30 seconds) being lesser than the free-flow travel time (133 seconds) of the shortest link (2 km) of the Sioux Falls network, satisfies the CFL conditions of LTM and stops vehicle leakage during the analysis. Three levels demand are loaded through the source nodes over the first 15 minutes (30 timesteps) of the analysis period. The total demand between each OD-pair is shown in Table 3.5.

The proposed multi-OD model is implemented on an Intel(R) Core(TM) i7-6700

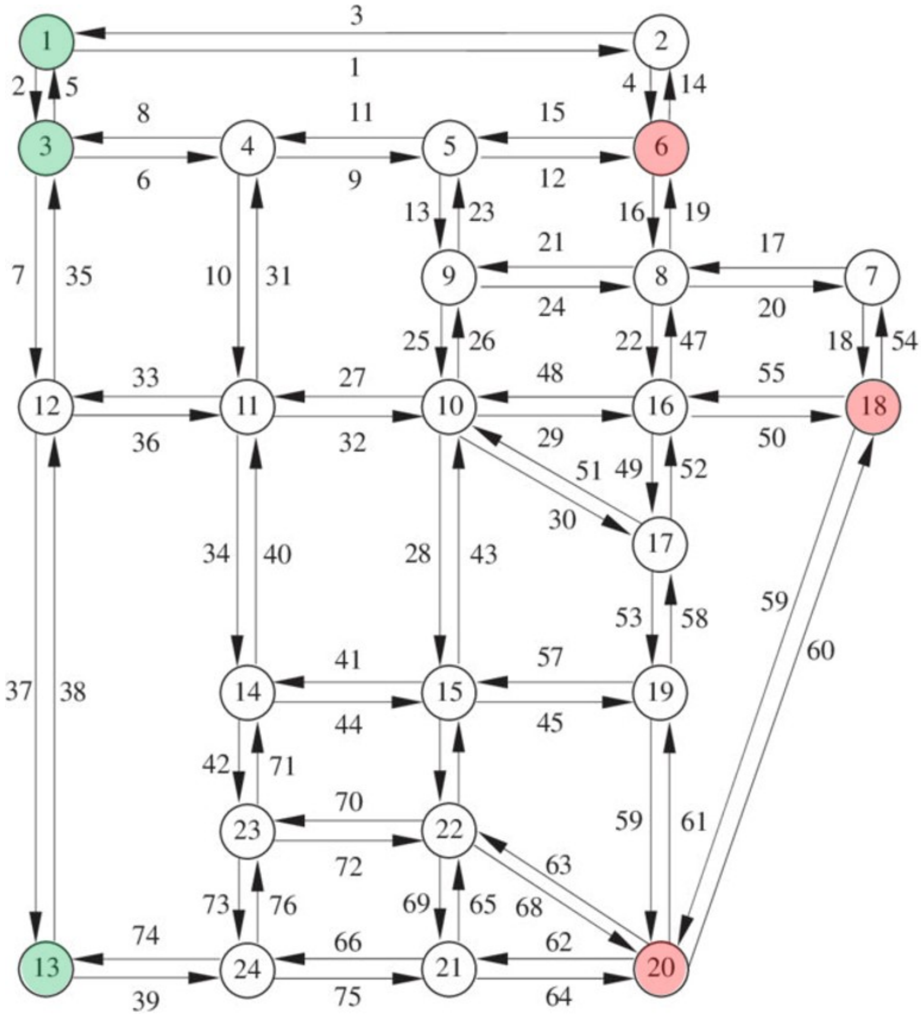

Figure 3.2: Sioux Falls network

@3.40GHz CPU with 16GB RAM. The solve times for low, medium and high demand are found to be 12.83, 17.45 and 20.54 minutes respectively with minimised TSTT of 8331530, 10746400 and 13421300 vehicle-seconds. To understand the traffic flow propagation in the network, N-curves are plotted for each OD-pair at each demand level and presented in Figures 3.3 – 3.5. The blue and red lines in the plots represent cumulative inflows at the origins and cumulative outflows at the destinations at each timestep respectively. The cumulative inflows are equal to the time-varying demand given as an input to the model whereas cumulative outflows shows the flow pattern in the network depending on the congestion level. As demand increases, the effect

Table 3.4: Data for Sioux Falls Network

Link	Capacity (veh/hr)	Length (km)	Link	Capacity (veh/hr)	Length (km)
1	25900.2	6	**39**	5091.3	4
2	23403.5	4	**40**	4876.5	4
3	25900.2	6	**41**	5127.5	5
4	4958.2	5	**42**	4924.8	4
5	23403.5	4	**43**	13512.0	6
6	17110.5	4	**44**	5127.5	5
7	23403.5	4	**45**	14564.8	3
8	17110.5	4	**46**	9599.2	3
9	17782.8	2	**47**	5045.8	5
10	4908.8	6	**48**	4854.9	4
11	17782.8	2	**49**	5229.9	2
12	4948.0	4	**50**	19679.9	3
13	10000.0	5	**51**	4993.5	8
14	4958.2	5	**52**	5229.9	2
15	4948.0	4	**53**	4824.0	2
16	4898.6	2	**54**	23403.5	2
17	7841.8	3	**55**	19679.9	3
18	23403.5	2	**56**	23403.5	4
19	4898.6	2	**57**	14564.8	3
20	7841.8	3	**58**	4824.0	2
21	5050.2	10	**59**	5002.6	4
22	5045.8	5	**60**	23403.5	4
23	10000.0	5	**61**	5002.6	4
24	5050.2	10	**62**	5059.9	6
25	13915.8	3	**63**	5075.7	5
26	13915.8	3	**64**	5059.9	6
27	10000.0	5	**65**	5229.9	2
28	13512.0	6	**66**	4885.4	3
29	4854.9	4	**67**	9599.2	3
30	4993.5	8	**68**	5075.7	5
31	4908.8	6	**69**	5229.9	2
32	10000.0	5	**70**	5000.0	4
33	4908.8	6	**71**	4924.8	4
34	4876.5	4	**72**	5000.0	4
35	23403.5	4	**73**	5078.5	2
36	4908.8	6	**74**	5091.3	4
37	25900.2	3	**75**	4885.4	3
38	25900.2	3	**76**	5078.5	2

of congestion in the network is clearly visible based on the increasing offset distance of the red lines (outflows) from the blue lines (inflows). Vehicles starting early from the origins always found the network in free-flow condition as expected, after which

Table 3.5: Origin-destination matrices (origins at 1, 3, 13 and destinations at 6, 18, 20)

Origin\Destination	6	18	20
Low demand			
1	0	875	1000
3	625	0	1250
13	875	1125	0
Medium demand			
1	0	1050	1200
3	750	0	1500
13	1050	1350	0
High demand			
1	0	1225	1400
3	875	0	1750
13	1225	1575	0

congestion started to play a role with vehicles spending more time in the network. With low demand, all the vehicles reach their destinations within 80 timesteps which increases to more than 100 timesteps for high demand. The vehicles from nodes 3 to 6 are found to be the first ones to complete their trips whereas vehicles from nodes 1 and 3 to 20 takes maximum time to reach their destinations in all the three levels of demand. This is found to be quite obvious looking at the network structure in Figure 3.2 with nodes 3 and 6 being quite closer to each other compared to the node 20 from 1 and 3. In the low demand case, a definitive horizontal step in the outflow curve is observed around the 60th timestep for OD-pair 13-18, indicating a bottleneck either at the origin or somewhere inside the network (explored in detail with Figures 3.6 – 3.8). This bottleneck is found to be less prominent in the medium demand level and shifted to timestep 70 for the high demand case. In both low and high demand case, the duration of the bottleneck is around 10 timesteps (5 minutes).

As mentioned earlier, the objective function of the proposed model minimises waiting times at the origins as well. If the capacities of the links, immediately connected to the origins cannot handle the incoming demand, vehicles are forced to wait creating bottlenecks at the origins. Hence, although Figures 3.3 – 3.5 depict the overall traffic flow pattern in the network, N-curves without these wait times at the origin would provide additional information about congestion effects on traffic flow such as bottlenecks. The following Figures 3.6 – 3.8 presents the N-curves from the time vehicles get an entry into the network at each demand level.

The blue lines in Figures 3.6 – 3.8 represent cumulative inflows into the network with quite a few horizontal steps representing the wait time at origins. The red lines in these plots remain the same representing the cumulative outflows at destinations. These red lines are devoid of horizontal steps for almost all the plots indicating steady flow of traffic inside the network even though the network is loaded gradually with the travel demand. Figure 3.6 provides additional information about the horizontal step in the cumulative outflow curve for OD-pair 13-18 along with the inference from Figure 3.3. The bottleneck indicated by this step most likely occurs at the origin 13 due to the capacity constraints on the immediate outgoing links (38, 39).

Interestingly, these N-curves in Figures 3.3 – 3.8 could inform about trip times between each OD-pair as well. These trip times are denoted by the horizontal offset distance between the cumulative inflows at origins (blue line) and cumulative outflows at destinations (red lines). Figures 3.9 and 3.10 presents these trip times including and excluding wait times at origins respectively. Note that, the trip times in Figures 3.9 and 3.10 are estimated by tracking the cumulative flows from origins to destinations. This process involves approximating travel times due to timestep discretisation as well as due to the reason that cumulative vehicle number at the origin may not match the cumulative vehicle number at destination exactly.

Figure 3.9 depicts the OD-specific trip times including the wait times at origins. As mentioned earlier, the time-varying demand is loaded into the network through the first 15 minutes of analysis. It is observed from this figure that the free-flow travel time is always experienced at the initial timestep. The trip times are found to be increasing during the subsequent timesteps. OD-pair 3-6 and 1-20 are observed to experience the minimum ($\approx$17 minutes) and maximum ($\approx$37 minutes) trip times respectively.

Figure 3.10 presents OD-specific trip times excluding wait times at origins providing additional information on congestion. It is observed that even though the demand is loaded at the origins within the first 15 minutes, vehicles keep entering into the network gradually for 27 minutes due to the capacity constraints of initial links. After entering into the network, trip times increases during the initial periods and decreases subsequently. The free-flow times are observed in both initial and final timesteps.

To speed up the computation time of the proposed model, another variation of the multi-OD base model of LTM-SODTA could be obtained with a destination-based approach. Here, the three sets of decision variables of the traffic flow model would be indexed for destinations instead of OD pairs. However a comparatively simplified model like this would pay the price of this simplification by compromising on OD-specific tracking of vehicles which might be essential in estimating endogenous travel times between each OD-pair as presented in Chapter 4. The destination-based approach is adopted in Chapter 5 to expedite the traffic flow model involving multiple vehicle classes.

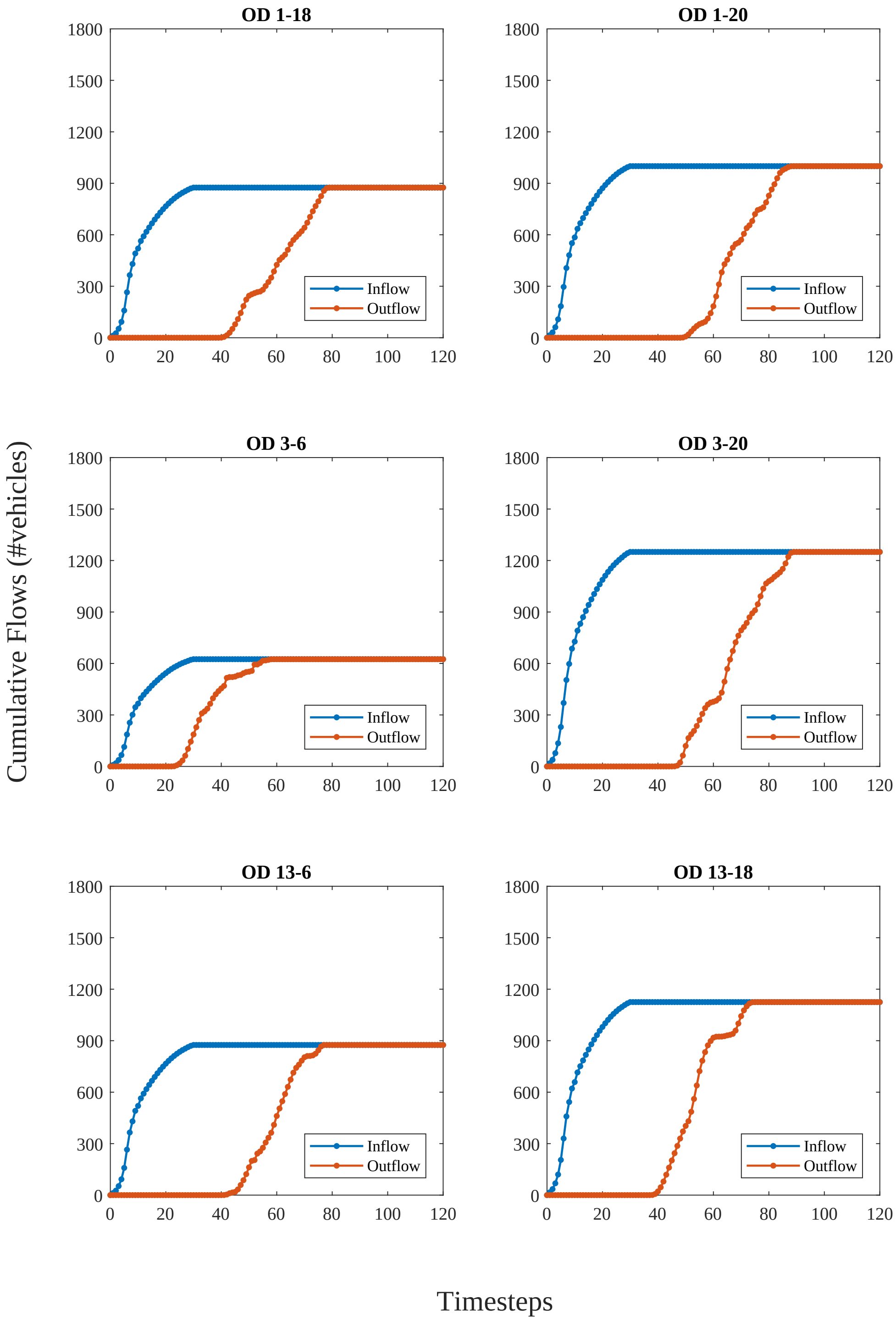

Figure 3.3: N-curves for low demand with waiting at source

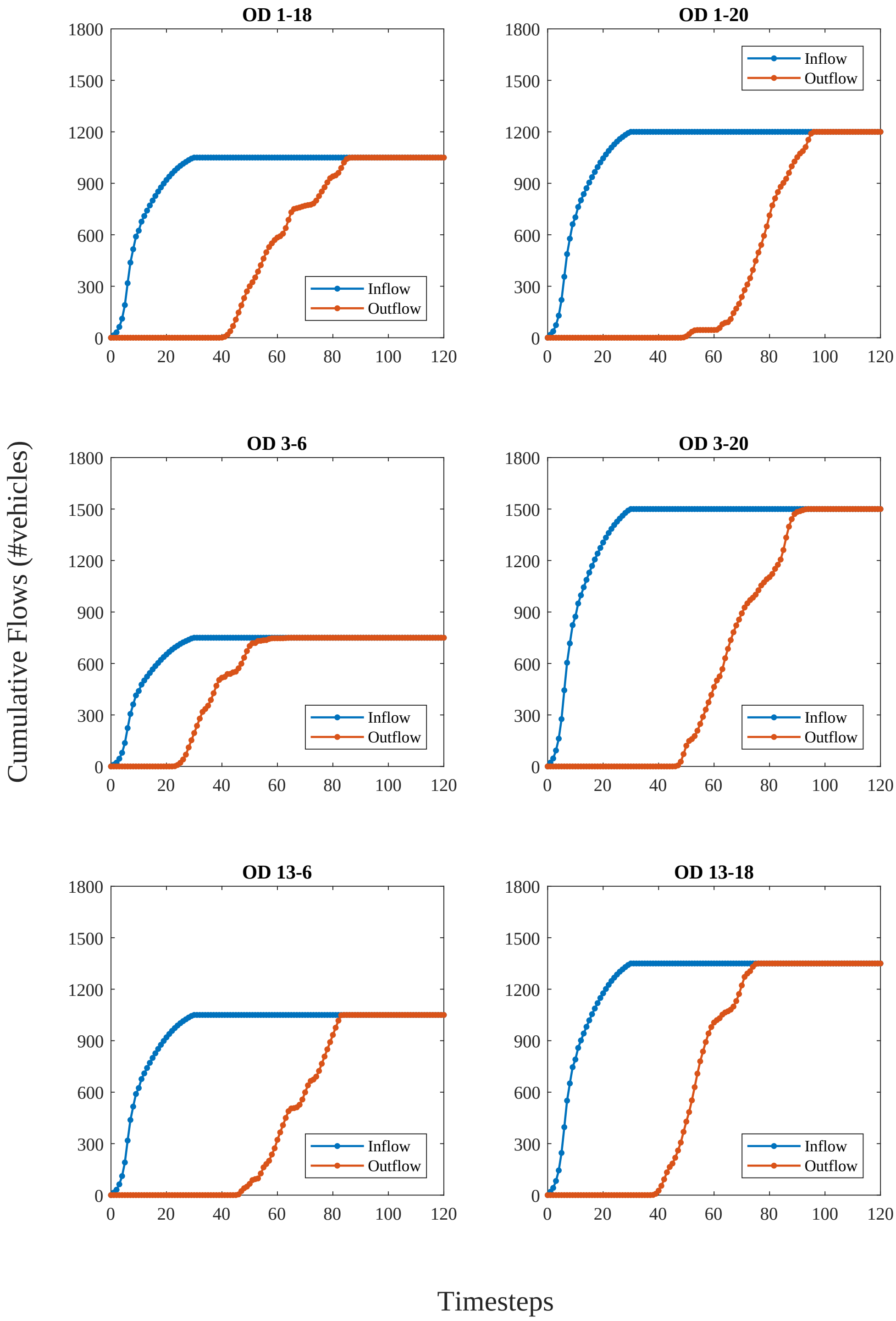

Figure 3.4: N-curves for medium demand with waiting at source

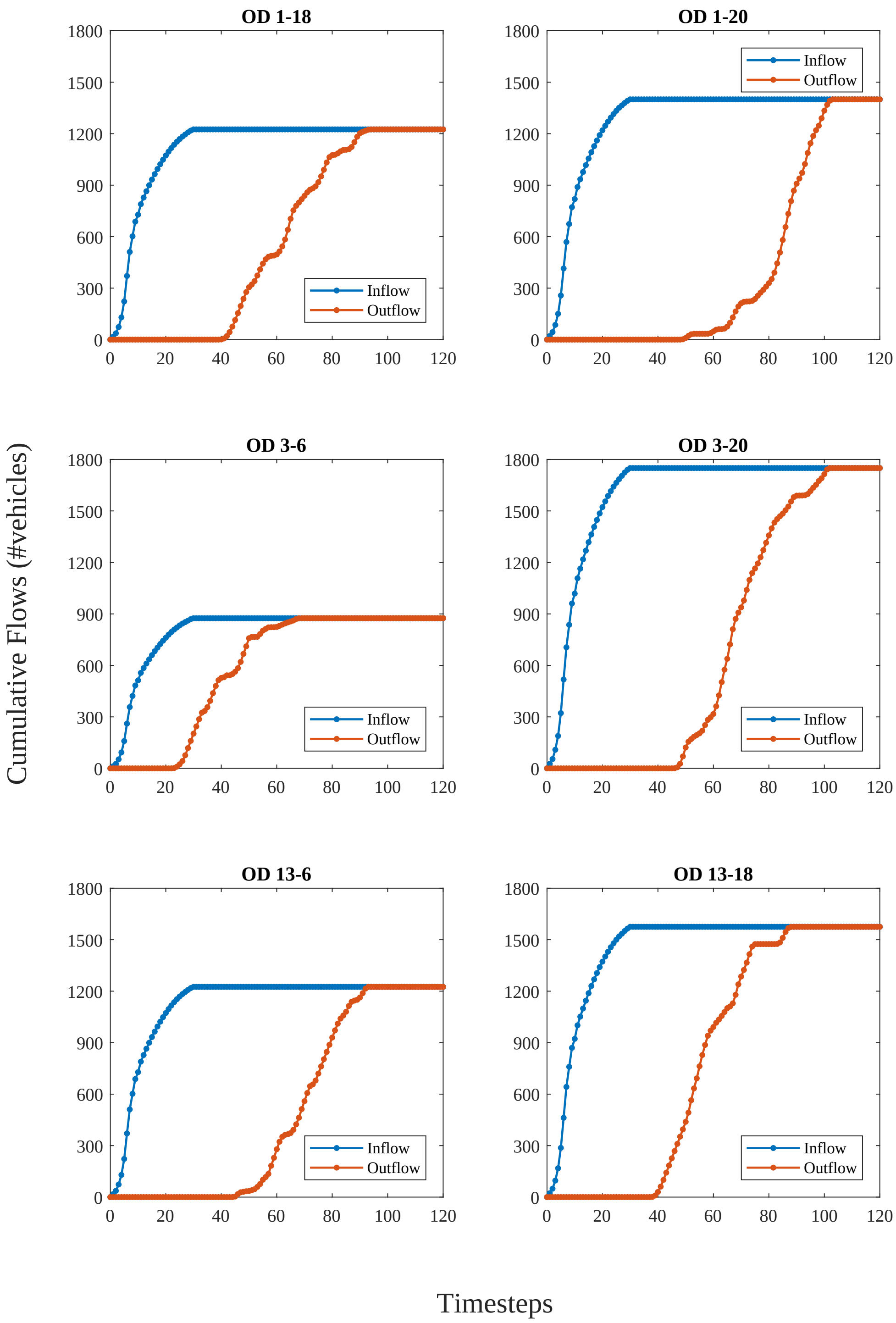

Figure 3.5: N-curves for high demand with waiting at source

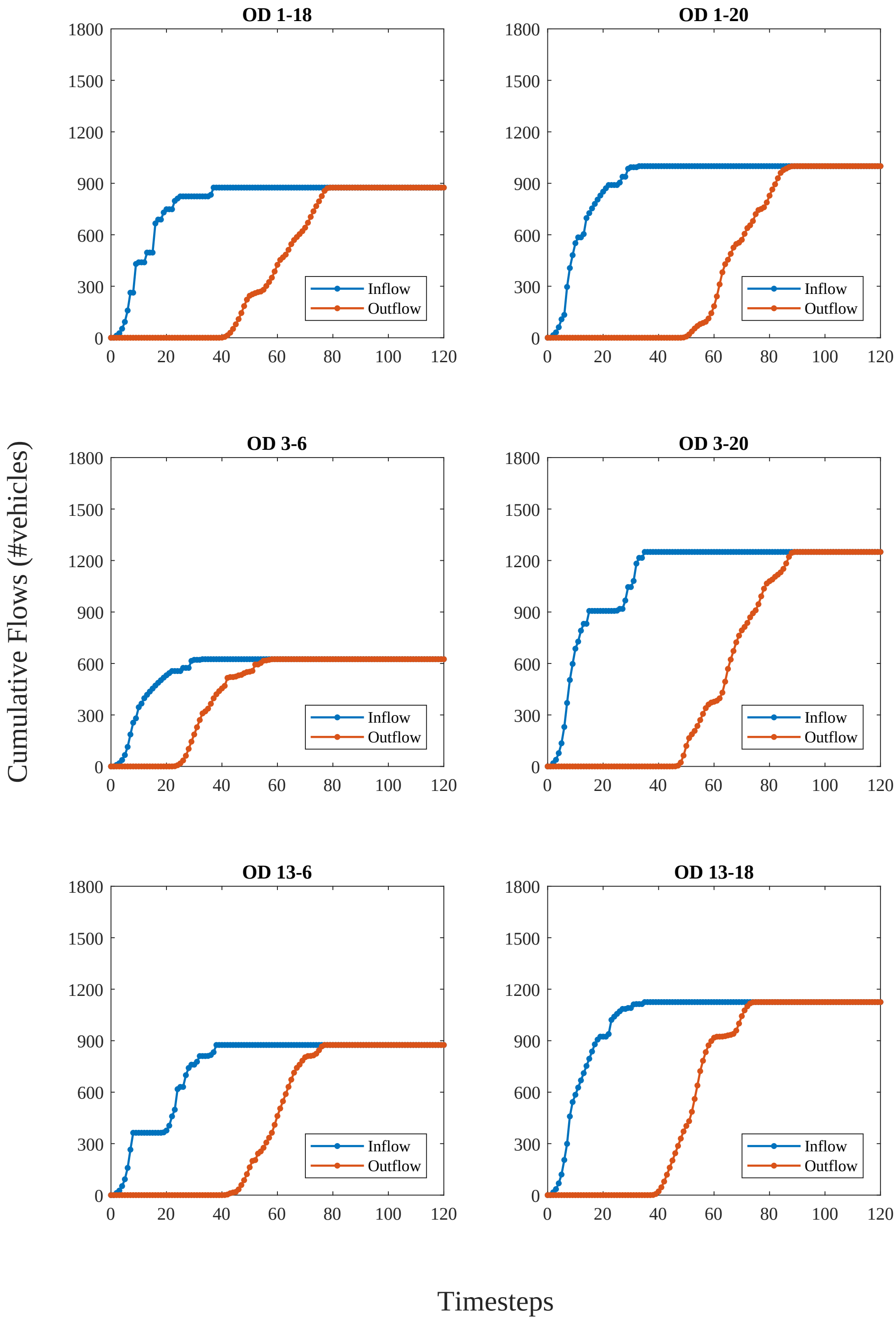

Figure 3.6: N-curves for low demand without waiting at source

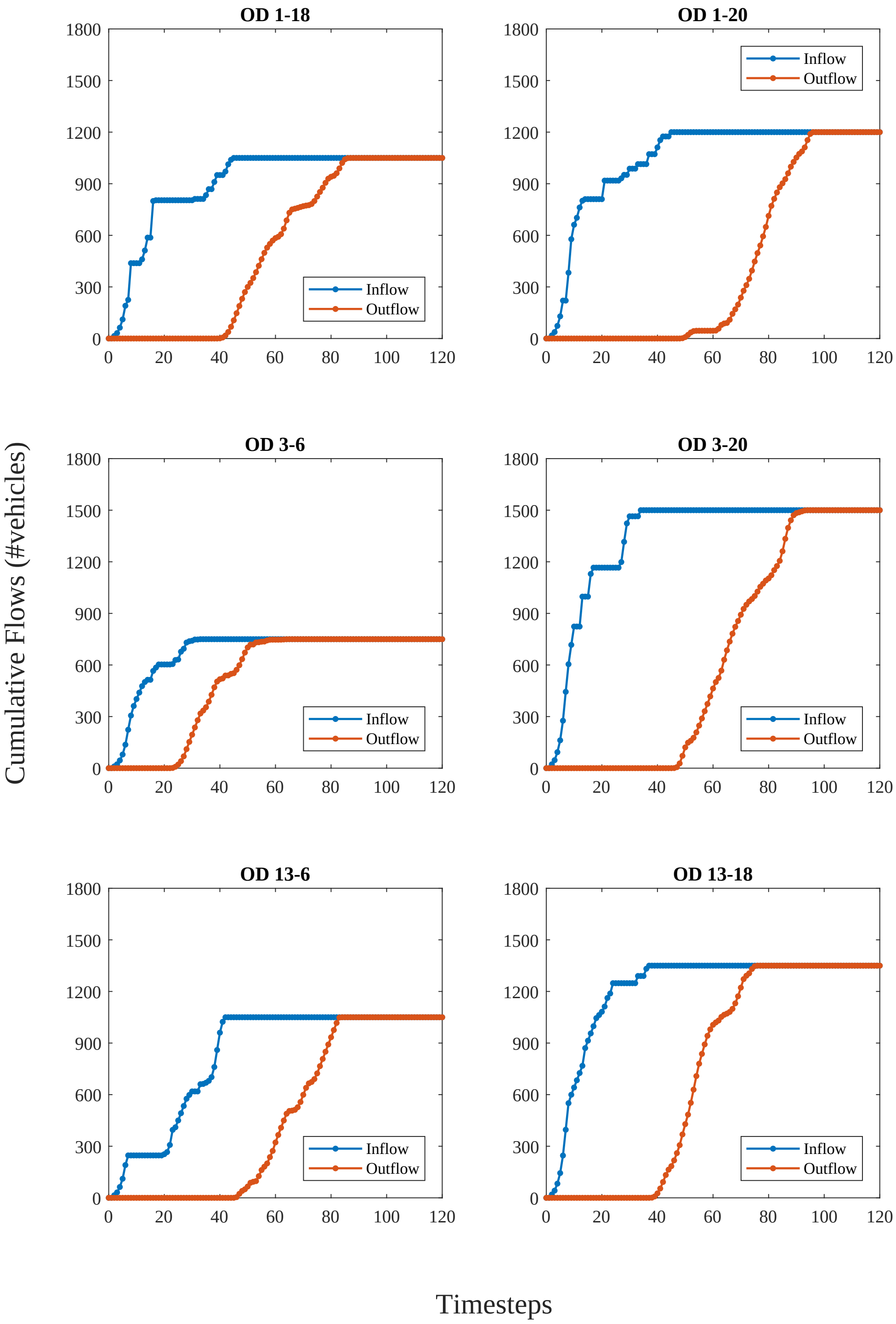

Figure 3.7: N-curves for low demand without waiting at source

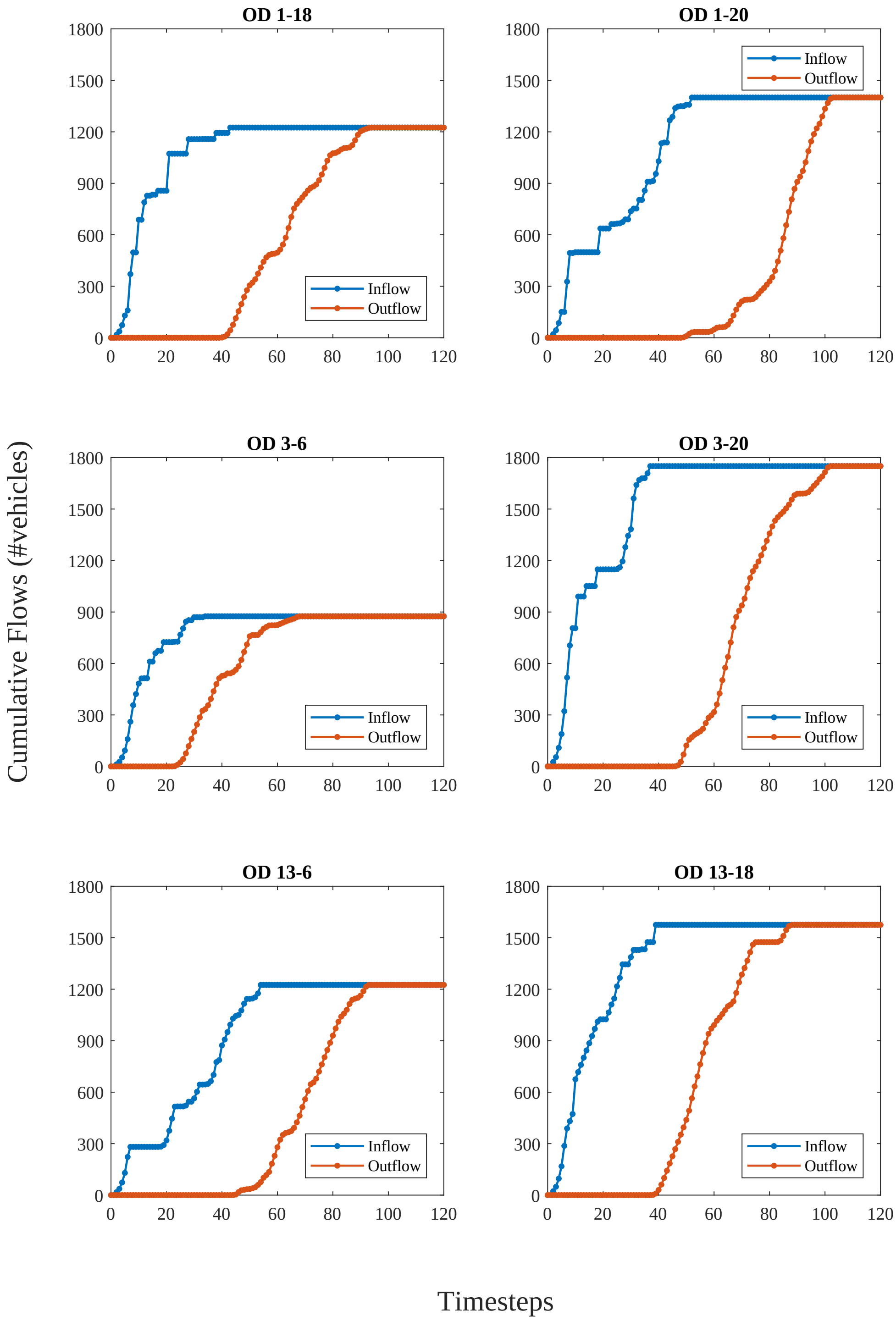

Figure 3.8: N-curves for high demand without waiting at source

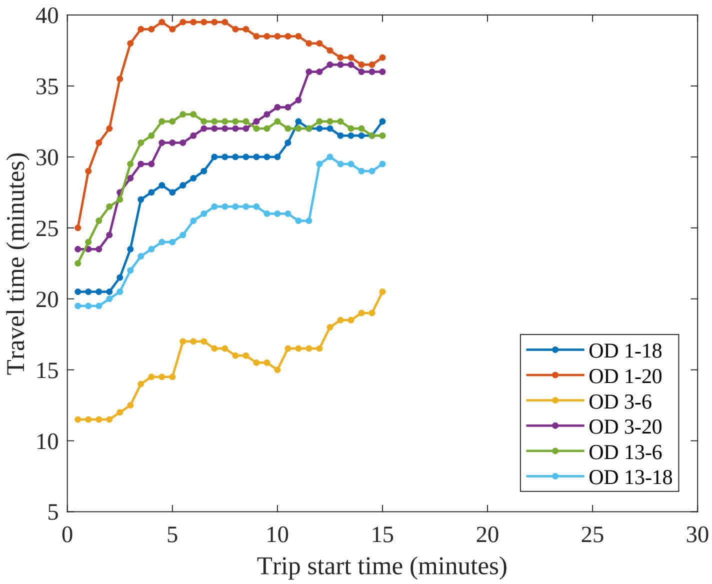

Figure 3.9: Travel times based on departure time (with wait time at origin)

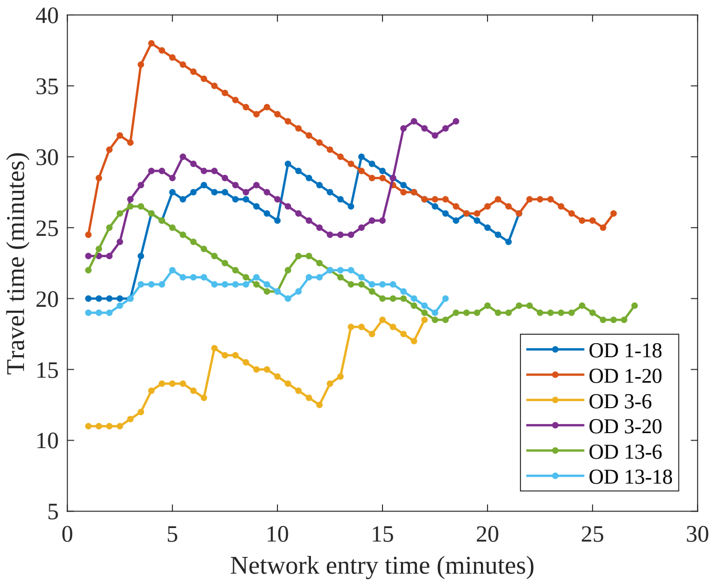

Figure 3.10: Travel times based on network entry time (without wait time at origin)

3.3 Conclusion

In this chapter, an LTM based LP formulation is proposed and implemented on a single-OD and a multi-OD network to solve a DTA model under SO traffic conditions. Compared to the original LTM, the proposed LTM-SODTA approach optimizes the turning ratios at each node of the network at each time step for an optimal system performance. The optimal solution flows from the two formulations are found to be consistent for describing free-flow as well as congested states of traffic accurately in terms of propagation of backward shock-waves, queuing of vehicles and optimal TSTT value. The objective function of the model also accounts for waiting time of the vehicles queued up at the entrance of a network. The proposed LP relaxes the strict minimum constraints on sending and receiving flows described in the original LTM theory using linear inequalities. The objective function handles this relaxation by penalizing vehicles' travel time in the network. During implementation of the LTM-SODTA on the example networks, the set of linear inequalities are found to be a suitable alternative to the strict minimum constraints, thus providing a valid LTM flow pattern.

The proposed LP can be applied for a range of network problems, including evaluating emergency evacuation strategies to a super-destination, and developing dynamic pricing policies. The implementation on the Sioux Falls network infers on the scalability of the proposed model. The proposed LTM-SODTA model in this chapter provides the foundation of this thesis. The subsequent chapters are built on this foundation to analyse various network design strategies such as capacity enhancement, departure-time incentive design, shared mobility service design and optimal lane allocation for autonomous vehicles in a mixed traffic network.

Chapter 4

System-level Mobility Analysis for Congestion Mitigation

In this chapter, the LTM-SODTA model proposed in the previous chapter is implemented for system-level mobility analysis for congestion mitigation with three strategies: network design with capacity enhancement and travel demand management with departure-time incentives and shared mobility service. The LTM-SODTA model is modified accordingly while implementing these three strategies with a limited budget. Network design with capacity enhancement is modelled by updating the sending and receiving flow constraints in the base model with a scope for improvement in capacity of selected links that will have the maximum effect on system-level performance. In this case, a comparison is made with the popular cell transmission model (CTM) while solving the network design problem (NDP). CTM is found to involve more number of variables compared to LTM due to its cell-based structure. From network design perspective, it is inferred that the cell representation of a network may not necessarily provide additional valuable degrees of freedom for NDP as improving only a segment of a link may not increase the throughput of vehicles in the network. This chapter also includes evaluating two travel demand management strategies where a novel method of endogenous estimation of travel time is designed and adopted for incentivising commuters to shift their departure-time and share their rides in two separate formulations.

4.1 Introduction

For analysing various major transportation infrastructure investments, transportation planners should be equipped with simple analytical tools to carry out a smooth and efficient decision making process. In the current era of innovative technologies and improved computational resources, development of these simple yet contemporary tools for dynamic transportation networks has gained attention from researchers all over the world. In solving network design problems (NDP), the methodologies around developing these tools has generally been focussed on improving efficiency of transportation networks with optimal usage of transport infrastructure.

There exists a number of ways to effectively improve efficiency of a transport network: efficient capacity improvements, peak-hour demand spreading with staggering work hours, shared mobility services are a few of them. In this chapter, we explore the versatility of the LTM-SODTA model, developed in Chapter 3, in evaluating various congestion mitigation strategies for overall system-level benefits. The proposed model is adapted for network design with capacity enhancement as well as evaluating two travel demand management strategies: departure-time incentives and shared mobility service.

This chapter is organized as follows. Section 4.2 presents the LTM-SODTA model for network design with capacity enhancement. Section 4.3 and 4.4 introduce the departure-time incentive and shared mobility service formulations for travel demand management with endogenous travel time estimation. Section 4.5 summarises the findings.

4.2 LTM-SODTA for network design

In this section, the proposed LTM-SODTA is adapted for providing a scope for capacity improvements of selected links leading to improvement in overall system-level performance by solving a network design problem (NDP). In any NDP, it is critical to find the locations where providing additional capacities would reap the maximum system-level benefits. To find these critical locations, the base model of LTM-SODTA for single-OD network (Equations (3.14)-(3.25)) is modified to accommodate a budget, allocated wisely among links for capacity improvements. For this purpose, the capacity and jam density constraints are improvised as follows.

To incorporate the scope for potential capacity improvement of f per unit of allocated budget b, Equation (3.19) is modified as follows:

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \leq \delta(q_i + f_i b_i) \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.1}$$

The Equations (3.20) and (3.21) are modified as follows to allow possible improvement in capacity (f) and jam density (e) per unit of allocated budget b.

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \leq \left(K_{jam} L_j + e_j b_j L_j - (z_j^+(t) - z_j^-(t_r))\right) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{4.2}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \leq \delta(q_j + f_j b_j) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{4.3}$$

Finally the budget constraint is presented in Equation (4.4) which specifies the upper limit of the sum of allocated budget as the total available budget B for network design.

$$\sum_{i \in A_C} b_i \leq B \tag{4.4}$$

$$b_i \geq 0 \qquad \forall i \in A_C \tag{4.5}$$

4.2.1 Model summary

The LTM-SODTA model for network design is presented by Equations (4.6) to (4.19) as follows.

$$\min TSTT = \min \delta \sum_{i \in A \setminus A_r} \sum_{t \in T} \left(z_i^+(t) - z_i^-(t)\right) \tag{4.6}$$

subject to,

$$z_i^+(t) = \sum_{t'<t} D(t') \qquad \forall i \in A_r, \forall t \in T \tag{4.7}$$

$$z_i^+(t) = \sum_{t'<t} \sum_{h \in \Gamma^-(i)} y_{h,i}(t') \qquad \forall i \in A \setminus A_r, \forall t \in T \tag{4.8}$$

$$z_i^-(t) = \sum_{t'<t} \sum_{j \in \Gamma^+(i)} y_{i,j}(t') \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.9}$$

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \leq \big(z_i^+(t_s) - z_i^-(t)\big) \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.10}$$

$$\sum_{j \in \Gamma^+(i)} y_{i,j}(t) \leq \delta(q_i + f_i b_i) \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.11}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \leq \big(K_{jam} L_j + e_j b_j L_j - (z_j^+(t) - z_j^-(t_r))\big) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{4.12}$$

$$\sum_{i \in \Gamma^-(j)} y_{i,j}(t) \leq \delta(q_j + f_j b_j) \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{4.13}$$

$$z_i^+(\bar{t}) = \sum_{t \in R} D(t) \qquad \forall i \in A_s \tag{4.14}$$

$$\sum_{i \in A \setminus A_s} b_i \leq B \tag{4.15}$$

$$y_{i,j}(t) \geq 0 \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.16}$$

$$z_i^+(t) \geq 0 \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.17}$$

$$z_i^-(t) \geq 0 \qquad \forall i \in A \setminus A_s, \forall t \in T \tag{4.18}$$

$$b_i \geq 0 \qquad \forall i \in A \setminus A_s \tag{4.19}$$

4.2.2 Numerical experiment and comparison with CTM-SODTA

In this section, the proposed LTM-SODTA for network design is implemented on the same single-OD network as presented in Chapter 3 (Figure 3.1). The increase in jam density (e_i) and capacity (f_i) per one unit of allocated budget (b_i) are link specific parameters with assumed value of 0.33 for links 2,3,4 and 9 and 0.25 for links 5,6,7 and 8. Total 9 units of budget is made available for the network design process.

Table 4.1: Optimal occupancy (in number of vehicles) with capacity enhancement

Links	**Time steps**														
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	40	30.4	20.8	0	0	0	0	0	0	0	0	0	0	0
2	0	0	19.6	39.2	58.8	39.2	19.6	0	0	0	0	0	0	0	0
3	0	0	10	20	21.2	11.2	1.5	0	0	0	0	0	0	0	0
4	0	0	0	0	0	9.6	19.2	28.8	19.2	9.6	0	0	0	0	0
5	0	0	0	0	0	10	20	30	30	20	10	0	0	0	0
6	0	0	0	0	0	0.4	0.4	0.4	0.4	0	0	0	0	0	0
7	0	0	0	0	0	9.6	19.2	20.8	20.8	11.2	1.5	0	0	0	0
8	0	0	0	0	0	0	0	0	9.6	19.6	29.2	29.2	19.6	9.6	0
9	0	0	0	0	0	0	0	0	0	19.6	39.2	50.8	31.2	11.5	0

Table 4.1 presents the optimal occupancy at each link of the network at each time step in terms of number of vehicles. The time-varying demand of 80 vehicles is loaded into the network. It is observed that vehicles queue up in link 2 reaching a value of 58.8 at timestep 4 and then clearing up completely by the end of the 6th timestep. This queue spillover phenomenon is observed in the other links in the network as well except link 6 which experiences an insignificant amount of flow throughout the analysis period. It should be noted that the demand at the first timestep (40) is more than the total capacity of the two downstream links (20). As a result, the capacity of link 2 is increased from 10 to 19.6 veh/timestep. This is detailed in Table 4.2 which gives the optimal budget allocation. At the optimum, 2.88 units of budget are allocated to link 2 and 1.38, 1.85 and 2.88 units are allocated to links 4, 7 and 9 respectively.

We compare the efficiency and the accuracy of this formulation with a CTM based SODTA formulation. We assume that the cells inside the same link have uniform traffic characteristics. In this scenario, we postulate that the budget will be allocated uniformly among the cells in a same link in the network. This claim is based on the properties of the transfer flow between two adjacent cells in the network. According to the CTM theory, the number of vehicles to be transferred (y_{ij}) from cell i to cell j should be less than or equal to the capacities of both the links Q_i and Q_j respectively at each timestep. Hence, if the budget is allocated non-uniformly among the cells representative of a link then there must exist at least one pair of adjacent cells with different capacities. For this pair of cells, the flow transferred is at most the minimum of these capacities, hence the throughput of their corresponding link will remain limited by the smallest cell capacity of this link. This implies that a non-uniform allocation of the budget among the cells representative of a link may result in sub-optimal network performance. We analyze this claim by implementing CTM based NDP on the same network as follows.

The cell representation of the example network is presented in Figure 4.1. The CTM network consists of 30 cells each with a length of 300 m which is equal to the distance travelled by a vehicle in one timestep at free-flow speed. The network characteristics remain unchanged. In order to obtain equivalent e_i and f_i values for the CTM-NDP formulation, these values are multiplied by the number of cells in each link. The LTM-SODTA results in a program with 595 variables, 899 constraints and 99 dual simplex iterations and 0.006s to converge to a TSTT value of 8919.23 veh-seconds. Whereas, the CTM-SODTA consists of significantly higher number of variables (897), constraints (2031) and takes 244 dual simplex iterations and 0.012s to converge to a same solution (TSTT = 8919.23 veh-seconds). Further, the sum of the optimal cell occupancies in a link are found to be exactly same as the optimal link occupancies obtained from the LTM-SODTA formulation, thus verifying the consistency of both the models. The allocated budget for each cell is presented in Table 4.2.

It is evident from Table 4.2 that the total budget allocated to a link based on

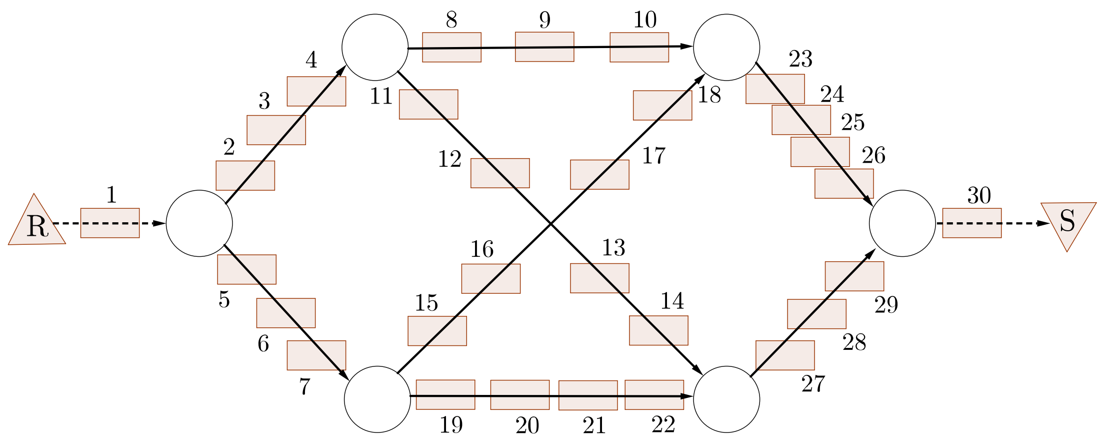

Figure 4.1: Example network for CTM-NDP

LTM-NDP formulation is the sum of the allocated budget in its consisting cells. From these results, it is inferred that the cell representation of a network may not necessarily provide additional valuable degrees of freedom for NDP as improving only a segment of a link may not increase the throughput of vehicles in the network.

4.2.3 Summary

In this section, an LTM based LP formulation is proposed and compared with its CTM counterpart on an example network to solve a dynamic NDP under SO traffic conditions. Compared to the single-destination SODTA based on the CTM, the LTM-NDP formulation requires considerably less decision variables, thus potentially providing a more scalable approach. Further, the model output of the CTM-NDP showed that there is no incentive to allocate non-uniform budget (leading to non-uniform capacity improvement) to the cells of the same link as the cell transfer flow is limited by the cell with the smallest capacity. This finding further advocates the use of LTM-NDP over CTM-NDP in terms of optimum budget allocation for a network design problem.

Table 4.2: Budget allocation in each link and corresponding cells

Links	Allocated budget (unit)	Cells	Allocated budget (unit)
1	0.00	1	0.00
2	2.88	2	0.96
		3	0.96
		4	0.96
3	0.00	5	0.00
		6	0.00
		7	0.00
4	1.38	8	0.46
		9	0.46
		10	0.46
5	0.00	11	0.00
		12	0.00
		13	0.00
		14	0.00
6	0.00	15	0.00
		16	0.00
		17	0.00
		18	0.00
7	1.85	19	0.46
		20	0.46
		21	0.46
		22	0.46
8	0.00	23	0.00
		24	0.00
		25	0.00
		26	0.00
9	2.88	27	0.96
		28	0.96
		29	0.96
10	0.00	30	0.00

4.3 Departure-time incentives based on endogenous travel time

In this section, the concept of departure-time incentives is explored for congestion mitigation. Departure-time incentives may ease up traffic congestion significantly. Commuters entering into a network at different times could be incentivised to change their departure times such that the overall system performance is improved. Here, the assumption is that a certain percentage of the entire demand is compliant to shift their departure times. However, one can analyse the effect of this compliance rate with different percentages of the demand willing to shift their departure times. This analysis is outside the scope of this thesis and will be explored in future research.

4.3.1 Model overview

The proposed model in this section is developed to incentivise commuters to shift their departure times for an overall improvement in system-level performance. It is evident that shifting departure times may cause inconvenience with regards to commuter's daily schedule. This inconvenience is modelled with an incentive scheme where an individual receives monetary benefits by the network operator for changing his/her departure-time for a trip. The amount of incentives is estimated based on the duration by which the departure-time is shifted as well as the increment in arrival time of a commuter due to the departure-time shift. Generally in LTM, travel times are estimated while post-processing the cumulative inflows and outflows of each link or an OD pair. In this study, these travel times are estimated endogenously in the model to infer upon the departure-time shift. This novel approach enriches the incentivisation process by penalising departure-time shift of commuters in a sensible manner.

4.3.2 Departure-time shift formulation

A variable $d(t)$ is introduced that represents the optimum demand split at each timestep keeping the total demand fixed. This optimum demand split leads to the minimum value of TSTT for a fixed total demand. The original time varying demand at each time step ($D(t)$) is incentivised to match this optimum demand split ($d(t)$) where a variable $u^{o,d}(t',t)$ represents the percentage of demand shift from departure time t to departure time t'. The solution to the proposed formulation includes the TSTT and percentage demand shift within timesteps with allocated incentives. Based on the available budget for incentives and desired level of improvement in TSTT, different proportion of demand shifts has been observed.

Here, the objective function remains the same as the base multi-OD model (Equation (4.20)), which is minimising the TSTT.

$$\min TSTT = \delta \min \sum_{(o,d)\in K} \sum_{i\in A\setminus A_s} \sum_{t\in T} \left(z_i^{o,d+}(t) - z_i^{o,d-}(t) \right) \tag{4.20}$$

However, additional variables and constraints are required to account for the incentive-based departure-time shift of the travel demand for an enhanced system performance. In this model, the new variable $u^{o,d}(t',t)$ accounts for the incentivised fraction of demand moving from one timestep to the other. In Constraint (4.21), the total fraction of demand shift from each timestep (t') to all the other timesteps (t) is equated to 1 for each OD-pair.

$$\sum_{t\in R} u^{o,d}(t',t) = 1 \qquad \forall (o,d) \in K, \forall t' \in R \tag{4.21}$$

Depending on the fractions of demand ($u^{o,d}(t',t)$) shifted from t' to t, the final demand which is the product of this fraction and the time-varying demand ($D^{o,d}(t')$), is loaded as the cumulative inflow of source centroid connectors ($z_o^{o,d+}(t)$).

$$z_o^{o,d+}(t) = \sum_{t'\in R} \sum_{t_1<t\in R} u^{o,d}(t',t_1) D^{o,d}(t') \quad \forall (o,d) \in K, \forall t \in T \tag{4.22}$$

Equation (4.22) ensures that the demand for $(o, d) \in K$ loaded at $t \in T$ accounts for the departure time shifts from t' to t. This assumption might be quite restrictive as a certain percentage of commuters could be reluctant to leave early or later even with incentives. To relax this assumption up to a certain extent, Equation (4.22) is modified to incentivise only a known portion of the demand, represented by θ in Equation (4.23).

$$z_o^{o,d+}(t) = \sum_{t'<t} D^{o,d}(t')(1-\theta) + \sum_{t' \in R} \sum_{t_1 < t \in R} u^{o,d}(t', t_1) D^{o,d}(t')\theta \quad \forall (o, d) \in K, \forall t \in T \tag{4.23}$$

The first term on the right hand side of Equation (4.23) represents the fixed portion of the entire demand which is not incentivised, whereas, the second term represents the incentivised travel demand. The constraints related to the underlying traffic flow model of LTM remain the same as the multi-OD version of the base model (Equations (3.28) to (3.34)).

4.3.3 Endogenous travel-time estimation

In LTM, traffic flow propagates through a network at discretised timesteps. Constraints on cumulative inflows and outflows updated at each of these timesteps dictate the traffic flow through the network. These cumulative flows on each link at each timestep could infer on link travel times depending on the number of timesteps vehicles spend on a link. With an OD-based representation of these flows, OD travel times can be estimated as well by matching a specific cumulative inflow number at an origin with the cumulative outflow number at a destination during post-processing of model outputs. In Chapter 3, a set of travel time plots (Figures 3.9 and 3.10) is obtained following the same method. Note that, these travel times are still an approximation due to timestep discretisation and the tolerance level considered while matching the cumulative flow values at origin and destination.

In this chapter, a novel methodology is adopted to estimate OD-based travel times and arrival times of vehicles endogenously. Subsequently, these arrival times are used as decision variables to penalise demand shift between timesteps. The integration of this travel time estimation component enriches the model in terms of estimating penalties for demand shift compared to a fixed penalty model.

LTM tracks vehicular flow in a network in terms of cumulative inflows and outflows at the entry and exit of each link, respectively. In this chapter, this characteristic of LTM is leveraged to estimate OD-based travel times as well as arrival times of vehicles endogenously at each departure timestep. Essentially, the number of timesteps required to match the cumulative inflow at an origin with the cumulative outflow at a destination for a departure timestep would represent the travel time in terms of timesteps between the corresponding O-D pair. To match these cumulative inflows and outflows for each departure timestep, a binary variable, $\psi^{o,d}(t, t_1)$ is introduced in the model. This variable takes the value 1 whenever it finds a cumulative inflow number at an origin equal or within a tolerance level (ϵ) with the cumulative outflow at its destination and 0 otherwise. In Equation (4.24), the binary property of $\psi^{o,d}(t, t_1)$ is implemented with a big-M method. The left hand side of the Equation (4.24) represents the absolute difference between cumulative inflows ($z_o^{o,d}(t)$) at origin o for an OD pair $(o, d) \in K$ at timestep t with cumulative outflows ($z_d^{o,d}(t_1)$) at destination d at all timesteps $t_1 > t$. In the right hand side, $\psi^{o,d}(t, t_1)$ finds the specific timestep (t_1) at which this absolute difference between cumulative flows at origin and destination is within a tolerance level (ϵ). The big-M ($M^{o,d}$) in Equation (4.24) makes this constraint redundant for all other timesteps.

$$\left| z_d^{o,d}(t_1) - z_o^{o,d}(t) \right| \leq \epsilon \psi^{o,d}(t, t_1) + (1 - \psi^{o,d}(t, t_1)) M^{o,d} \quad \forall (o, d) \in K, t \in R, t_1 \in T \tag{4.24}$$

As the absolute function on variables brings non-linearity in the model, Equation (4.24) is split into two Equations, (4.25) and (4.26) as follows with the same interpretation. These two equations are associated with the positive and negative components of the absolute difference of cumulative flows.

$$z_d^{o,d}(t_1) - z_o^{o,d}(t) \leq \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \quad \forall (o,d) \in K, t \in R, t_1 \in T \tag{4.25}$$

$$-(z_d^{o,d}(t_1) - z_o^{o,d}(t)) \leq \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \quad \forall (o,d) \in K, t \in R, t_1 \in T \tag{4.26}$$

For an OD pair ($(o,d) \in K$), at each timestep of demand loading ($t \in R$), equation (4.27) restricts the sum of $\psi^{o,d}(t,t_1)$ over all other timesteps (t_1) greater than (t) to 1. This will make sure that a cumulative inflow number, representing the entry of a vehicle at origin is matched with only one cumulative outflow number at destination, representing the exit of that vehicle.

$$\sum_{t_1 > t \in T} \psi^{o,d}(t,t_1) = 1 \qquad \forall (o,d) \in K, \forall t \in R \tag{4.27}$$

4.3.4 Budget estimation for departure-time incentives

In this section, the formulation of departure-time incentives is described. The incentives for departure-time shift are designed to depend on two factors. If a commuter's departure time is shifted from t' to t, these two factors are: the absolute difference in departure-time shift ($|t-t'|$) and the absolute difference in arrival times at t ($\tau^{o,d}(t)$) and t' ($\tau^{o,d}(t')$). The difference in arrival times is represented by $\Delta^{o,d}(t',t)$. The total budget, B, is designed to be the sum of these two absolute differences multiplied by the shifted travel demand ($D^{o,d}(t')\theta u^{o,d}(t',t)$) summed over all timesteps and OD-pairs.

The total budget, B, is indicative and expressed in terms of the product of number of commuters with departure time shift and shifted timesteps. To obtain the monetary representation, the total budget can be multiplied by a constant term depicting the value of time (VOT) of commuters. The incentive received by each commuter with shifted departure time could also be represented with monetary amount based on their VOT. However, as this constant term will not affect the output, it is not included in the proposed model. Equation (4.28) presents this budget constraint and Equation (4.29) expands it further.

$$\sum_{(o,d)\in K}\sum_{t'\in R}\sum_{t\in R:t\neq t'} D^{o,d}(t')\theta u^{o,d}(t',t)\big(|t-t'|+|\Delta^{o,d}(t',t)|\big)\leq B \tag{4.28}$$

$$\sum_{(o,d)\in K}\sum_{t'\in R}\sum_{t\neq t'\in R} D^{o,d}(t')\theta\big(u^{o,d}(t',t)|t-t'|+|u^{o,d}(t',t)\Delta^{o,d}(t',t)|\big)\leq B \tag{4.29}$$

Equations (4.30) define the arrival time difference ($\Delta^{o,d}(t',t)$) based on a commuter's departure time at t' or t. In Equation (4.31), these arrival times for the departure times t' and t for each OD pair are presented as functions of the binary variable $\psi^{o,d}(t,t_1)$ as explained in the previous section.

$$\Delta^{o,d}(t',t)=\tau^{o,d}(t)-\tau^{o,d}(t') \tag{4.30}$$

$$\Delta^{o,d}(t',t)=\sum_{t_1>t}\psi^{o,d}(t,t_1)t_1-\sum_{t_2>t'}\psi^{o,d}(t',t_2)t_2 \quad \forall(o,d)\in K,\forall t\in T \tag{4.31}$$

Substituting the expression for $\Delta^{o,d}(t',t)$ in Equation (4.29), Equation (4.32) is obtained.

$$\sum_{(o,d)\in K}\sum_{t'\in R}\sum_{t\neq t'\in R} D^{o,d}(t')\theta\Big(u^{o,d}(t',t)|t-t'| + \Big|\sum_{t_1>t}u^{o,d}(t',t)\psi^{o,d}(t,t_1)t_1-\sum_{t_2>t'}u^{o,d}(t',t)\psi^{o,d}(t',t_2)t_2\Big|\Big)\leq B \tag{4.32}$$

Equation (4.32) brings in non-linearity in the model in two ways. The first non-linearity is caused by the product of a continuous variable, $u^{o,d}(t',t)$ and a binary variable, $\psi^{o,d}(t,t_1)$ and the second one is due to the presence of an absolute function on arrival time variables. To linearise the product of the continuous and binary variables, another continuous variable, $\phi^{o,d}(t',t,t_1)$ is introduced as presented in Equation (4.33) and a standard linearisation method is adopted, presented by the set of Equations

(4.35) to (4.38). Equation (4.34) presents the bounds of the continuous variable $u^{o,d}(t',t)$, describing the fraction of demand shift between timesteps.

$$\phi^{o,d}(t',t,t_1) = u^{o,d}(t',t)\psi^{o,d}(t,t_1) \qquad \forall(o,d)\in K, \forall t,t' \in R : t \neq t', t_1 \in T \tag{4.33}$$

$$u^{o,d}(t',t) \in [\underline{u},\overline{u}] = [0,1] \qquad \forall(o,d)\in K, \forall t,t' \in R : t \neq t' \tag{4.34}$$

$$\phi^{o,d}(t',t,t_1) \leq \psi^{o,d}(t,t_1) \qquad \forall(o,d)\in K, \forall t,t' \in R : t \neq t', t_1 \in T \tag{4.35}$$

$$\phi^{o,d}(t',t,t_1) \geq \underline{u}\psi^{o,d}(t,t_1) \qquad \forall(o,d)\in K, \forall t,t' \in R : t \neq t', t_1 \in T \tag{4.36}$$

$$\phi^{o,d}(t',t,t_1) \leq u^{o,d}(t',t) - \underline{u}(1-\psi^{o,d}(t,t_1)) \quad \forall(o,d)\in K, \forall t,t' \in R : t \neq t', t_1 \in T \tag{4.37}$$

$$\phi^{o,d}(t',t,t_1) \geq u^{o,d}(t',t) - (1-\psi^{o,d}(t,t_1)) \quad \forall(o,d)\in K, \forall t,t' \in R : t \neq t', t_1 \in T \tag{4.38}$$

The product of $u^{o,d}(t',t)$ and $\psi^{o,d}(t,t_1)$ is substituted by $\phi^{o,d}(t',t,t_1)$ in Equation (4.32) as shown in Equation (4.39).

$$\begin{aligned}\sum_{(o,d)\in K}\sum_{t'\in R}\sum_{t\neq t'\in R} D^{o,d}(t')\Big(u^{o,d}(t',t)|t-t'| \\ + \Big|\sum_{t_1>t}\phi^{o,d}(t',t,t_1)t_1 - \sum_{t_2>t'}\phi^{o,d}(t',t,t_2)t_2\Big|\Big) \leq B\end{aligned} \tag{4.39}$$

In Equation (4.32), the non-linearity due to the presence absolute function still remains. To linearise this second non-linearity, another variable $\alpha^{o,d}(t',t)$ is introduced as presented in Equation (4.40). This variable is constrained by Equations (4.41) and (4.42), winding up the integer-linear formulation of the departure-time incentive model.

$$\sum_{(o,d)\in K}\sum_{t'\in R}\sum_{t\neq t'\in R} D^{o,d}(t')\big(u^{o,d}(t',t)|t-t'| + \alpha^{o,d}(t',t)\big) \leq B \tag{4.40}$$

$$\alpha^{o,d}(t',t) \geq \sum_{t_1>t\in T} \phi^{o,d}(t',t,t_1)t_1 - \sum_{t_2>t'\in T} \phi^{o,d}(t',t,t_2)t_2 \quad \forall (o,d)\in K, \forall t',t \in R \tag{4.41}$$

$$\alpha^{o,d}(t',t) \geq \sum_{t_2>t'\in T} \phi^{o,d}(t',t,t_2)t_2 - \sum_{t_1>t\in T} \phi^{o,d}(t',t,t_1)t_1 \quad \forall (o,d)\in K, \forall t',t \in R \tag{4.42}$$

4.3.5 Model summary

The entire departure-time incentive model is presented by Equations (4.43) to (4.69) as follows.

$$\min TSTT = \delta \min \sum_{(o,d)\in K} \sum_{i\in A\setminus A_s} \sum_{t\in T} \left(z_i^{o,d+}(t) - z_i^{o,d-}(t) \right) \tag{4.43}$$

subject to,

$$\sum_{t\in R} u^{o,d}(t',t) = 1 \qquad \forall (o,d)\in K, \forall t' \in R \tag{4.44}$$

$$z_o^{o,d+}(t) = \sum_{t'<t} D^{o,d}(t')(1-\theta) + \sum_{t'\in R} \sum_{t_1<t\in R} u^{o,d}(t',t_1) D^{o,d}(t')\theta \qquad \forall (o,d)\in K, \forall t\in T \tag{4.45}$$

$$z_i^{o,d+}(t) = \sum_{t'<t} \sum_{h\in\Gamma^-(i)} y_{h,i}^{o,d}(t') \qquad \forall i \in A\setminus A_r, \forall (o,d)\in K, \forall t\in T \tag{4.46}$$

$$z_i^{o,d-}(t) = \sum_{t'<t} \sum_{j\in\Gamma^+(i)} y_{i,j}^{o,d}(t') \qquad \forall i \in A\setminus A_s, \forall (o,d)\in K, \forall t\in T \tag{4.47}$$

$$\sum_{j\in\Gamma^+(i)} y_{i,j}^{o,d}(t) \leq \left(z_i^{o,d+}(t_s) - z_i^{o,d-}(t) \right) \qquad \forall i \in A\setminus A_s, \forall (o,d)\in K, \forall t\in T\setminus\{t_n\} \tag{4.48}$$

$$\sum_{(o,d)\in K} \sum_{j\in\Gamma^+(i)} y_{i,j}^{o,d}(t) \leq \delta q_i \qquad \forall i \in A\setminus A_s, \forall t\in T \tag{4.49}$$

$$\sum_{(o,d)\in K} \sum_{i\in\Gamma^-(j)} y_{i,j}^{o,d}(t) \leq K_{jam} L_j - \sum_{(o,d)\in K} \left(z_j^{o,d+}(t) - z_j^{o,d-}(t_r) \right) \qquad \forall j \in A\setminus\{A_r, A_s\}, \forall t\in T \tag{4.50}$$

$$\sum_{(o,d)\in K}\sum_{i\in\Gamma^-(j)} y_{i,j}^{o,d}(t) \leq \delta q_j \qquad \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \tag{4.51}$$

$$z_d^{o,d+}(\bar{t}) = \sum_{t\in R} D^{o,d}(t) \qquad \forall (o,d) \in K \tag{4.52}$$

$$z_d^{o,d}(t_1) - z_o^{o,d}(t) \leq \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \qquad \forall (o,d) \in K, t \in R, t_1 \in T \tag{4.53}$$

$$-(z_d^{o,d}(t_1) - z_o^{o,d}(t)) \leq \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \qquad \forall (o,d) \in K, t \in R, t_1 \in T \tag{4.54}$$

$$\sum_{t_1>t\in T} \psi^{o,d}(t,t_1) = 1 \qquad \forall (o,d) \in K, \forall t \in R \tag{4.55}$$

$$\sum_{(o,d)\in K}\sum_{t'\in R}\sum_{t\neq t'\in R} D^{o,d}(t')\big(u^{o,d}(t',t)|t-t'| + \alpha^{o,d}(t',t)\big) \leq B \tag{4.56}$$

$$\alpha^{o,d}(t',t) \geq \sum_{t_1>t\in T} \phi^{o,d}(t',t,t_1)t_1 - \sum_{t_2>t'\in T} \phi^{o,d}(t',t,t_2)t_2 \qquad \forall (o,d) \in K, \forall t', t \in R \tag{4.57}$$

$$\alpha^{o,d}(t',t) \geq \sum_{t_2>t'\in T} \phi^{o,d}(t',t,t_2)t_2 - \sum_{t_1>t\in T} \phi^{o,d}(t',t,t_1)t_1 \qquad \forall (o,d) \in K, \forall t', t \in R \tag{4.58}$$

$$\phi^{o,d}(t',t,t_1) \leq \psi^{o,d}(t,t_1) \qquad \forall (o,d) \in K, \forall t', t \in R(t' \neq t), t_1 \in T \tag{4.59}$$

$$\phi^{o,d}(t',t,t_1) \geq \underline{u}\psi^{o,d}(t,t_1) \qquad \forall (o,d) \in K, \forall t', t \in R(t' \neq t), t_1 \in T \tag{4.60}$$

$$\phi^{o,d}(t',t,t_1) \leq u^{o,d}(t',t) - \underline{u}(1-\psi^{o,d}(t,t_1)) \qquad \forall (o,d) \in K, \forall t', t \in R(t' \neq t), t_1 \in T \tag{4.61}$$

$$\phi^{o,d}(t',t,t_1) \geq u^{o,d}(t',t) - (1-\psi^{o,d}(t,t_1)) \qquad \forall (o,d) \in K, \forall t', t \in R(t' \neq t), t_1 \in T \tag{4.62}$$

$$y_{i,j}^{o,d}(t) \geq 0 \qquad \forall i \in A \setminus A_s, j \in \Gamma^+(i), \forall(o,d) \in K, \forall t \in T \tag{4.63}$$

$$z_i^{o,d+}(t) \geq 0 \qquad \forall i \in A, \forall(o,d) \in K, \forall t \in T \tag{4.64}$$

$$z_i^{o,d-}(t) \geq 0 \qquad \forall i \in A \setminus A_s, \forall(o,d) \in K, \forall t \in T \tag{4.65}$$

$$u^{o,d}(t',t) \in [0,1] \qquad \forall(o,d) \in K, \forall t', t \in R \tag{4.66}$$

$$\psi^{o,d}(t,t_1) \in \{0,1\} \qquad \forall(o,d) \in K, \forall t \in R, t_1 > t \in T \tag{4.67}$$

$$\phi^{o,d}(t',t,t_1) \geq 0 \qquad \forall(o,d) \in K, \forall t', t \in R, t_1 \in T \tag{4.68}$$

$$\alpha^{o,d}(t',t) \geq 0 \qquad \forall(o,d) \in K, \forall t', t \in R \tag{4.69}$$

The departure-time incentive model is implemented on a multi-OD Nguyen-Dupuis network in the next section.

4.3.6 Numerical experiments

In this section, we consider a multi-OD Nguyen-Dupuis network to implement the departure-time incentive model. In this study, it is assumed that only 50% of the entire demand agree to receive incentives for a possible departure time shift ($\theta = 0.5$). The network consists of 19 links as shown in Figure 4.2 and is loaded with a time-varying demand. The demand matrix with total demand and the link characteristics are presented in Tables 4.3 and 4.4 respectively.

Table 4.3: Demand matrix for Nguyen-Dupuis network

Origin\Destination	2	3
1	0	1200
4	800	0

The free-flow speed and backward wave speed in the network is considered as 54 and 36 km/hr respectively. The timestep, δ, is considered as 30 seconds satisfying the CFL condition (Courant et al., 1928) which states that this timestep should be lesser than the free-flow travel time of the shortest link in the network (180 seconds).

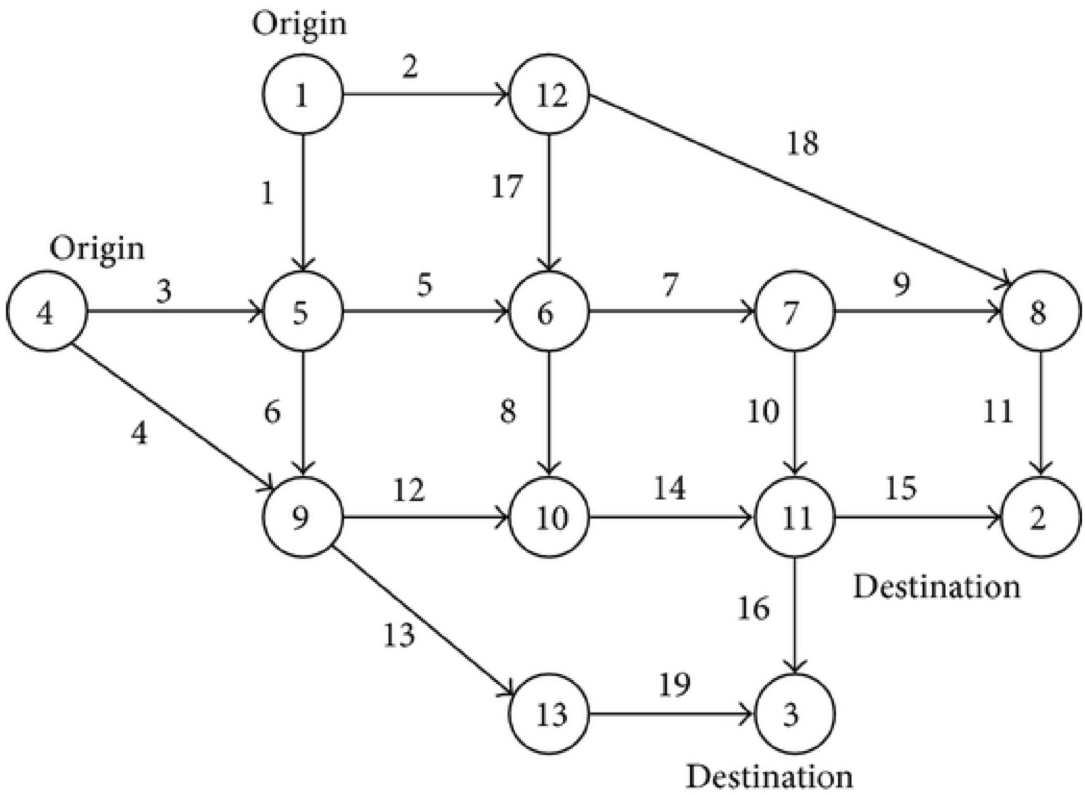

Figure 4.2: Nguyen-Dupuis network

Table 4.4: Link characteristics for Nguyen-Dupuis network

Link	**Capacity (veh/hr)**	**Length (km)**	**Link**	**Capacity (veh/hr)**	**Length (km)**
1	3600	8.1	**11**	2160	7.2
2	3600	6.3	**12**	2160	8.1
3	3600	8.1	**13**	3600	12.6
4	3600	6.3	**14**	3600	4.5
5	2160	8.1	**15**	2160	11.7
6	3600	8.1	**16**	2160	8.1
7	2160	10.8	**17**	3600	9
8	3600	8.1	**18**	3600	9.9
9	3600	2.7	**19**	2160	4.5
10	3600	5.4			

The time-varying demand is loaded into the network through first 10 timesteps for an analysis period of 120 timesteps equivalent to 1 hour.

The proposed model is implemented on an Intel(R) Core(TM) i7-6700 @3.40GHz CPU with 16GB RAM and CPLEX 12.10.0.0 as solver with an increasing budget (B) to understand the effect of incentives on the network performance. The computation time is found to be significantly depended upon the budget, ranging from less than a minute (for $B = 0$) to 57 minutes (for $B = 8000$). Figures 4.3 and 4.4 present the effect of incentivised departure-time shift on travel times for OD pairs 1-3 and 4-2 respectively. These incentives are estimated endogenously in the model. From these Figures 4.3 and 4.4, it is observed that the free-flow travel time is experienced only for the first departure-time. These free-flow travel times are 38 and 35 minutes for OD-pair 1-3 and 4-2 respectively. The travel times is found to increase in the subsequent departure times for both OD pairs reaching a maximum of 55 minutes. As the budget increases, the OD-based travel time decreases due to the departure-time shifts of commuters. Interestingly, for OD-pair 1-3, travel time corresponding to the last timestep of demand loading remains unaffected by incentives at around 55 minutes. Hence, even with incentives, the network remain congested for the last departure-time for OD-pair 1-3. In case of OD-pair 4-2, a similar trend is observed for a budget of 0, 2000 and 10000 where the travel time (55 minutes) corresponding to the last departure-time is found to be unaffected by incentives. However, overall, increment of budget is found to reduce significantly the travel times for OD-pair 4-2 as well. This reduction in travel times of both OD pairs translate to a steady reduction in TSTT (from 5336100 to 5128060 veh-secs) with increasing budget as represented in Figure 4.5.

Figures 4.6 and 4.7 show the change in demand profile due to the departure-time incentivisation with different budget for both the OD-pairs. The increment in budget is found to create a flattening effect on the demand loading curve. Due to more

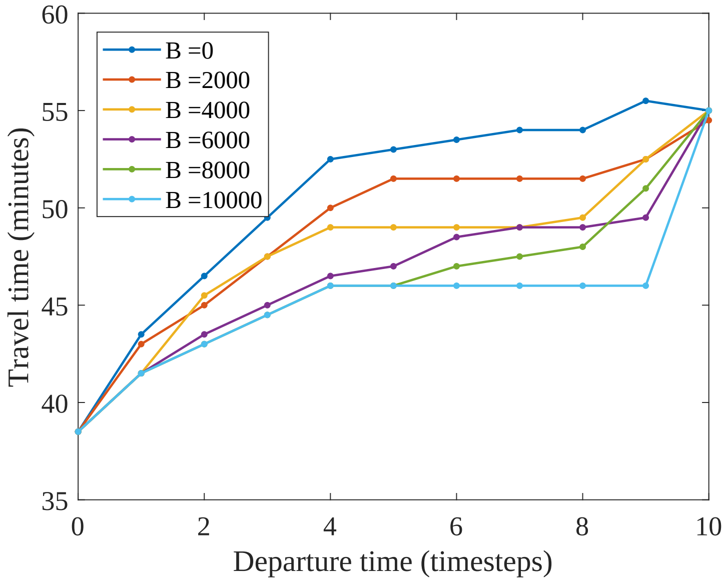

Figure 4.3: Change in travel times with incentives for OD 1-3

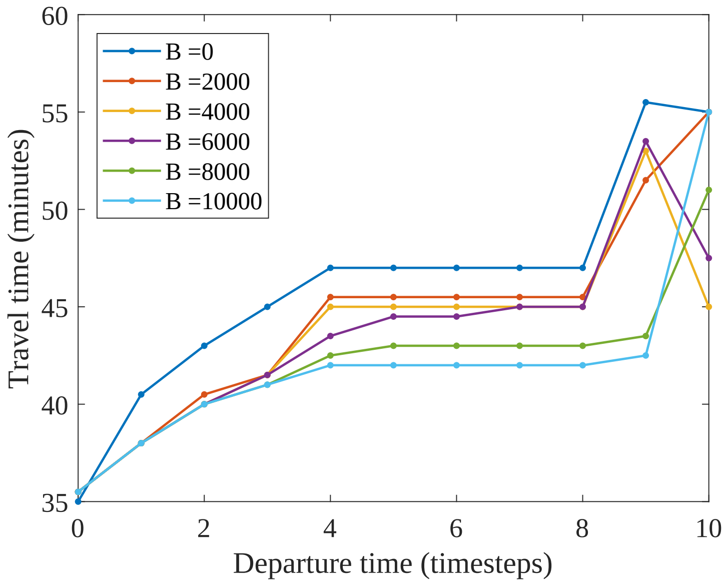

Figure 4.4: Change in travel times with incentives for OD 4-2

flexibility in changing the departure-times of commuters with increasing demand, the network found to be loaded gradually to reduce congestion.

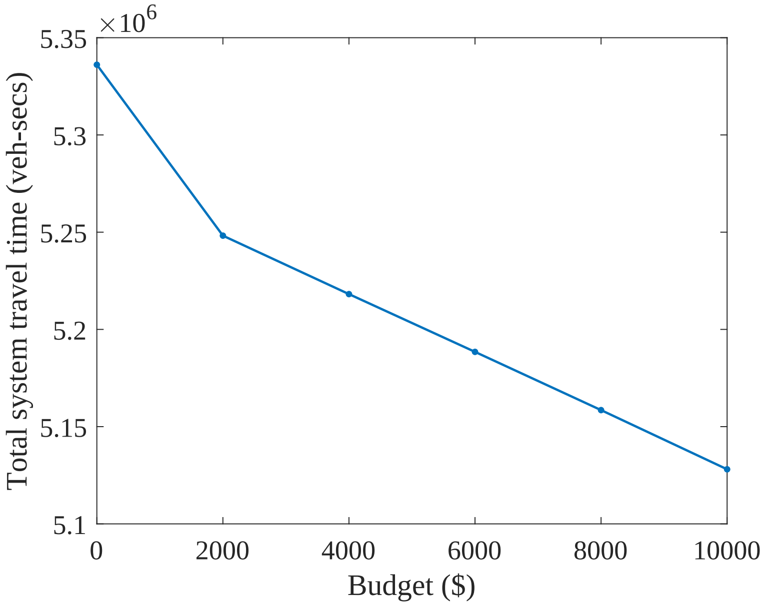

Figure 4.5: Change in TSTT with budget

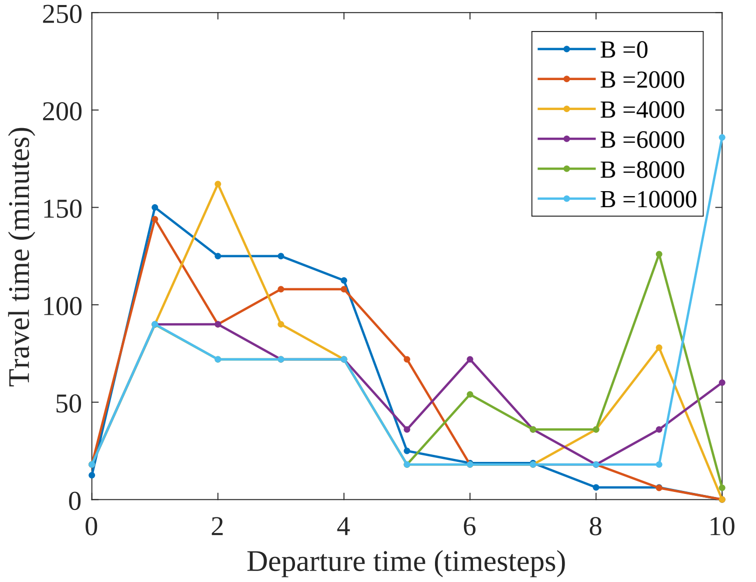

Figure 4.6: Demand profile with incentives for OD 1-3

Figures 4.8 and 4.9, present the output related to the departure-time shift variable $u^{o,d}(t', t)$ for different budget. Here, the rows and columns represent initial intended

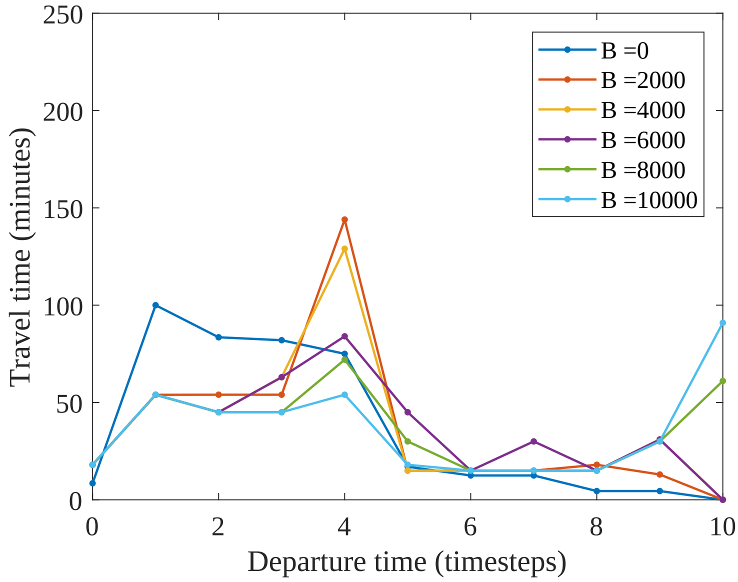

Figure 4.7: Change in demand profile with incentives for OD 4-2

departure-time and incentivised departure-time of commuters respectively. The values of $u^{o,d}(t', t)$ are presented in percentages. Note that, only 50% of the entire demand is made available for incentivisation. A significant spread of the demand is observed with increment in budget for both OD-pairs. Interestingly, as the budget increased, most of the demand is shifted to the last timestep. This may have happened to spread out the demand evenly including the portion which is not incentivised.

4.3.7 Summary

In this section, the LTM-SODTA model is implemented for departure-time incentives on a multi-OD test network. A novel method of endogenous estimation of OD travel time is developed and adopted for incentivising the departure-time shift of commuters for an overall improvement in system performance. Only 50% of the demand is made available to be incentivised for leaving early or late from their respective origins. The model output shows that incentives have significant effect in reducing OD travel

Incentivised departure-time (timesteps)

Initial departure-time (timesteps)

B=0

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	0	100	0	0	0	0	0	0	0	0	0
2	0	0	100	0	0	0	0	0	0	0	0
3	0	0	0	100	0	0	0	0	0	0	0
4	0	0	0	0	100	0	0	0	0	0	0
5	0	0	0	0	0	100	0	0	0	0	0
6	0	0	0	0	0	0	100	0	0	0	0
7	0	0	0	0	0	0	0	100	0	0	0
8	0	0	0	0	0	0	0	0	100	0	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=2000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	7.1	92	0	0	0	0	0	0	1	0	0
2	0	0	44	41	0	0	0	0	15	0	0
3	0	0	0	32	68	0	0	0	0	0	0
4	0	0	0	0	16	84	0	0	0	0	0
5	0	0	0	0	0	100	0	0	0	0	0
6	0	0	0	0	0	0	92	8	0	0	0
7	0	0	0	0	0	0	0	84	16	0	0
8	0	0	0	0	0	0	0	0	100	0	0
9	6.2	0	0	0	0	0	0	0	0.2	92	1.6
10	0	0	0	0	0	0	0	0	0	0	100

B=4000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	7.3	20	49	0	0	0	12	0	12	0	0
2	0	0	100	0	0	0	0	0	0	0	0
3	0	0	0	44	0	8.8	0	14	33	0	0.1
4	0	0	0	0	28	0	0	0	0	72	0
5	0	0	0	0	0	0	0	0	0	100	0
6	0	0	0	0	0	0	0	0	0	100	0
7	0	0	0	0	0	0	0	0	0	100	0
8	0	0	0	0	0	0	0	0	100	0	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=6000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	7.3	20	0	0	0	0	50	23	0	0	0
2	0	0	44	0	0	0	41	0	0	0	15
3	0	0	0	15	0	0	0	0	19	33	33
4	0	0	0	0	28	20	0	0	0	0	52
5	0	0	0	0	0	100	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	100	0
7	0	0	0	0	0	0	0	100	0	0	0
8	0	0	0	0	0	0	0	0	100	0	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=8000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	2	20	0	0	0	0	0	36	43	0	0
2	6.4	0	15	0	0	0	0	0	0	78	0
3	0	0	0	15	25	8.8	49	0	1.5	0	0
4	0	0	0	0	0	0	0	0	0	100	0
5	0	0	0	0	0	0	35	0	0	16	48
6	0	0	0	0	0	0	100	0	0	0	0
7	0	0	0	0	0	0	0	0	0	100	0
8	0	0	0	0	0	0	0	0	0	100	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=10000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	7.3	20	13	0	2.4	7.3	0	12	19	20	0
2	0	0	0	0	0	0	0	0	0	0	100
3	0	0	0	15	22	0	0	0	0	0	63
4	0	0	0	0	0	0	0	0	0	0	100
5	0	0	0	0	0	0	0	0	0	0	100
6	0	0	0	0	0	0	92	0	8	0	0
7	0	0	0	0	0	0	0	0	0	0	100
8	0	0	0	0	0	0	0	0	0	0	100
9	0	0	0	0	0	0	0	0	0	0	100
10	0	0	0	0	0	0	0	0	0	0	100

Figure 4.8: Change in demand profile with incentives for OD 4-2

times. The OD-demand profiles are found to have a flattening effect as the total budget for incentive increases. The departure-time shift variable, $u^{o,d}(t', t)$, is found to shift most of the demand to the last departure-time in case of maximum budget, indicating an even spread of incentivised and non-incentivised demand throughout the demand loading period. In the next section, the LTM-SODTA model is modified to design shared mobility services.

Incentivised departure-time (timesteps)

Initial departure-time (timesteps)

B=0

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	0	100	0	0	0	0	0	0	0	0	0
2	0	0	100	0	0	0	0	0	0	0	0
3	0	0	0	100	0	0	0	0	0	0	0
4	0	0	0	0	100	0	0	0	0	0	0
5	0	0	0	0	0	100	0	0	0	0	0
6	0	0	0	0	0	0	100	0	0	0	0
7	0	0	0	0	0	0	0	100	0	0	0
8	0	0	0	0	0	0	0	0	100	0	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=2000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	19	7.9	0	0	55	0	14	5	0	0	0.1
2	0	0	29	0	33	0	0	0	38	0	0
3	0	0	0	32	68	0	0	0	0	0	0
4	0	0	0	0	100	0	0	0	0	0	0
5	0	0	0	0	0	76	24	0	0	0	0
6	0	0	0	0	0	0	0	0	0	100	0
7	0	0	0	0	0	0	0	100	0	0	0
8	0	0	0	0	0	0	0	0	0	100	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=4000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	19	7.9	0	0	0	0	13	35	26	0	0
2	0	0	6.7	0	0	87	5.9	0	0	0	0
3	0	0	0	54	46	0	0	0	0	0	0
4	0	0	1.2	0	73	0	0	0	0	25	0.1
5	0	0	0	0	0	0	0	0	0	100	0
6	0	0	0	0	0	0	0	0	0	100	0
7	0	0	0	0	0	0	0	100	0	0	0
8	0	0	0	0	0	0	0	0	0	100	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=6000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	19	7.4	0	0	30	0	5	18	21	0	0.1
2	0	0	7.8	0	92	0	0	0	0	0	0
3	0	0	0	54	46	0	0	0	0	0	0
4	0	0	0	0	51	0	0	0	0	49	0
5	0	3.1	0	0	0	76	0	0	0	20	0
6	0	0	0	0	0	0	100	0	0	0	0
7	0	0	0	0	0	0	0	0	0	100	0
8	0	0	0	0	0	0	0	0	100	0	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=8000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	19	7.9	0	0	0	0	1.1	0	21	51	0
2	0	0	7.8	0	0	0	0	6	0	0	86
3	0	0	0	9.8	0	29	0	0	0	0	61
4	0	0	0	0	92	2.8	5.2	0	0	0	0
5	0	0	0	0	0	100	0	0	0	0	0
6	0	0	0	0	0	0	100	0	0	0	0
7	0	0	0	0	0	0	0	100	0	0	0
8	0	0	0	0	0	0	0	0	100	0	0
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

B=10000

	0	1	2	3	4	5	6	7	8	9	10
0	100	0	0	0	0	0	0	0	0	0	0
1	19	8.1	0	0	0	0	0	0	0	0	73
2	0	0	7.8	0	0	2.4	21	21	31	17	0
3	0	0	0	9.8	0	0	0	0	0	45	46
4	0	0	0	0	44	0	0	0	0	0	56
5	0	0	0	0	0	100	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	100
7	0	0	0	0	0	0	0	0	0	0	100
8	0	0	0	0	0	0	0	0	0	0	100
9	0	0	0	0	0	0	0	0	0	100	0
10	0	0	0	0	0	0	0	0	0	0	100

Figure 4.9: Change in demand profile with incentives for OD 4-2

4.4 Shared mobility services under endogenous vehicle occupancy

In this section, the usefulness of the LTM-SODTA model is further explored with shared mobility services for congestion mitigation. Shared mobility generally refers to the shared use of transport modes on short-terms on an on-demand basis. However, it is an umbrella term for various forms of car-sharing, bike-sharing, ride-sharing (carpooling and van-pooling), and on-demand ride services. It can also include alternative transit services, such as para-transit, shuttles, which can supplement fixed route bus and rail services. In this study, the definition of shared mobility service is limited to an incentive-based ride-sharing system where a network operator designs a fleet of vehicles for servicing a network.

4.4.1 Model overview

The proposed model in this section is developed for a shared mobility service operator to design a fleet of vehicles for servicing a network. The fleet is assumed to consist of passenger cars only and designed based on commuters' choices whether or not to share their rides with other fellow passengers sharing same origins, destinations and departure-times. Based on the level of sharing, vehicle occupancies are estimated endogenously while minimising the TSTT of the network. It is evident that a model with a system level travel time minimisation objective would maximise the vehicle occupancies to reduce the vehicular demand in the network. However, a model like this would completely disregard inconvenience caused to the commuters while sharing their rides during their trips. To strike a trade-off between this inconvenience and network-wise system performance, a set of budget constraints is incorporated in the proposed model which require the service operator to incentivise the commuters to share their rides. These incentives depend on the gain in arrival times due to sharing rides as well as the value of time (V) of commuters. However, the model assumes that the entire travel demand is willing to share their rides. This assumption can be relaxed by providing bounds on the vehicle occupancy variable or by allowing a fixed portion of demand to share their rides. The novelty of the proposed model lies in the estimation of endogenous vehicle occupancies for an overall improvement in system performance while respecting a limited budget constraint that incentivises ride-sharing in the network with an arrival time based incentive scheme. The proposed model would be exceptionally useful in understanding the effect of shared mobility services on traffic conditions in a network while accounting for commuters' willingness to share their rides.

4.4.2 Endogenous vehicle occupancy estimation

In this model, the objective function remains the same as the multi-OD LTM-SODTA as presented in Equation (4.70).

$$\min TSTT = \delta \min \sum_{(o,d)\in K} \sum_{i\in A\setminus A_s} \sum_{t\in T} \left(z_i^{o,d+}(t) - z_i^{o,d-}(t) \right) \tag{4.70}$$

A variable, $r^{o,d}(t')$, is introduced in the model to endogenously capture vehicle occupancies. To determine the vehicular demand entering into the network, the OD-based passenger demand $D^{o,d}(t')$ is divided by $r^{o,d}(t')$ and loaded as cumulative inflow ($z_o^{o,d+}(t)$) through the source centroid connectors as presented in Equation (4.71).

$$z_o^{o,d+}(t) = \sum_{t'<t} \frac{D^{o,d}(t')}{r^{o,d}(t')} \qquad \forall (o,d) \in K, \forall t \in T \tag{4.71}$$

However, the structure of Equation (4.71) creates non-linearity in the model due to the cross-multiplication product of two variables $z_o^{o,d+}(t)$ and $r^{o,d}(t')$. To handle this non-linearity, $r^{o,d}(t')$ is replaced with $\rho^{o,d}(t')$ which represents the inverse of $r^{o,d}(t')$

as shown in Equation (4.72).

$$z_o^{o,d+}(t) = \sum_{t'<t} D^{o,d}(t')\rho^{o,d}(t') \qquad \forall (o,d) \in K, \forall t \in T \tag{4.72}$$

4.4.3 Budget estimation for shared mobility services

It is evident that sharing rides with other passengers in single vehicle may create inconvenience during a trip. To account for this inconvenience, an incentive scheme is designed in the model where commuters are incentivised to share their rides based on their gain in arrival time and their value of time (V). Equation (4.73) presents the budget constraint based on arrival times ($\tau^{o,d}(t)$) of commuters sharing their rides ($D^{o,d}(t)(1-\rho^{o,d}(t))$) and V. Here, B represents the total budget available for implementing the incentivisation scheme. Equation (4.74) and (4.75) expand the budget constraint further to explore the presence of any non-linearity in the model.

$$\sum_{(o,d)\in K} \sum_{t\in T} D^{o,d}(t)(1-\rho^{o,d}(t))\tau^{o,d}(t)V \leq B \tag{4.73}$$

$$\sum_{(o,d)\in K} \sum_{t\in T} D^{o,d}(t)(1-\rho^{o,d}(t))\tau^{o,d}(t) \leq \frac{B}{V} \tag{4.74}$$

$$\sum_{(o,d)\in K} \sum_{t\in T} D^{o,d}(t)\tau^{o,d}(t) - \sum_{(o,d)\in K} \sum_{t\in T} D^{o,d}(t)\rho^{o,d}(t)\tau^{o,d}(t) \leq \frac{B}{V} \tag{4.75}$$

The arrival times of commuters starting their trips at time t ($\tau^{o,d}(t)$) are estimated based on the endogenous travel time estimation method developed in Section 4.3. Equations (4.76) to (4.79) present this estimation process of arrival times.

$$\tau^{o,d}(t) = \sum_{t_1>t} \psi^{o,d}(t,t_1)t_1 \qquad \forall (o,d) \in K, \forall t \in T \tag{4.76}$$

$$\sum_{t_1\in T} \psi^{o,d}(t,t_1) = 1 \qquad \forall (o,d) \in K, \forall t \in R \tag{4.77}$$

$$z_d^{o,d}(t_1) - z_o^{o,d}(t) \leq \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \quad \forall (o,d) \in K, t \in R, t_1 \in T \tag{4.78}$$

$$-(z_d^{o,d}(t_1) - z_o^{o,d}(t)) \leq \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \quad \forall (o,d) \in K, t \in R, t_1 \in T \tag{4.79}$$

Substituting the expression for arrival times ($\tau^{o,d}(t)$) in terms of the binary travel time estimation variable ($\psi^{o,d}(t,t_1)$) in Equation (4.75), Equation (4.80) is obtained.

$$\sum_{(o,d)\in K}\sum_{t\in T} D^{o,d}(t)\Big(\sum_{t_1>t}\psi^{o,d}(t,t_1)t_1\Big) - \sum_{(o,d)\in K}\sum_{t\in T} D^{o,d}(t)\Big(\sum_{t_1>t}\rho^{o,d}(t)\psi^{o,d}(t,t_1)t_1\Big) \leq \frac{B}{V} \quad (4.80)$$

Here, a non-linearity is encountered due to the product of vehicle occupancy ($\rho^{o,d}(t)$), a continuous variable, and binary travel time estimation variable, $\psi^{o,d}(t,t_1)$. To linearise this product, a standard linearisation technique is adopted, similar to the one presented in Section 4.3. A continuous variable, $\gamma^{o,d}(t,t_1)$, is introduced to replace the product of these continuous and binary variables (Equation (4.81)) along with a set of constraints (Equations (4.83) to (4.86)) to perform the linearisation. Equation (4.82) presents the upper and lower bounds of the vehicle occupancy variable ($\rho^{o,d}(t)$).

$$\gamma^{o,d}(t,t_1) = \rho^{o,d}(t)\psi^{o,d}(t,t_1) \qquad \forall(o,d)\in K, \forall t,t_1\in T \quad (4.81)$$

$$\rho^{o,d}(t) \in [\underline{\rho},\overline{\rho}] \qquad \forall(o,d)\in K, \forall t\in T \quad (4.82)$$

$$\gamma^{o,d}(t,t_1) \leq \psi^{o,d}(t,t_1) \qquad \forall(o,d)\in K, \forall t,t_1\in T \quad (4.83)$$

$$\gamma^{o,d}(t,t_1) \geq \underline{\rho}\psi^{o,d}(t,t_1) \qquad \forall(o,d)\in K, \forall t,t_1\in T \quad (4.84)$$

$$\gamma^{o,d}(t,t_1) \leq \rho^{o,d}(t) - \underline{\rho}(1-\psi^{o,d}(t,t_1)) \qquad \forall(o,d)\in K, \forall t,t_1\in T \quad (4.85)$$

$$\gamma^{o,d}(t,t_1) \geq \rho^{o,d}(t) - (1-\psi^{o,d}(t,t_1)) \qquad \forall(o,d)\in K, \forall t,t_1\in T \quad (4.86)$$

The final linear budget constraint is presented in Equation (4.87) as follows.

$$\sum_{(o,d)\in K}\sum_{t\in T} D^{o,d}(t)\Big(\sum_{t_1>t}\psi^{o,d}(t,t_1)t_1\Big) - \sum_{(o,d)\in K}\sum_{t\in T} D^{o,d}(t)\Big(\sum_{t_1>t}\phi^{o,d}(t,t_1)t_1\Big) \leq \frac{B}{V} \quad (4.87)$$

4.4.4 Model summary

The entire model for incentive-based shared mobility service is presented by Equations (4.88) to (4.110) as follows.

$$\min TSTT = \delta \min \sum_{(o,d)\in K} \sum_{i\in A\setminus A_s} \sum_{t\in T} \left(z_i^{o,d+}(t) - z_i^{o,d-}(t) \right) \tag{4.88}$$

subject to,

$$z_o^{o,d+}(t) = \sum_{t'<t} D^{o,d}(t')\rho^{o,d}(t') \qquad \forall (o,d)\in K, \forall t\in T \tag{4.89}$$

$$z_i^{o,d+}(t) = \sum_{t'<t} \sum_{h\in\Gamma^-(i)} y_{h,i}^{o,d}(t') \qquad \forall i\in A\setminus A_r, \forall (o,d)\in K, \forall t\in T \tag{4.90}$$

$$z_i^{o,d-}(t) = \sum_{t'<t} \sum_{j\in\Gamma^+(i)} y_{i,j}^{o,d}(t') \qquad \forall i\in A\setminus A_s, \forall (o,d)\in K, \forall t\in T \tag{4.91}$$

$$\sum_{j\in\Gamma^+(i)} y_{i,j}^{o,d}(t) \le \left(z_i^{o,d+}(t_s) - z_i^{o,d-}(t) \right) \qquad \forall i\in A\setminus A_s, \forall (o,d)\in K, \forall t\in T\setminus\{t_n\} \tag{4.92}$$

$$\sum_{(o,d)\in K} \sum_{j\in\Gamma^+(i)} y_{i,j}^{o,d}(t) \le \delta q_i \qquad \forall i\in A\setminus A_s, \forall t\in T \tag{4.93}$$

$$\sum_{(o,d)\in K} \sum_{i\in\Gamma^-(j)} y_{i,j}^{o,d}(t) \le K_{jam}L_j - \sum_{(o,d)\in K} \left(z_j^{o,d+}(t) - z_j^{o,d-}(t_r) \right) \qquad \forall j\in A\setminus\{A_r, A_s\}, \forall t\in T \tag{4.94}$$

$$\sum_{(o,d)\in K} \sum_{i\in\Gamma^-(j)} y_{i,j}^{o,d}(t) \le \delta q_j \qquad \forall j\in A\setminus\{A_r, A_s\}, \forall t\in T \tag{4.95}$$

$$z_d^{o,d+}(\bar{t}) = \sum_{t\in R} D^{o,d}(t) \qquad \forall (o,d)\in K \tag{4.96}$$

$$z_d^{o,d}(t_1) - z_o^{o,d}(t) \le \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \qquad \forall (o,d)\in K, t\in R, t_1\in T \tag{4.97}$$

$$-(z_d^{o,d}(t_1) - z_o^{o,d}(t)) \le \epsilon\psi^{o,d}(t,t_1) + (1-\psi^{o,d}(t,t_1))M^{o,d} \qquad \forall (o,d)\in K, t\in R, t_1\in T \tag{4.98}$$

$$\sum_{t_1>t\in T} \psi^{o,d}(t,t_1) = 1 \qquad \forall (o,d)\in K, \forall t\in R \tag{4.99}$$

$$\sum_{(o,d)\in K}\sum_{t\in T} D^{o,d}(t)\Big(\sum_{t_1>t}\psi^{o,d}(t,t_1)t_1\Big) - \sum_{(o,d)\in K}\sum_{t\in T} D^{o,d}(t)\Big(\sum_{t_1>t}\phi^{o,d}(t,t_1)t_1\Big) \leq \frac{B}{V} \quad (4.100)$$

$$\gamma^{o,d}(t,t_1) \leq \psi^{o,d}(t,t_1) \qquad \forall (o,d)\in K, \forall t, t_1 \in T \quad (4.101)$$

$$\gamma^{o,d}(t,t_1) \geq \underline{\rho}\psi^{o,d}(t,t_1) \qquad \forall (o,d)\in K, \forall t, t_1 \in T \quad (4.102)$$

$$\gamma^{o,d}(t,t_1) \leq \rho^{o,d}(t) - \underline{\rho}(1-\psi^{o,d}(t,t_1)) \qquad \forall (o,d)\in K, \forall t, t_1 \in T \quad (4.103)$$

$$\gamma^{o,d}(t,t_1) \geq \rho^{o,d}(t) - (1-\psi^{o,d}(t,t_1)) \qquad \forall (o,d)\in K, \forall t, t_1 \in T \quad (4.104)$$

$$y^{o,d}_{i,j}(t) \geq 0 \qquad \forall i \in A \setminus A_s, j \in \Gamma^+(i), \forall (o,d)\in K, \forall t \in T \quad (4.105)$$

$$z^{o,d+}_{i}(t) \geq 0 \qquad \forall i \in A, \forall (o,d)\in K, \forall t \in T \quad (4.106)$$

$$z^{o,d-}_{i}(t) \geq 0 \qquad \forall i \in A \setminus A_s, \forall (o,d)\in K, \forall t \in T \quad (4.107)$$

$$\rho^{o,d}(t) \in [\underline{\rho}, \overline{\rho}] \qquad \forall (o,d)\in K, \forall t \in R \quad (4.108)$$

$$\psi^{o,d}(t,t_1) \in \{0,1\} \qquad \forall (o,d)\in K, \forall t \in R, t_1 > t \in T \quad (4.109)$$

$$\gamma^{o,d}(t,t_1) \geq 0 \qquad \forall (o,d)\in K, \forall t \in R, t_1 > t \in T \quad (4.110)$$

4.4.5 Numerical experiments

The proposed model for shared mobility service is implemented on the same multi-OD Nguyen-Dupuis as presented in Figure 4.2 with same network characteristics presented in Table 4.4. The total demand for shared mobility service is considered to be 150 and 135 for OD 1-3 and 4-2 respectively. The average vehicle occupancies of each passenger car is considered to vary between 1 and 4 which sets the bounds of the inverse of average vehicle occupancy variable, $\rho^{o,d}(t)$ as $[0.25, 1]$. The model is implemented with an increasing demand to understand the network performance with shared mobility services.

Figure 4.10 presents a steady decline in TSTT with an increasing increment in budget (from 604665 to 145534 veh-secs).

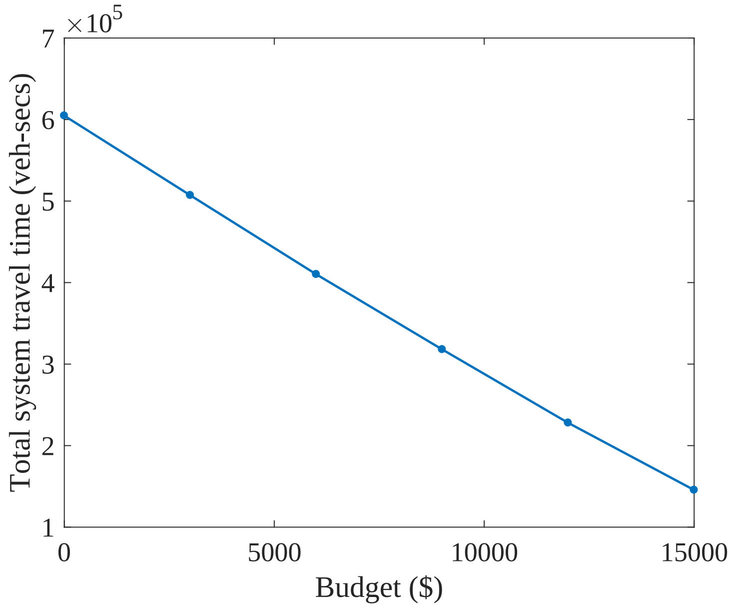

Figure 4.10: Change in demand profile with incentives for OD 4-2

Figures 4.11 presents the change in vehicle occupancy with increasing budget (B). As expected, with zero budget for incentives, no commuter is willing to share their rides. Hence, average vehicle occupancy remains 1 for all departure times. As budget for incentive increases \$3000, the vehicle occupancies start to increase with average vehicle occupancy of 3.33 for departure timestep 2 for OD 1-3. With further increment in budget, the vehicle occupancies are found to increase reaching the maximum value of 4 with budget value of \$15000.

Figure 4.12 shows the change in travel time with an increasing budget. These travel times are estimated endogenously and plays a crucial role in ride-sharing by the commuters. With zero budget, the travel times for both OD pairs are found to vary between 34.5-37.5 minutes. With maximum budget, the travel times for departure time 3, 4 and 5 reduces to 35 minutes.

4.4.6 Summary

In this section, the LTM-SODTA model is modified to design a shared mobility service. The commuters are incentivised to share their rides based on the increment in their arrival time. These arrival times are calculated based on the endogenously estimated travel times. The model is implemented on a multi-OD test network. The model output shows that with increasing budget for incentivisation, more number of commuters share their rides leading to less number of cars in the network with better system performance.

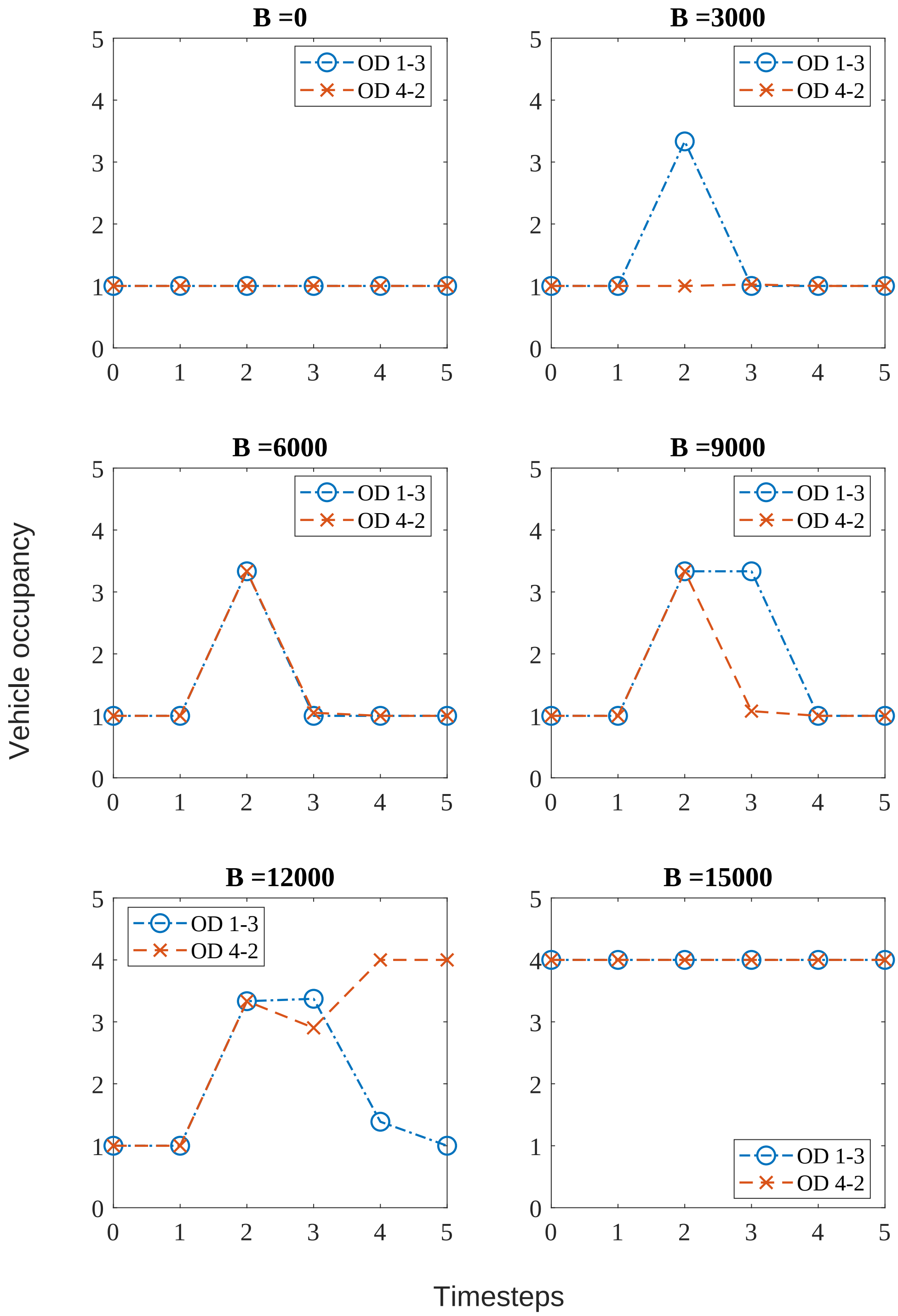

Figure 4.11: Change in demand profile with incentives for OD 4-2

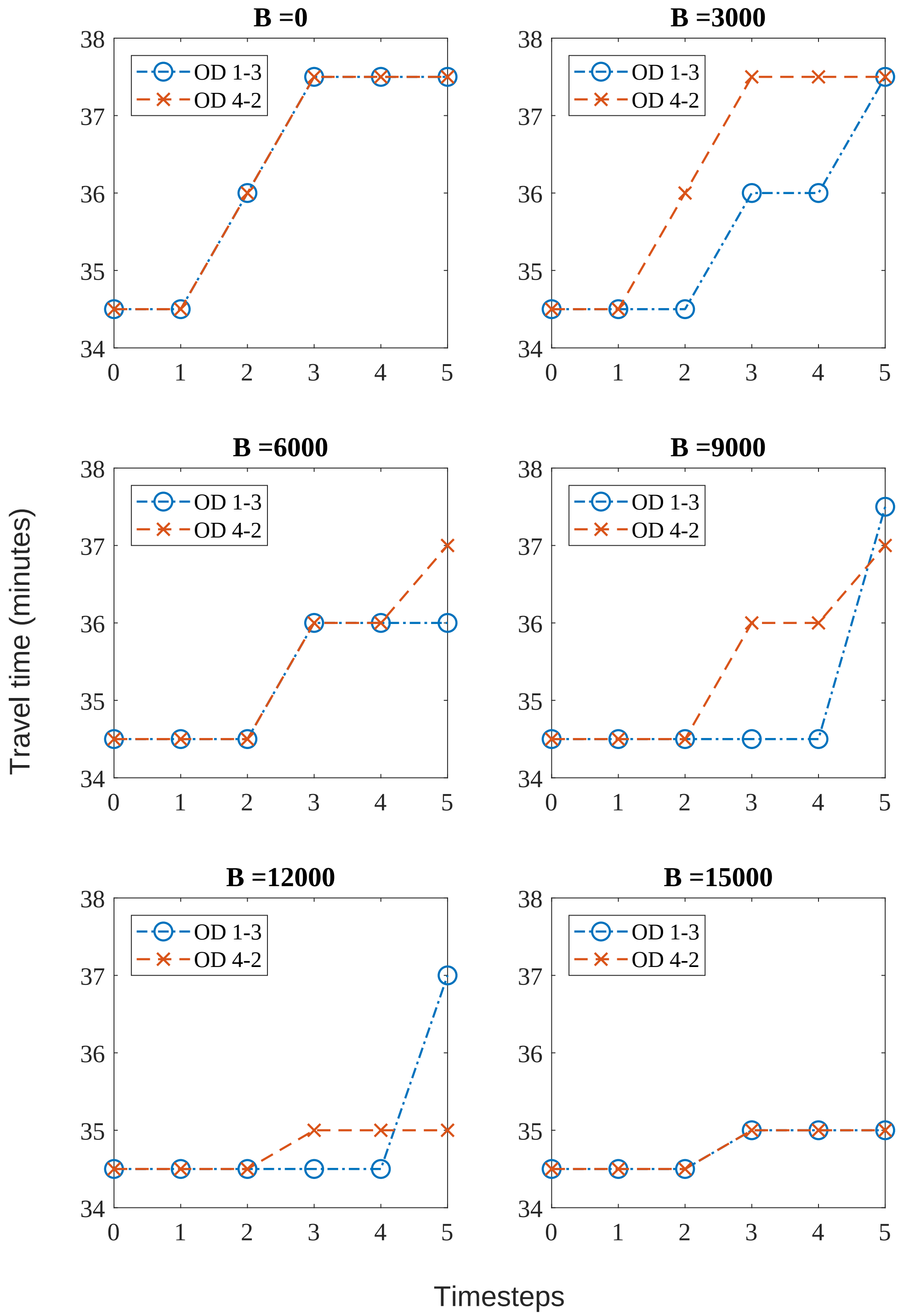

Figure 4.12: Change in demand profile with incentives for OD 4-2

4.5 Conclusion

In this chapter, the versatility of the LTM-SODTA model, proposed in Chapter 3, is explored for three different applications: network design, departure time incentives and shared mobility services. The first application involves solving a network design problem (NDP) with potential capacity enhancement and compared with its cell transmission model (CTM) based NDP on an example network. Compared to the single-destination SODTA based on the CTM, the LTM-NDP formulation is found to involve considerably less decision variables, thus potentially providing a more scalable approach. Further, the model output of the CTM-NDP showed that there is no incentive to allocate non-uniform budget (leading to non-uniform capacity improvement) to the cells of the same link as the cell transfer flow is limited by the cell with the smallest capacity. This finding further advocates the use of LTM-NDP over CTM-NDP in terms of optimum budget allocation for a network design problem.

In the second application, the LTM-SODTA model is implemented for departure-time incentives on a multi-OD test network. A novel method of endogenous estimation of travel time is developed and adopted for incentivising the departure-time shift of commuters for an overall improvement in system performance. Only 50% of the demand is incentivised for leaving early or late from their respective origins. The model output shows that incentives have significant effect in reducing OD travel times. The OD-demand profiles are found to have a flattening effect as the total budget for incentive increases. The departure-time shift variable, $u^{o,d}(t', t)$, is found to shift most of the demand to the last departure-time in case of maximum budget, indicating an even spread of incentivised and non-incentivised demand throughout the demand loading period.

In the third application, the LTM-SODTA model is modified to design a shared mobility service. The commuters are incentivised to share their rides based on the increment in their arrival time if they choose to shift their departure time. These arrival times are calculated based on the endogenously estimated travel times. The model is implemented on a multi-OD test network. The model output shows that with increasing budget for incentivisation, more number of commuters share their rides leading to less number of cars in the network with better system performance.

In the next chapter, the LTM-SODTA model is implemented for optimal lane allocation of autonomous vehicles in a mixed traffic scenario.

Chapter 5

Freeway Network Design under Endogenous Automated Mobility Demand

In this chapter, an integrated mixed-integer programming framework is proposed for optimal exclusive lane design for vehicles equipped with cooperative adaptive cruise control (CACC) on a freeway network which accounts for commuters' demand split among CACC and legacy vehicles via a logit model incorporating class-based utilities. The link transmission model (LTM) is incorporated as the underlying traffic flow model due to its computational efficiency for system optimum dynamic traffic assignment. The LTM is modified to integrate two vehicle classes namely, legacy vehicles and vehicles with CACC with a lane-based approach. The presence of binary variables to represent lane design and the logit model for endogenous demand estimation results in a nonconvex mixed-integer nonlinear program (MINLP) formulation. To tackle this challenging optimization problem, a Benders' decomposition approach is adopted. The proposed approach iteratively explores possible lane designs in the Benders' master problem and, at each iteration, solves a sequence of system-optimum dynamic traffic assignment (SODTA) problems which is shown to converge to fixed-points representative of logit-compatible demand splits. Further, it is proven that the proposed solution method converges to a local optima of the nonconvex problem and conditions are identified under which this local optima is a global solution. The proposed approach is implemented on two hypothetical freeway networks with single and multiple origins and destinations. The numerical results reveal that the optimal lane design of freeway network is non-trivial and can inform on the value of accounting for endogenous demand in the proposed freeway network design.

5.1 Introduction

In this chapter, the applications of the LTM-SODTA model is explored further with a multi-class formulation involving vehicles with level 1 automation on the Society of Automotive Engineers automation scale (SAE, 2013). At this level the automation mainly involves: cooperative adaptive cruise control (CACC), speed harmonization and cooperative merging. These automated vehicles (AV) have the potential to provide various benefits in terms of faster traffic flow propagation due to vehicle-to-vehicle communication, safer manoeuvres leading to traffic safety and overall system-level benefits in terms of reduction in congestion and vehicular emissions. Regardless of the numerous benefits of AVs, one would be too naive to assume that AVs will be immediately adopted by legacy vehicle (LV) owners in near future. A more reasonable assumption would be the existence of a transition period where interactions between LVs and AVs exist, leading to a gradual increment in market penetration of AVs. This transition period will be crucial as safety might be compromised in mixed operations of LVs and AVs, especially in case of arterial networks, involving pedestrians, cyclists and signalized intersections.

A system of judicially designed AV-exclusive lanes could facilitate a smooth transition eliminating the critical interaction between AVs and LVs and familiarise the current transport network with such disruptive automation technology. In a futuristic transport system where fleets of AVs are operated as a service to meet the travel demand of a network, it would be crucial to know the location of such AV-exclusive lanes along with the fleet size of AVs. For this purpose, a mixed integer non-linear LP (MINLP) model is developed for optimal AV-exclusive lane design along with endogenous estimation of AV demand. This MINLP is developed in three stages and implemented on a freeway network. To begin with, the LP framework for a lane-based LTM formulation, presented in Chapter 3, is adopted with fixed AV-exclusive lanes, fixed proportion of AVs on a freeway network with a system level objective. Further, a binary variable is introduced to obtain optimal lane design for AVs for an improved system performance, resulting in a mixed-integer linear program (MILP). Finally, a logit model is brought in to estimate the endogenous demand for each vehicle class which introduces non-linearity in the model, resulting in an MINLP. This non-linearity is circumvented with a fixed-point algorithm along with method of successive averages (MSA) to obtain convergence of the fixed-point.

This chapter is organized into five sections. Section 5.2 presents the problem formulation followed by the solution methodology in Section 5.3. The proposed formulation is studied on two numerical networks, presented in Section 5.4. Section 5.5 presents the key findings of the study along with future research directions.

5.2 Freeway network design problem

5.2.1 Model overview

As mentioned earlier, the model developed in this chapter consists of two vehicle classes: LVs and AVs. The LVs are privately owned regular vehicles without any automation features for vehicle-to-vehicle communication. Whereas, AVs are equipped with connected and automated features such as CACC, speed harmonisation and cooperative merging. These AVs are not privately owned but belong to a fleet of vehicles servicing a network. In a mixed traffic network with these two vehicle classes, an AV can fully utilise its automated features only if the leading/following vehicles are AVs as well. This study refrains from modelling this mixed-traffic interactions and proposes a lane-based approach with dedicated lanes for AVs. These lanes restrict entry of LVs, allowing AVs to get full benefits of their automated features. The regular lanes allow both the vehicle classes where AVs behave like LVs with restrained automated features. The AV-exclusive lanes in the network are not fixed rather allocated by the overall objective function of the model which minimises the total system travel time (TSTT) involving both vehicle classes. As providing more number of these exclusive lanes might affect the traffic flow of LVs, they are provided only at those crucial locations which would improve the overall system performance.

The proposed model in this study also designs the fleet of AVs to service a network. A logit model is adopted for this purpose which estimates the AV proportion in the network based on the gain/loss in utilities of AVs compared to the LV mode. The utility function for the logit functions include mode-wise travel time, waiting time (0 for LVs) and travel fare. This logit model is embedded in a fixed-point algorithm, solved iteratively along with the multi-OD base model of LTM-SODTA to find a proportion of AVs satisfying a mode-choice equilibrium. The entire formulation involving AV-exclusive lane allocation, multi-OD LTM-SODTA and the fixed-point algorithm is decomposed with Benders' decomposition method to disentangle the binary lane allocation variables with the rest of the formulation. With an extensive review of existing literature, it is observed that an integrated framework of this kind has not been explored before.

The following assumptions are made while developing this formulation. The algorithm is designed for a multi-OD freeway network with single path between each OD pair, hence route choice modelling is averted. Further, the lane-changing behaviour and vehicle holding issues are not modelled as well.

5.2.2 Problem formulation

In the freeway network design problem, the multi-OD SODTA formulation is modified with destination-based variables instead of OD-based ones to speed up the computation process. Here, the total system travel time (TSTT) consists of travel times of two vehicle classes: LV and AV.

Table 5.1: Mathematical notations for multi-class SODTA

Sets	
A	set of all lanes and centroid connectors
A_r	set of source centroid connectors
A_s	set of sink centroid connectors
A_c	set of source centroid connectors and physical lanes
A_{av}	set of candidate AV-exclusive lanes
K	set of origin-destination pairs
$\Gamma^-(i)$	set of predecessor lanes of lane $i \in A$
$\Gamma^+(i)$	set of successor lanes of lane $i \in A$
T	set of discretized time steps for traffic flow propagation $(t_0, t_1, ..., t_n)$
R	set of discretized time steps for demand loading
Parameters	
$D^{o,d}(t)$	total demand from o to d at t
q_{lv}	capacity of a regular lane
q_{av}	capacity of an AV-exclusive lane
L_i	length of lane $i \in A$
K_{jam}	jam density
v	free-flow speed
w_{lv}	backward shockwave speed on a regular lane
w_{av}	backward shockwave speed on an AV-exclusive lane
δ	discretized time step for traffic flow propagation
$\beta \geq 0$	total amount of utility gained while making a trip
$\beta^{o,d}_{\tau_{lv}} \geq 0$	disutilities per unit travel time of LVs for $(o,d) \in K$
$\beta^{o,d}_{\tau_{av}} \geq 0$	disutilities per unit travel time of AVs for $(o,d) \in K$
Variables	
$y^k_{i,j,lv}(t) \geq 0$	transfer flow of LVs from lane $i \in A$ to lane $j \in A$ destined to $k \in A_s$ at time $t \in T$
$y^k_{i,j,av}(t) \geq 0$	transfer flow of AVs from lane $i \in A$ to lane $j \in A$ destined to $k \in A_s$ at time $t \in T$
$z^{k+}_{i,lv}(t) \geq 0$	cumulative inflow of LVs on lane $i \in A \setminus A_s$ destined to $k \in A_s$ at time $t \in T$
$z^{k+}_{i,av}(t) \geq 0$	cumulative inflow of AVs on lane $i \in A \setminus A_s$ destined to $k \in A_s$ at time $t \in T$
$z^{k-}_{i,lv}(t) \geq 0$	cumulative outflow of LVs on lane $i \in A \setminus A_s$ destined to $k \in A_s$ at time $t \in T$
$z^{k-}_{i,av}(t) \geq 0$	cumulative outflow of AVs on lane $i \in A \setminus A_s$ destined to $k \in A_s$ at time $t \in T$
$b_i \in \{0,1\}$	binary variable indicating whether a lane $i \in A_{av}$ is AV-exclusive (1) or not (0)
$p^{o,d} \in [0,1]$	fraction of AV demand for $(o,d) \in K$
$\tau^{o,d}_{lv} \geq 0$	average path travel time for LVs for $(o,d) \in K$
$\tau^{o,d}_{av} \geq 0$	average path travel time for AVs for $(o,d) \in K$

The objective function in the proposed formulation minimizes this TSTT as shown in Eq. (5.1).

$$\min\ (\text{TSTT}_{lv} + \text{TSTT}_{av}) \tag{5.1}$$

TSTT_{lv} and TSTT_{av} can be obtained from the difference in cumulative inflows and outflows of each link, representing the number of LVs/AVs present on that link at each timestep and the number of timesteps they spend on that link. The underlying LTM provides these cumulative inflows and outflows as output. The total vehicular demand in the network is the sum of LVs and AVs in the network.

5.2.3 Endogenous demand model

The time-varying total vehicle demand at time t between each origin-destination pair in the network is denoted by $D^{o,d}(t)$ and it is assumed to be fixed in the proposed formulation. $D^{o,d}(t)$ is presented as a sum of the demands of two vehicle classes, LVs and AVs, in Eq. (5.2).

$$D^{o,d}(t) = D^{o,d}_{lv}(t) + D^{o,d}_{av}(t) \qquad \forall(o,d) \in K, \forall t \in R \tag{5.2}$$

Though, the total demand in the network is fixed, the demand corresponding to each vehicle class varies depending on the proportions of AVs ($p^{o,d}$) between each OD pair in the network and they are obtained from Eqs. (5.3a) and (5.3b).

$$D^{o,d}_{lv}(t) = D^{o,d}(t)(1 - p^{o,d}) \qquad \forall(o,d) \in K, \forall t \in R \tag{5.3a}$$

$$D^{o,d}_{av}(t) = D^{o,d}(t)p^{o,d} \qquad \forall(o,d) \in K, \forall t \in R \tag{5.3b}$$

In the proposed formulation, the demand for each vehicle class is endogenous to the proportions of AVs ($p^{o,d}$), which is obtained based on the attractiveness of the modes in the network. A logit model is adopted to quantify this attractiveness, as summarized in Eq. (5.4).

$$p^{o,d} = \frac{e^{U^{o,d}_{av}}}{e^{U^{o,d}_{lv}} + e^{U^{o,d}_{av}}} \tag{5.4}$$

It is assumed that the utility of each mode depends on the average travel times of all the vehicles of that mode between each OD pair, mode-wise travel fare and waiting time based on the availability of each mode (0 for LVs). The average travel times of

LV and AV are denoted by $\tau_{lv}^{o,d}$ and $\tau_{av}^{o,d}$ and the utility of each mode is obtained from Eqs. (5.5a) and (5.5b).

$$U_{lv}^{o,d} = \beta - \beta_{\tau_{lv}}^{o,d}\tau_{lv}^{o,d} - \beta_f^{o,d}F_{lv}^{o,d} \qquad \forall(o,d) \in K \tag{5.5a}$$

$$U_{av}^{o,d} = \beta - \beta_{\tau_{av}}^{o,d}\tau_{av}^{o,d} - \beta_f^{o,d}F_{av}^{o,d} - \beta_w^{o,d}W_{av} \qquad \forall(o,d) \in K \tag{5.5b}$$

Here, β represents the total amount of utility gained while making a trip, whereas, $\beta_{\tau_{lv}}^{o,d}$ and $\beta_{\tau_{av}}^{o,d}$ are the disutilities created per unit travel time by LVs and AVs respectively. The utility of each mode is also impacted by travel fare, F_{lv} and F_{av}, corresponding to LVs and AVs respectively. Along with the travel fare, the utility function of AVs includes a waiting time component (W_{av}) as a user might have to wait for a while to get serviced.

The logit model presented in Eq. (5.4) is modified with the difference in utilities between the modes, as presented in Eq. (5.6).

$$p^{o,d} = \frac{1}{e^{\left(\beta_{\tau_{av}}^{o,d}\tau_{av}^{o,d} - \beta_{\tau_{lv}}^{o,d}\tau_{lv}^{o,d} + \beta_f(F_{av}-F_{lv}) + \beta_w W_{av}\right)} + 1} \qquad \forall(o,d) \in K \tag{5.6}$$

5.2.4 Network dynamics

The LTM, proposed by Yperman et al. (2005), is a numerical solution method for dynamic network loading, developed based on the first order kinematic wave theory Lighthill and Whitham (1955); Richards (1956). In this study, LTM is chosen as it involves fewer variables per link compared to models such as cell transmission model (CTM) Daganzo (1994). Here, the conventional LTM is adapted to accommodate two vehicle classes by introducing two types of lanes: AV-exclusive and regular lanes. An AV-exclusive lane differs from a regular lane in terms of following headway, capacity and speed of backward shockwave propagation. The difference in these traffic flow characteristics affects the fundamental diagrams of traffic flow significantly as explained in the following subsection.

5.2.4.1 Fundamental diagram

The fundamental diagram of traffic flow reflects the relationship among the macroscopic traffic flow parameters of a network: traffic flow, density and speed. These relationships approximate all possible stationary traffic states during the analysis period and provide significant insights regarding the overall behaviour of traffic in a network.

The shape of the fundamental diagrams depicting these relationships may vary depending on the assumptions and approximations of a study. Greenshields et al. (1935) was the first to propose a parabolic relationship between traffic flow and density. Later on, Newell (1993b) provided a simplified approach to the kinematic wave theory of traffic flow and developed a triangular shaped fundamental diagram, defined by three parameters: a fixed free-flow speed (v_f), capacity or maximum flow (q) and a jam density (K_{jam}). Yperman et al. (2005) adopted this simplified fundamental diagram while developing the LTM which is the underlying traffic flow model in the proposed formulation.

In a network with AV-exclusive lanes, the macroscopic traffic flow parameters may be significantly affected by faster reaction times of AVs leading to reduced following headway, increased throughput and faster propagation of backward shockwave due to congestion. Levin and Boyles (2016b) found considerable difference in the shape of the fundamental diagram for different reaction times of a characteristic vehicle. This study also showed how capacity and wave speed increase as the AV proportion increases with the human drivers having double the reaction time of AVs. Tientrakool et al. (2011) demonstrated that due to tighter time and space headways among vehicles, the capacity of a lane could be approximately tripled by converting it into an AV-exclusive lane. Hence, while comparing traffic flow on AV-exclusive and regular lane, the shape of the triangular fundamental diagram will be significantly different due to the changes in capacity (q) and backward wave speed (w) leading to the same jam density (K_{jam}).

In this study, the AV-exclusive lanes are considered to have double the capacity ($q_{av} = 2q_{lv}$) and backward wave speed ($w_{av} = 2w_{lv}$) of that of the regular lane while the free-flow speed (v_f) and jam density (K_{jam}) are kept the same for both lane types. The fundamental diagrams of traffic flow on both of these lane types is shown in Figure 5.1.

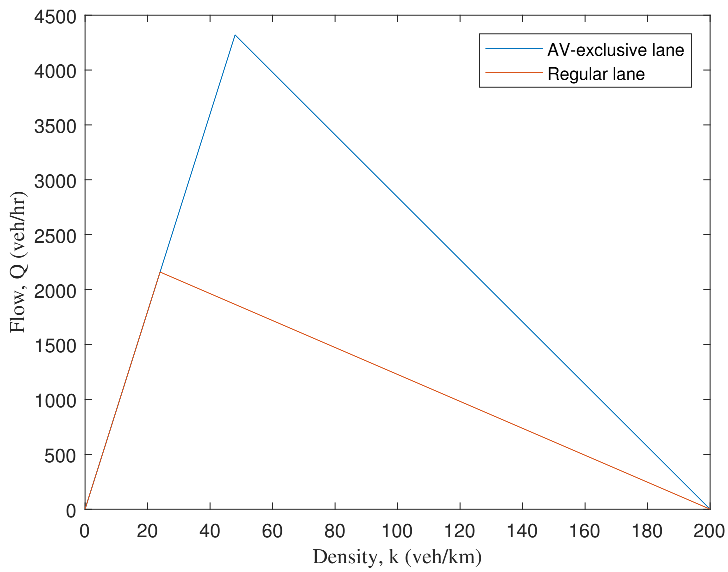

Figure 5.1: Fundamental diagrams of traffic flow for two lane types

5.2.4.2 Traffic flow propagation

The LTM keeps track of the vehicular flow in the network with cumulative inflows and outflows of each link at each time-step. Here, a lane-based LTM is developed where the vehicle class-specific cumulative inflows and outflows from each lane i towards destination k at time t are denoted by $z^{k+}_{i,lv}(t)\big(z^{k+}_{i,av}(t)\big)$ and $z^{k-}_{i,lv}(t)\big(z^{k-}_{i,av}(t)\big)$ respectively. The demand corresponding to each vehicle class is loaded into the network as the cumulative inflow to the source centroid connectors as shown in Eqs. (5.7a) and

(5.7b).

$$z^{k+}_{i,lv}(t) = \sum_{t'<t} D^{i,k}(t')(1 - p^{i,k}) \qquad \forall i \in A_r, \forall (i,k) \in K, \forall t \in T \tag{5.7a}$$

$$z^{k+}_{i,av}(t) = \sum_{t'<t} D^{i,k}(t')p^{i,k} \qquad \forall i \in A_r, \forall (i,k) \in K, \forall t \in T \tag{5.7b}$$

The cumulative inflow to the other lanes at time t is defined as the sum of transfer flows from all the incoming lanes predecessor (Γ^-) to that lane over all the timesteps up until t. These are defined in Eqs. (5.8a) and (5.8b).

$$z^{k+}_{i,lv}(t) = \sum_{t'<t} \sum_{h \in \Gamma^-(i)} y^k_{h,i,lv}(t') \qquad \forall i \in A \setminus \{A_r, A_s\}, \forall k \in A_s, \forall t \in T \tag{5.8a}$$

$$z^{k+}_{i,av}(t) = \sum_{t'<t} \sum_{h \in \Gamma^-(i)} y^k_{h,i,av}(t') \qquad \forall i \in A \setminus \{A_r, A_s\}, \forall k \in A_s, \forall t \in T \tag{5.8b}$$

Similarly, the cumulative outflows ($z^{k-}_{i,lv}(t), z^{k-}_{i,av}(t)$) from a lane at time t is defined as the sum of transfer flows to all the outgoing lanes successor (Γ^+) to that lane over all the timesteps up until t. These are defined in Eqs. (5.9a) and (5.9b).

$$z^{k-}_{i,lv}(t) = \sum_{t'<t} \sum_{j \in \Gamma^+(i)} y^k_{i,j,lv}(t') \qquad \forall i \in A \setminus A_r, \forall k \in A_s, \forall t \in T \tag{5.9a}$$

$$z^{k-}_{i,av}(t) = \sum_{t'<t} \sum_{j \in \Gamma^+(i)} y^k_{i,j,av}(t') \qquad \forall i \in A \setminus A_r, \forall k \in A_s, \forall t \in T \tag{5.9b}$$

The LTM has been built based on three flow components: sending flow, receiving flow and transfer flow. Sending flow is defined as the amount of vehicular flow allowed to go out from link i to link j respecting its flow capacity. Yperman et al. (2005) derived the equation of sending flow based on the propagation of a free-flow traffic state at the upstream boundary of a link transmitting to the downstream boundary $\frac{L_i}{v_{f,i}}$ (link free-flow travel time) time units later. In the proposed lane-based LTM formulation, this concept is implemented for each of the vehicle classes as presented in Eqs. (5.10a) and (5.10b).

$$\sum_{j\in\Gamma^+(i)} y^k_{i,j,lv}(t) \leq \left(z^{k+}_{i,lv}(t_s) - z^{o,d-}_{i,lv}(t)\right) \qquad \forall i \in A\setminus A_s, \forall k\in A_s, \forall t\in T\setminus\{t_n\}$$

$$\text{where, } t_s = t+\delta-\frac{L_i}{v_{f,i}} \tag{5.10a}$$

$$\sum_{j\in\Gamma^+(i)} y^k_{i,j,av}(t) \leq \left(z^{k+}_{i,av}(t_s) - z^{k-}_{i,av}(t)\right) \qquad \forall i \in A\setminus A_s, \forall k\in A_s, \forall t\in T\setminus\{t_n\}$$

$$\text{where, } t_s = t+\delta-\frac{L_i}{v_{f,i}} \tag{5.10b}$$

Eqs. (5.11a) and (5.11b) present the capacity constraints on sending flow for regular and candidate AV lanes respectively. On regular lanes, the total flow of LVs and AVs is restricted to the capacity of regular lanes (δq_{lv}) whereas, on candidate AV lanes, a binary variable $b_i \in \{0,1\}$, $\forall i \in A_{av}$ is introduced to detect whether a lane is regular or AV-exclusive lane. If lane i is a regular lane (AV-exclusive lane), *i.e.,* $b_i = 0$ (1), this sending flow is restricted to the capacity of a regular lane, δq_{lv} (AV-exclusive lane, δq_{av}).

$$\sum_{k\in K}\sum_{j\in\Gamma^+(i)} \left(y^k_{i,j,lv}(t) + y^k_{i,j,av}(t)\right) \leq \delta q_{lv} \qquad \forall i\in A\setminus A_{av}, \forall t\in T \tag{5.11a}$$

$$\sum_{k\in K}\sum_{j\in\Gamma^+(i)} \left(y^k_{i,j,lv}(t) + y^k_{i,j,av}(t)\right) \leq \delta q_{lv}(1-b_i) + \delta q_{av} b_i \quad \forall i\in A_{av}, \forall t\in T \tag{5.11b}$$

In the LTM, the receiving flow is defined as the amount of vehicular flow allowed to be received at link j from link i depending on the congestion level and the capacity of link j. The receiving flow constraint, as presented in Eq. (5.12), is derived based on the backward propagation of a congested traffic state from the downstream boundary of a link which reaches the upstream boundary $\frac{L_i}{w_{av}}$ time units later. Here, w_{av} denotes the backward wave speed of the congested traffic state.

$$\sum_{i\in\Gamma^-(j)} \sum_{(o,d)\in K} \left(y^k_{i,j,lv}(t) + y^k_{i,j,av}(t)\right) \\ \leq K_{jam}L_j - \sum_{(o,d)\in K} \left(\left(z^{o,d+}_{j,lv}(t) - z^{o,d-}_{j,lv}(t_{r,lv})\right) + \left(z^{o,d+}_{j,av}(t) - z^{o,d-}_{j,av}(t_{r,av})\right)\right) \\ \forall j \in A \setminus \{A_r, A_s\}, \forall t \in T \text{ where, } t_{r,lv} = t + \delta - \frac{L_i}{w_{lv}}, t_{r,av} = t + \delta - \frac{L_i}{w_{av}} \quad (5.12)$$

Similar to Eqs. (5.11a) and (5.11b), Eqs. (5.13a) and (5.13b) represent the capacity constraint on receiving flow of a link with the binary parameter, b, depending on the lane being a regular or candidate AV lane.

$$\sum_{k\in A_s} \sum_{i\in\Gamma^-(j)} \left(y^k_{i,j,lv}(t) + y^k_{i,j,av}(t)\right) \leq \delta q_{lv} \qquad \forall j \in A \setminus A_{av}, \forall t \in T \quad (5.13a)$$

$$\sum_{k\in A_s} \sum_{i\in\Gamma^-(j)} \left(y^k_{i,j,lv}(t) + y^k_{i,j,av}(t)\right) \leq \delta q_{lv}(1 - b_j) + \delta q_{av} b_j \qquad \forall j \in A_{av}, \forall t \in T \quad (5.13b)$$

In the proposed formulation, LVs are restricted from entering an AV-exclusive lane. A set of integer-linear constraints is formulated to implement this restriction in the proposed model as presented in Eqs. (5.14a) and (5.14b). Using the binary lane design variable (b_i), the transfer flow of LVs at any timestep is either restricted or kept free, *i.e.*, equal to the capacity of regular lane, for a downstream AV-exclusive or regular lane respectively.

$$\sum_{(o,d)\in K} \sum_{j\in\Gamma^+(i)} y^k_{i,j,lv}(t) \leq (1 - b_i)\delta q_{lv} \qquad \forall i \in A_{av}, \forall t \in T \quad (5.14a)$$

$$\sum_{(o,d)\in K} \sum_{i\in\Gamma^-(j)} y^k_{i,j,lv}(t) \leq (1 - b_j)\delta q_{lv} \qquad \forall j \in A_{av}, \forall t \in T \quad (5.14b)$$

Eqs. (5.15a) and (5.15b) conclude the lane-based LTM formulation ensuring the exit of all the vehicles that entered into the network and reaching their respective destinations at the end of the last timestep ($\bar{t}$).

$$z_{k,lv}^{k+}(\bar{t}) = \sum_{t \in R \setminus \bar{t}} D^{o,k}(t)(1 - p^{o,k}) \qquad \forall k \in A_s, \forall (o,k) \in K \tag{5.15a}$$

$$z_{k,av}^{k+}(\bar{t}) = \sum_{t \in R \setminus \bar{t}} D^{o,k}(t)p^{o,k} \qquad \forall k \in A_s, \forall (o,k) \in K \tag{5.15b}$$

5.2.5 MINLP formulation

In the LTM, the cumulative inflows and outflows of each lane at each timestep track the vehicular flow in the network. The difference between these inflows and outflows of a lane at each timestep represents the number of vehicles present in that lane for δ time units, where δ is the duration of each timestep. Hence, the sum of these differences over all the lanes, OD pairs and timesteps will provide the TSTT of each vehicle class in the network as follows.

$$\text{TSTT}_{lv} = \delta \sum_{i \in A \setminus A_s} \sum_{(o,d) \in K} \sum_{t \in T} \left(z_{i,lv}^{k+}(t) - z_{i,lv}^{k-}(t) \right) \tag{5.16a}$$

$$\text{TSTT}_{av} = \delta \sum_{i \in A \setminus A_s} \sum_{(o,d) \in K} \sum_{t \in T} \left(z_{i,av}^{k+}(t) - z_{i,av}^{k-}(t) \right) \tag{5.16b}$$

The average travel times are estimated based on the number of timesteps each vehicle spends on each link, averaged over the total demand of that vehicle class as presented in Eqs. (5.17a) and (5.17b). Let $L^{o,d}$ be the set of links belonging to the path of OD $(o, d) \in K$, and let $a_l^{o,d}$ be a binary parameter indicating whether link l is on the path of OD pair (o, d) or not. The class-based average OD travel times are:

$$\tau_{lv}^{o,d} = \sum_{l \in L^{o,d}} \left(\frac{\sum_{i \in l} \sum_{k \in A_s} \sum_{t \in T} \left(z_{i,lv}^{k+}(t) - z_{i,lv}^{k-}(t) \right) \delta}{\sum_{(o',d') \in K} \sum_{t \in R \setminus t_n} D^{o',d'}(t)(1 - p^{o',d'}) a_l^{o',d'}} \right) \qquad \forall (o,d) \in K \tag{5.17a}$$

$$\tau_{av}^{o,d} = \sum_{l \in L^{o,d}} \left(\frac{\sum_{i \in l} \sum_{k \in A_s} \sum_{t \in T} \left(z_{i,av}^{k+}(t) - z_{i,av}^{k-}(t) \right) \delta}{\sum_{(o',d') \in K} \sum_{t \in R \setminus t_n} D^{o',d'}(t) p^{o',d'} a_l^{o',d'}} \right) \qquad \forall (o,d) \in K \tag{5.17b}$$

The objective function of the proposed MINLP is rewritten and presented in Eq. (5.1) as follows.

$$\min \delta \sum_{i\in A\setminus A_s} \sum_{(o,d)\in K} \sum_{t\in T} \left(z^{k+}_{i,lv}(t) + z^{k+}_{i,av}(t) - z^{k-}_{i,lv}(t) - z^{k-}_{i,av}(t) \right) \tag{5.18}$$

Note that, as LVs are restricted on AV-exclusive lanes, at least one path with regular lanes is kept fixed between each OD-pair in the model for movement of LVs. This constraint is imposed by defining the solution space of the binary lane design variable (b_i) in such a way that each link consists of at least one lane which is not a candidate for AV-exclusive lane conversion. The resulting MINLP formulation `FNDP` represents the freeway network design problem.

$$\textbf{Model 1 } (\texttt{FNDP}). \begin{cases} \min \ TSTT \quad (5.18) \\ \text{s.t.:} \\ \textit{Endogenous demand} \quad (5.6), (5.17a), (5.17b) \\ \textit{Network dynamics} \quad (5.7a) \text{ - } (5.15b) \\ \boldsymbol{y} \in \mathcal{Y}, \boldsymbol{z} \in \mathcal{Z}, \boldsymbol{b} \in \mathcal{B}, \boldsymbol{\tau} \in \mathcal{T}, \boldsymbol{p} \in \mathcal{P} \end{cases}$$

As presented above, `FNDP` involves five sets of variables: transfer flows ($\boldsymbol{y}$) with domain $\mathcal{Y} = \mathbb{R}_+^{|A_c||\Gamma^+(A_c)||A_s||T|}$, cumulative inflows and outflows ($\boldsymbol{z}$) with domain $\mathcal{Z} = \mathbb{R}_+^{|A||A_s||T|}$, binary variables for lane allocation ($\boldsymbol{b}$) with domain: $\mathcal{B} = \{0,1\}^{|A_{av}|}$, class-wise travel times ($\boldsymbol{\tau}$) with domain $\mathcal{T} = \mathbb{R}_+^{2|K|}$, and OD proportion of AVs ($\boldsymbol{p}$) domain $\mathcal{P} = [0,1]^{|K|}$. Due to the integer variables for lane design ($\boldsymbol{b}$) and the nonlinear logit model, `FNDP` may lead to computational tractability issues for bigger networks. In Section 5.3, these non-linearity issues are handled by introducing a Benders decomposition approach with an embedded fixed-point algorithm and implement it on a freeway network in Section 5.4.

The outputs of Model `FNDP` can be interpreted as follows. The main output are the lane design variables $\boldsymbol{b}$ which indicate which candidate lane should be AV-exclusive in the freeway network. The remaining variables are used to account for congestion effects and endogenous travel demand. Travel demand is loaded into the network through the source centroid connectors as expressed in Eqs. (5.3a) and (5.3b). At the completion of the trips, Eqs. (5.15a) and (5.15b) ensures that vehicles leave the network through the sink centroid connectors. On a freeway, these source and sink centroid connectors represent on- and off-ramps respectively. If the network is unable to accept demand due to congestion on the freeway, vehicles may be held at on-ramps which are assumed to have sufficiently large capacities. Since waiting time is penalized in the objective function, the outputs of the proposed formulation can be interpreted as the level of control at the freeway on-ramps.

The proposed model is analysed in the following sections.

5.2.6 Fixed-point analysis

To motivate the design of a dedicated solution method and to provide insights into the behavior of `FNDP`, a simplified version of the model is considered wherein the endogenous demand $\boldsymbol{p}$ and the lane design $\boldsymbol{b}$ are assumed fixed. This simplified model is called $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$ and presented below.

Model 2 ($\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$)**.**
$$\begin{cases} \min \; TSTT \quad (5.18) \\ \text{s.t.:} \\ \textit{Network dynamics} \quad \text{(5.7a) - (5.15b)} \\ \boldsymbol{y} \in \mathcal{Y}, \boldsymbol{z} \in \mathcal{Z} \end{cases}$$

The variables involved in Model $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$ are transfer flows $\boldsymbol{y} \in \mathcal{Y}$ and cumulative inflows and outflows $\boldsymbol{z} \in \mathcal{Z}$. Here a single-OD network is considered with two links with fixed AV-exclusive lanes as illustrated in Figure 5.2, where the fixed AV lanes are shown in blue. The first link consists of three regular lanes and one AV-exclusive lane followed by a capacity drop on the second link which has one LV and one AV-exclusive lane. The $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$ is solved for a series of OD proportions of AVs ($p^{o,d}$) and calculate the corresponding logit-derived proportion of AVs ($p^{o,d}_{logit}$) using Eq. (5.6) based on the optimal solution of $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$. Note that if $p = p_{logit}$, then the demand splits are logit-compatible, i.e. equilibrated, and this solution corresponds to a fixed-point. This fixed-point is defined as follows.

Definition 1 (Fixed-point)**.** *Let* $F : \mathcal{P} \to \mathcal{P}$ *be a continuous function of the OD proportion vector* $\boldsymbol{p} \in \mathcal{P}$*.* $\boldsymbol{p}$ *is considered as a fixed-point if* $F(\boldsymbol{p}) = \boldsymbol{p}$*.*

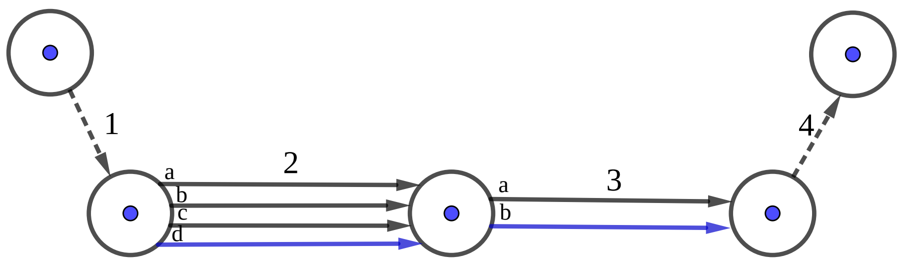

Figure 5.2: Single OD case study network

The regular and AV-exclusive lanes differ from each other in terms of capacity and backward wave speed of congestion propagation. In this study network, the capacity (4320 veh/hr) and backward wave speed (28.4 km/hr) of an AV-exclusive lane is taken as double that of an regular lane due to the inter-connectivity of AVs leading to better traffic flow and faster congestion propagation. The length and jam density of the links are 800m and 200 veh/km respectively with a free-flow speed of 90 km/hr for both vehicle classes. The demand is loaded through the source centroid connector 1 into the network. The capacity and jam density of the source and sink centroid connectors are set to very high values with a negligible length for instantaneous loading of demand into the network based on available network capacity. The values of these network parameters are provided in Table 5.2.

This experiment is started by varying the proportion of AVs ($p^{o,d}$) from 0.5 to 0.99 in steps of 0.01 and solve Model $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$ at each value of $p^{o,d}$. For each step, the average OD travel time of each vehicle class ($\tau_{lv}^{o,d}, \tau_{av}^{o,d}$) is obtained using Eqs. (5.17a) and (5.17b). Next, $p_{logit}^{o,d}$ is calculated by substituting $\tau_{lv}^{o,d}$ and $\tau_{av}^{o,d}$ in Eq. (5.6) along

Table 5.2: Single OD network characteristics

Parameters	Source (1)	Lane 2a	Lane 2b	Lane 2c	Lane 2d	Lane 3a	Lane 3b	Sink (4)
Length (km)	0.0001	0.8	0.8	0.8	0.8	0.8	0.8	0.0001
Free-flow speed (km/hr)	90	90	90	90	90	90	90	90
Backward wave speed (km/hr)	12.2	12.2	12.2	28.4	28.4	12.2	28.4	12.2
Capacity (veh/hr)	360000	2160	2160	4320	4320	2160	4320	360000
Jam density (veh/km)	100000	200	200	200	200	200	200	100000

with the coefficients of these travel times ($\beta^{o,d}_{\tau_{lv}}, \beta^{o,d}_{\tau_{av}}$). These coefficients are obtained from a previous study on route choice behaviour of LVs and AVs where the value of time for LV and AV users were found to be \$10 and \$6.5/hr Wong et al. (2018). In this study, it is assumed that this travel fare is equivalent in both vehicle classes as the additional fare of an AV-service might be compensated by vehicle maintenance and insurance cost of owning an LV. It is also assumed that the AVs are ubiquitous in the network, hence the average waiting time to get an AV-service tends to zero. Hence, the utility of each vehicle-class depends only on the travel times.

To identify fixed-points ($p^{o,d} = p^{o,d}_{logit}$), p against $p^{o,d}_{logit}$ is plotted for different values of $\beta^{o,d}_{\tau_{av}}$ while $\beta^{o,d}_{\tau_{lv}}$ remains fixed, as shown in Figure 5.3.

The dotted line in Figure 5.3 acts as a reference line to locate fixed-points. Interestingly, the line curve depicting the relationship between $p^{o,d}$ and $p^{o,d}_{logit}$ in Figure 5.3 is found to cross this reference line multiple times showing the existence of multiple fixed points in the problem. Figure 5.4 depicts the change in the value of the objective function (TSTT) with respect to p. Here, it is observed that all fixed-points have equal TSTT. This experiment highlights that fixed-points may be non-unique, but may also correspond to identical TSTT.

From Figure 5.3, it is observed that in each case, the line plot depicting the relationship between p and p_{logit} crosses the reference line at least once, referring to the existence of at least one fixed-point satisfying the logit model. This existence of

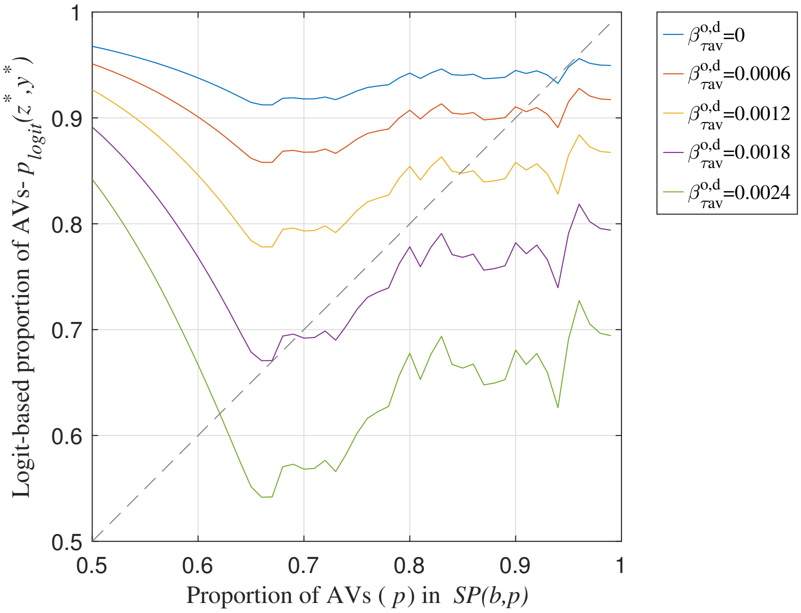

Figure 5.3: Multiple fixed points with fixed lane design

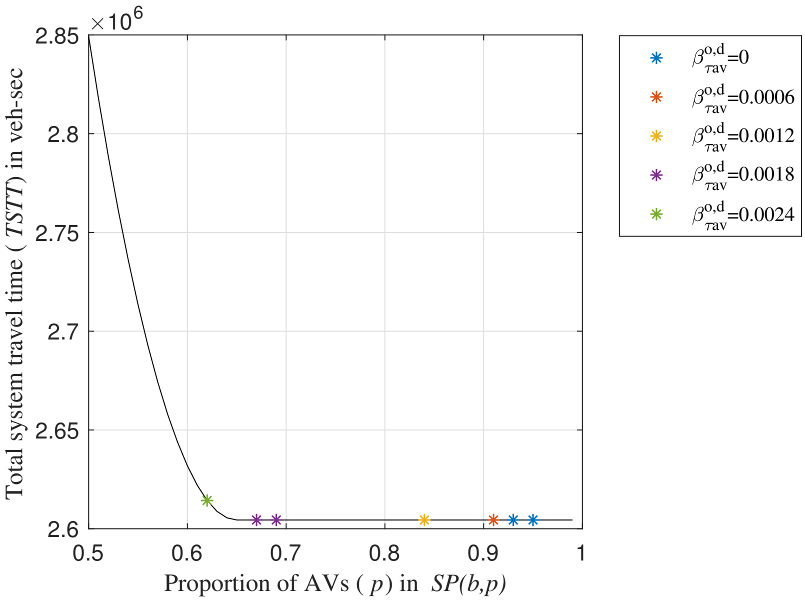

Figure 5.4: Change in TSTT with p

at least one fixed-point is proven with Proposition 1.

Proposition 1. *Existence of fixed-point: For a fixed lane design vector $\boldsymbol{b} \in \mathcal{B}$, there exists at least one fixed-point such that $F(\boldsymbol{p}) = \boldsymbol{p}$.*

Proof. It is shown that one can construct such a continuous function $F(\boldsymbol{p})$. Let $h(\boldsymbol{p})$ be the function mapping the OD proportion vector $\boldsymbol{p}$ to the optimal solution of the linear program $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$ for fixed lane design vector $\boldsymbol{b}$ as defined by (2). Let $\boldsymbol{z} \in \mathcal{Z}$ be the vector of cumulative inflow and outflow and let $\boldsymbol{y} \in \mathcal{Y}$ be the vector of transfer flows obtained after solving $\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$, formally:

$$h : \mathcal{P} \to \mathcal{Z} \times \mathcal{Y}$$
$$h(\boldsymbol{p}) = (\boldsymbol{z}, \boldsymbol{y})$$

Let $g(\boldsymbol{z}, \boldsymbol{y})$ be the function mapping the optimal cumulative inflows and outflows, and transfer flows to average, class-based (LV and AV) OD travel times $(\boldsymbol{\tau}_{lv}, \boldsymbol{\tau}_{av}) \in \mathcal{T}$ as defined in Eqs. (5.17a) and (5.17b):

$$g : \mathcal{Z} \times \mathcal{Y} \to \mathcal{T}$$
$$g(\boldsymbol{z}^{\star}, \boldsymbol{y}^{\star}) = (\boldsymbol{\tau}_{lv}, \boldsymbol{\tau}_{av})$$

Finally, let $f(\boldsymbol{\tau}_{lv}, \boldsymbol{\tau}_{av})$ be the function mapping average class-based OD travel times to OD proportion $\boldsymbol{p} \in \mathcal{P}$ via the proposed logit model as defined in (5.6):

$$f : \mathcal{T} \to \mathcal{P}$$
$$f(\boldsymbol{\tau}_{lv}, \boldsymbol{\tau}_{av}) = \boldsymbol{p}$$

Let $F(\boldsymbol{p}) = f(g(h(\boldsymbol{p})))$, F is a continuous function from the compact convex set $\mathcal{P}$ to itself. By Brouwer's theorem, there exists at least one fixed-point $\boldsymbol{p}$ such that $F(\boldsymbol{p}) = \boldsymbol{p}$. $\square$

In Section 5.3, a solution methodology is developed to solve the non-linear Model 1 with variable lane design and endogenous demand based on the embedded logit model.

5.3 Benders' decomposition approach

In this section, a decomposition approach is proposed with variable lane design and endogenous demand for each vehicle class. The purpose of this development is to identify the crucial locations in a network where providing an AV-exclusive lane will reap the maximum benefit for a given AV demand. As LVs are restricted on AV-exclusive lanes, it is necessary to deploy AV-exclusive lanes judiciously to cater to both vehicle classes keeping the social welfare into perspective. Here, Benders' decomposition method is introduced which iteratively explores possible lane designs in a master problem and, at each iteration, solves a sequence of SODTA problems which is shown to converge to fixed-points representative of logit-compatible demand splits.

Benders' decomposition approach eases up the computation burden of a mathematical model by partitioning the overall formulation into a relaxed master problem with mainly integer variables and a subproblem with all the continuous variables. For a detailed review of Benders' approach, one can refer to Rahmaniani et al. (2017). For problems with minimizing objective function, such as the proposed model in this study, the relaxed master problem provides a lower bound at each iteration of Benders' method. Whereas, the subproblem handles the complicated constraints of the original problem which is solved iteratively for each relaxed solution of the master problem. At each iteration until convergence, the Benders' method generates either a feasibility cut or an optimality cut towards obtaining the optimal solution. A feasibility cut is generated to eliminate an infeasible solution provided by the subproblem, preventing the master to produce it again. On the other hand, if the sub-problem solution is feasible, an upper bound is obtained and an optimality cut is derived towards closing the optimality gap.

In the decomposition of Model `FNDP`, only the binary lane design variables $\boldsymbol{b}$ are retained in the master problem `MP`, which is summarized below:

Model 3 (MP). $\begin{cases} \min Z \\ \text{s.t.:} \\ Z \geq \textit{Optimality cuts} \\ 0 \geq \textit{Feasibility cuts} \\ \boldsymbol{b} \in \mathcal{B},\ Z \geq 0 \end{cases}$

In the proposed model, the subproblem ($\texttt{SP}(\boldsymbol{b}, \boldsymbol{p})$) is initiated by fixing the set of binary lane design variables. Depending on the feasibility of the subproblem, dual prices or rays for each constraint are calculated, followed by solving the master problem which provides the values for the next set of binary lane design variables. This iterative process continues until an exact solution of the objective function is reached.

The subproblem in the proposed formulation consists of two components: the lane-based LTM with system-level objective function and an endogenous demand model. As the endogenous demand model introduces non-linearity in the formulation, a fixed-point algorithm is developed for solving the subproblem as explained in the following subsection.

Fixed-point algorithm

The endogenous demand model is crucial to study the effect of infrastructural changes such as AV-exclusive lanes on AV demand. Note that, the formulation without the endogenous demand model is a useful model by itself as it can estimate the progressive deployment of AV-exclusive lanes in a network corresponding to incremental penetration of AV demand. However, this model does not account for the effect of these AV-exclusive lanes on the demand of each vehicle class.

Here, the logit model is adopted for endogenously estimate the demand for each vehicle class. As the logit model is nonlinear, a fixed-point algorithm is used to separate the endogenous demand component from the SODTA formulation. The fixed-point algorithm estimates the proportion of AVs ($p^{o,d}$) in an iterative process involving the logit model, presented in Eq. (5.6). The Algorithm 1 presented in the next subsection, keeps the nonlinear logit model out of the MILP, keeping it a linear mathematical formulation. The new proportion of AV demand is obtained based on the difference in utilities between the modes, substituted in the logit model (as shown in Eq. (5.6)), which is fed back to Model 2 for subsequent iterations. Here, the method of successive averages (MSA) is adopted for convergence of the fixed-point algorithm which is based on a predetermined move size along the descent direction. Note that MSA is guaranteed to converge after a certain number of iterations by providing lesser weightage to subsequent solutions at each iteration. This iterative process of MSA may disregard the instability in the fixed-point solution. This instability is monitored by checking the value of $p^{o,d}$ before and after implementing MSA.

The Benders' decomposition algorithm along with fixed-point algorithm is presented in Algorithm 1. Next, it is proven that Algorithm 1 converges to a local optima of Model `FNDP` in Proposition 2 and identify under which conditions this local optima is a global solution in Proposition 3.

Definition 2 (Local optima of Model `FNDP`)**.** *A solution $\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{p}$ of Model* `FNDP` *which verifies all constraints of Model* `FNDP` *and for which $\boldsymbol{y}, \boldsymbol{z}$ is a minimizer of* `SP`$(\boldsymbol{b}, \boldsymbol{p})$ *is called a local optima of Model* `FNDP`*.*

Proposition 2. *Algorithm 1 converges to a local optima of Model* `FNDP`

Proof. At each Benders' iteration m (outer repeat loop), a lane design $\boldsymbol{b}^m$ and a lower bound Z^m are found. As shown by Proposition 1, there exists at least one fixed-point for any lane design. Hence, at each iteration m, the fixed-point procedure (inner repeat loop) converges to a fixed-point $F(\boldsymbol{p}^n) = \boldsymbol{p}^n$ where n is last iteration of this procedure. The optimal solution of `SP`$(\boldsymbol{b}^m, \boldsymbol{p}^n)$ yields an upper bound $TSTT^n$ on the optimal objective value of Model `FNDP`. Algorithm 1 terminates when the gap between the best upper bound $UB = \max_n(\text{TSTT}^n)$ and the lower bound Z^m

Algorithm 1: Benders decomposition with fixed-point algorithm for `FNDP`

$\boldsymbol{p}^0 \leftarrow \frac{1}{e^{\tau_{ff}^{o,d}\left(\beta_{\tau av}^{o,d}-\beta_{\tau lv}^{o,d}\right)}+1}$ (Solve logit model with free-flow travel times)
$\boldsymbol{b}^0 \leftarrow \mathbf{0}$ (No AV-exclusive lanes)
$m \leftarrow 0$
$n \leftarrow 0$
$UB \leftarrow \infty$
repeat
  $m \leftarrow m+1$
  repeat
    $n \leftarrow n+1$
    $\boldsymbol{y}^n, \boldsymbol{z}^n \leftarrow$ Solve $\texttt{SP}(\boldsymbol{b}^m, \boldsymbol{p}^n)$
    for $(o,d) \in K$ **do**
      $\tau_{lv}^{o,d,n} \leftarrow \sum_{l \in L^{o,d}} \left(\frac{\sum_{i \in l} \sum_{k \in A_s} \sum_{t \in T} \left(z_{i,lv}^{k+}(t) - z_{i,lv}^{k-}(t) \right) \delta}{\sum_{(o',d') \in K} \sum_{t \in R \setminus tn} D^{o',d'}(t)(1-p^{o',d',n}) a_l^{o',d'}} \right)$
      $\tau_{av}^{o,d,n} \leftarrow \sum_{l \in L^{o,d}} \left(\frac{\sum_{i \in l} \sum_{k \in A_s} \sum_{t \in T} \left(z_{i,av}^{k+}(t) - z_{i,av}^{k-}(t) \right) \delta}{\sum_{(o',d') \in K} \sum_{t \in R \setminus tn} D^{o',d'}(t) p^{o',d',n} a_l^{o',d'}} \right)$
      $p^{o,d} \leftarrow \frac{1}{e^{\left(\beta_{\tau av}^{o,d} \tau_{av}^{o,d,n} - \beta_{\tau lv}^{o,d} \tau_{lv}^{o,d,n}\right)}+1}$
      $p^{o,d,n+1} \leftarrow \frac{n}{n+1} p^{o,d,n} + \frac{1}{n+1} p^{o,d}$
  until $\sum_{(o,d) \in K} |p^{o,d,n+1} - p^{o,d,n}| \leq \epsilon_{MSA}$;
  if *SP*$(\boldsymbol{b}^m, \boldsymbol{p}^n)$ *is infeasible* **then**
    Generate feasibility cut
  else
    if $TSTT^n < UB$ **then**
      $UB \leftarrow \text{TSTT}^n$
      $\boldsymbol{b}^\star \leftarrow \boldsymbol{b}^m$
      $\boldsymbol{\tau}^\star \leftarrow \boldsymbol{\tau}^n$
      $\boldsymbol{p}^\star \leftarrow \boldsymbol{p}^n$
    $GAP \leftarrow \frac{UB - Z^m}{UB}$
    if $GAP > \epsilon$ **then**
      Generate optimality cut
  $\boldsymbol{b}^m, Z^m \leftarrow$ Solve `MP`
until $GAP \leq \epsilon$;
return $UB, \boldsymbol{b}^\star, \boldsymbol{p}^\star, \boldsymbol{\tau}^\star$

is below a predefined tolerance. Let $(\boldsymbol{z}^\star, \boldsymbol{y}^\star)$ be the optimal solution of $\texttt{SP}(\boldsymbol{b}^m, \boldsymbol{p}^n)$ corresponding to the best upper bound UB. The solution $(\boldsymbol{z}^\star, \boldsymbol{y}^\star)$ minimize TSTT for the fixed-point $\boldsymbol{p}^n = F(\boldsymbol{p}^n)$. Thus, upon termination, Algorithm 1 returns a lane design vector $\boldsymbol{b}^\star$ and an OD proportion vector $\boldsymbol{p}^\star$ which verifies all constraints of

Model `FNDP` and minimizes TSTT for this configuration. □

Proposition 3. *For any lane design vector $\boldsymbol{b}$, if all fixed-points $\boldsymbol{p} = F(\boldsymbol{p})$ have equal TSTT, then Algorithm 1 converges to a global optima of Model `FNDP`.*

Proof. If all fixed-points have equal TSTT, then Benders' cuts are guaranteed to never overestimate the lower bound Z obtained from solving the master problem `MP`. Hence, all local optima of Model `FNDP` have equal and minimal TSTT. □

Proposition 3 identifies the conditions under which the solution of the proposed algorithm is globally optimal. Although, it is not mathematically proven that in the presence of multiple fixed points all fixed points yield identical TSTT, in the investigations carried out in this study, this behaviour is consistently observed on congested freeway networks.

In the next section, this algorithm is implemented on a single-OD and a multi-OD network.

5.4 Numerical experiments

The proposed formulation is implemented on the same single OD network (Figure 5.2) presented in Section 5.2.6 along with a multi-OD freeway network (Figure 5.6).

5.4.1 Single OD freeway network with fixed lane design

The solution methodology presented in Section 5.3 is implemented on the single OD freeway network with fixed lane design for AVs. As mentioned earlier, the presence of the binary lane design variable makes the proposed formulation a mixed-integer linear program, whereas, the endogenous demand model introduces non-linearity due to the structure of the logit model. The fixed-point algorithm is developed to circumvent non-linearity and Benders decomposition method to handle the binary lane design variable in the master problem. To understand the performance of the fixed-point algorithm alone, at first Model 2 is implemented on the single OD network with fixed dedicated lanes for AVs. The algorithm is initialised with a proportion of AV obtained from Eq. (5.6) substituting the free-flow travel time between the OD pairs. Model 2 is solved at each iteration of this algorithm. The value of $p^{o,d}$ is updated at each iteration with the proportion (p_{logit}) obtained by the endogenous demand model until convergence where a fixed-point is reached ($p = p_{logit}$).

The network characteristics of this test network is same as presented in Table 5.2. A time-varying demand profile is selected for this analysis which is loaded into the network every 2 minutes for the first 8 minutes of a total analysis period of 100 minutes. The total demand is 2950 vehicles which includes both LVs and AVs. Note that, the objective function of the proposed formulation takes into account the waiting time of vehicles at the source centroid connector depending on the available capacity in the network. A timestep (δ) of 30 seconds is considered at which the cumulative inflows and outflows of each link are updated by the underlying LTM. For a single OD network with fixed dedicated lanes for AVs with $\beta^{o,d}_{\tau_{av}} = 0.0018$, this convergence of the fixed-point algorithm is plotted in Figure 5.5.

Figure 5.5 shows that the algorithm converges to a fixed-point after 67 iterations at $p = 0.68$ which is one of the fixed points presented by the purple dot in Figure 5.4. At convergence, the value of the objective function (TSTT) is found to be 2604430 veh-sec with a computation time of 24.067 seconds.

In this case study on a single OD network with fixed AV-exclusive lanes, it is observed that the fixed-point algorithm performs well with fast convergence. In Subsection 5.4.2, the Benders' decomposition method is introduced to handle variable lane design problem on a multi-OD network.

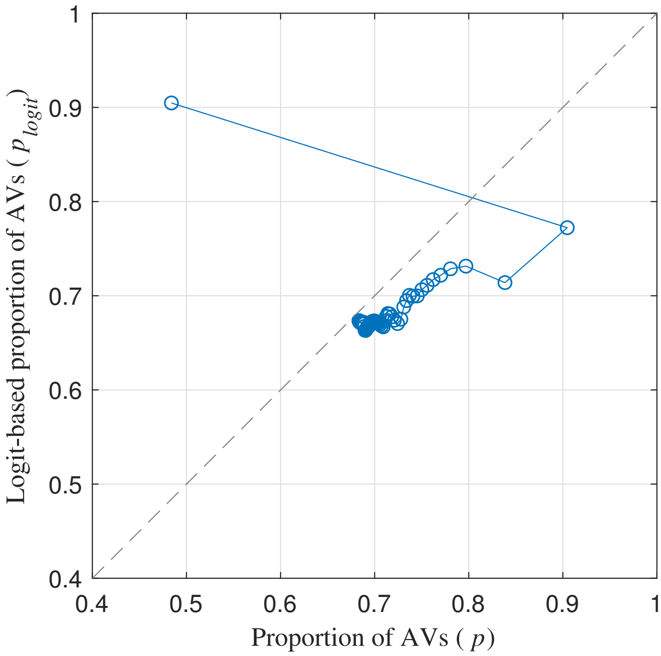

Figure 5.5: Convergence of FP algorithm
(with fixed lane design)

5.4.2 Multi-OD freeway network

The proposed model is implemented with Benders method and fixed-point algorithm on a multi-OD freeway network, presented in Figure 5.6. This 27km long freeway consists of 6 OD pairs, 9 links, 4 on-ramps and 4 off-ramps. Each of these links has either 2 or 3 lanes as shown in Figure 5.6 where one lane in each link, showed in green colour, belongs to the set of candidate AV lanes (A_{av}) which could potentially converted into an AV-exclusive lane for better system performance. As each link consists of only one candidate AV-lane, a subsequent conversion of these candidate AV-lanes into AV-exclusive lanes will not necessitate multiple lane changes from LVs to avoid entering designed AV-exclusive lanes. The total vehicular demand is considered to be 3000 with an analysis period of 50 mins where the demand is loaded into the network through the 4 on-ramps over a period of first 10 mins of the analysis. The free-flow speed (90 km/hr), capacities (2160 and 4320 veh/hr for regular and

AV-exclusive lanes respectively) and the backward wave speeds (12.2 and 28.4 km/hr for regular and AV-exclusive lanes respectively) are considered the same as the single OD network (Figure 5.2) to match the fundamental diagram of traffic flow, showed in Figure 5.1. The traffic flow propagation is captured every minute based on the lane-based LTM.

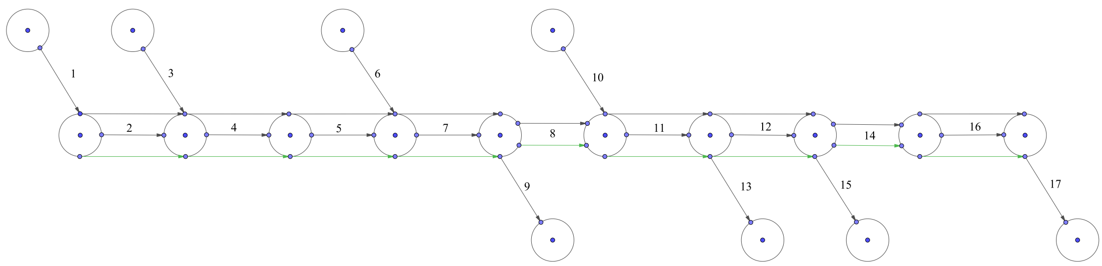

Figure 5.6: Multi-OD network

Here, 7 experiments are designed to study the performance of the proposed algorithm. Apart from the base case, the algorithm is implemented for different demand (±25%), capacity of AV lanes (±25%) and coefficient of average AV travel times (±25%). The performance of Algorithm 1 for these different scenarios is presented in Table 5.3.

Table 5.3: Performance of Algorithm 1 on a multi-OD freeway network for different scenarios

Parameter	Base	-25% Demand	+25% Demand	-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
Nb of converted AV lanes	7(9)	7(9)	6(9)	5(9)	8(9)	7(9)	7(9)
CPU time (mins)	119.88	71.57	144.52	26.51	255.94	99.98	98.45
Nb of Benders (m)	55	64	40	14	144	53	49
Nb of FP iterations (n)	269	166	395	73	817	261	237
Nb of FP per Benders (n/m)	4.89	2.59	9.88	5.21	5.67	4.92	4.84
TSTT (veh-sec)	2248830	1452190	3255250	2316410	2216510	2247660	2250430
%reduction in TSTT	11.43	9.41	12.16	8.77	12.71	11.48	11.37

Table 5.3 shows that it is not beneficial to convert all candidate AV lanes to AV-exclusive lanes. For example, 7 out of 9 candidate lanes are converted to AV-exclusive

lanes for the base case which remains the same for the reduced demand case as well. However, for an increased demand, a lesser number of lanes are converted to AV lanes (6). This could be due to increased LV demand which requires more regular lanes in the network. In case of an increased AV lane capacity, the system performance is significantly enhanced by allocating more AV-exclusive lanes (8) compared to the reduced AV-lane capacity case (5). On the other hand, the coefficient of average AV travel times is found to have no effect on AV lane design in the network. These deployed AV-exclusive lanes are found to decrease the TSTT by around 10% in all the scenarios, with a maximum improvement of 12.71% for increased AV-lane capacity. Hence, AV-exclusive lanes are always found to have a positive impact in network performance.

The computation time of the proposed algorithm varied from around 0.5 to 4 hours on an Intel(R) Core(TM) i7-6700 @3.40GHz CPU with 16GB RAM. The base case was found to converge within 2 hours with 55 iterations of Benders and a total of 269 fixed-point iterations. Interestingly, the change in the capacity of AV-exclusive lanes is found to have the maximum effect on computation time of the proposed algorithm with a 10 times increment in computation time for inflated capacity compared to the deflated capacity of AV lanes. Similar increment in the number of Benders and fixed-point iterations is also observed in these cases with inflated and deflated capacities.

Table 5.4: Optimal proportion of AVs for each OD pair for different scenarios

OD pairs	Initial $p^{o,d}$	Optimal $p^{o,d}$						
		Base	-25% Demand	+25% Demand	-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
(1,13)	0.33	0.73	0.70	0.76	0.74	0.72	0.82	0.62
(3,9)	0.41	0.63	0.63	0.62	0.64	0.63	0.68	0.57
(3,17)	0.28	0.76	0.75	0.75	0.76	0.75	0.83	0.64
(6,15)	0.38	0.66	0.65	0.70	0.66	0.67	0.74	0.57
(10,15)	0.44	0.65	0.58	0.68	0.63	0.64	0.69	0.61
(10,17)	0.38	0.70	0.64	0.73	0.68	0.70	0.76	0.64

Table 5.4 summarizes the optimal proportion of AVs ($p^{o,d}$) obtained from the endogenous demand model in different scenarios. The values of $p^{o,d}$ is initialised by substituting free-flow travel times of vehicles in the logit model Eq. (5.4). Due to the structure of the logit model, the initial $p^{o,d}$ values are found to be inversely proportional to the free-flow travel time between OD pairs and vary from 0.28 to 0.44. As the free-flow travel times on regular and AV-exclusive lanes are the same, this variation is solely due to the difference in the coefficient of travel times in the logit model. In congested traffic conditions the proposed algorithm estimates these $p^{o,d}$ to be around 0.57 to 0.83. In the base case, the OD pair (3,17) is found to have the maximum value of $p^{o,d}$ with a value of 0.76, whereas the OD pair (3,9) requires the least proportion with a value of 0.63. With a reduction in total demand, this proportion is found to be reduced by a maximum of 10%. Whereas, with increased demand, the maximum increment in $p^{o,d}$ is only around 4.6%. This could be due to the increased total demand which require fewer AV-exclusive lanes, as shown in Table 5.3, to cater for the increased LV demand. Hence, increasing $p^{o,d}$ may not provide much improvement in system level performance. On the other hand, $p^{o,d}$ values are not found to be affected significantly with inflated and deflated capacity of AV-exclusive lanes. However, the different coefficients of the average travel times of AVs (β_{av}) are found to considerably affect the AV proportions. A reduction in this coefficient means that average travel time will create less disutility for AVs rendering this mode more attractive than AVs and vice-versa. The output from the proposed algorithm shows similar effect with a maximum increment of 12% in $p^{o,d}$ for a 25% reduction in β_{av} and a maximum of 15.7% decrease with an increment in β_{av}.

The average travel times LVs and AVs on each link for all the 7 experiments are presented in Tables 5.5 and 5.6. As mentioned earlier, the proposed algorithm triggers 7 out 9 candidate AV lanes into AV-exclusive lanes. These exclusive lanes are found to have a significant effect on the average travel times of LVs with a maximum reduction of 45% for the base case. Similar improvement is observed for other demand scenarios as well with a maximum reduction of 41 and 46% respectively for LVs. These maximum reductions are mainly observed on the waiting times of vehicles on on-ramps. With a deflated capacity of the proposed AV lanes, the average reduction of average travel times on congested links is found to be around 8% with a maximum reduction of 23% in the first on-ramp of the network. Whereas, the inflated capacity provided a average reduction of 12% in the travel times with a maximum reduction of 52%. In the case with reduced $\beta^{o,d}_{\tau_{av}}$ value, the average reduction of travel times for LVs are found to be 5.6% compared to the case with increased $\beta^{o,d}_{\tau_{av}}$ value where this reduction is 14%. This output is quite intuitive as the reduced a $\beta^{o,d}_{\tau_{av}}$ will create less impedance on the mode choice decisions of users based on AV travel times, leading to more AVs in the network and less travel time benefits of for LV users.

Table 5.5: Changes in the average travel times of LVs in different traffic scenarios

Links	Base		-25% Demand		+25% Demand		-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
	Initial	Optimal	Initial	Optimal	Initial	Optimal	Optimal	Optimal	Optimal	Optimal
1	277.88	174.828	151.166	106.739	406.912	240.976	212.511	131.875	131.725	183.033
2	181.729	185.284	146.044	120.791	220.085	262.399	193.588	178.963	240.452	170.738
3	306.436	165.757	205.679	120.054	409.129	219.676	193.067	166.291	147.396	197.611
4	140.619	137.943	130.73	121.139	147.593	152.443	160.908	139.039	149.653	139.45
5	147.981	142.416	124.657	127.542	141.599	130.418	139.193	140.259	131.899	133.287
6	169.538	147.117	120.405	109.776	317.411	294.197	133.437	191.978	207.007	119.886
7	134.415	129.13	121.8	122.169	153.188	124.741	123.178	124.885	127.869	122.729
8	131.674	121.193	126.358	121.159	128.367	128.853	125.93	121.148	120	120
10	210.512	235.873	144.859	84.8162	384.315	352.003	225.173	205.47	247.179	227.634
11	120	120	120	120	120	120	120	120	120	120
12	120	120	120	120	120	120	120	120	120	120
14	120	120	120	120	120	120	120	120	120	120
16	120	120	120	120	120	120	120	120	120	120

With the proposed AV-exclusive lanes, the average travel times of AVs are found to be reduced by an average of 15%. Note that, these average travel times correspond to AVs on both regular and exclusive lanes. The maximum reduction of 60% is observed for the waiting time on source connector 10. The average reductions in travel times are around 16.4% and 13.4% for the reduced and increased demand cases respectively. The deflated and inflated capacities of the AV-exclusive lanes reduced the average AV travel times of AVs by 12.8 and 17.5% respectively with a maximum reduction of 64% for the inflated capacity case. With an decreased and increased $\beta^{o,d}_{\tau_{av}}$ the average reduction in the average travel times are around 16.4 and 15% respectively.

Table 5.6: Changes in the average travel times of AVs in different traffic scenarios

Links	Base		-25% Demand		+25% Demand		-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
	Initial	Optimal	Initial	Optimal	Initial	Optimal	Optimal	Optimal	Optimal	Optimal
1	228.002	194.321	107.748	83.7857	386.231	279.905	216.835	183.595	173.859	195.476
2	174.544	174.023	146.698	139.789	211.304	201.096	190.852	162.101	174.372	169.77
3	244.744	202.592	165.435	110.367	362.77	309.514	213.117	188.538	192.961	205.334
4	144.699	139.908	128.386	124.517	166.232	144.193	152.298	132.189	148.148	140.269
5	132.483	128.633	131.94	126.396	140.082	136.263	120.497	140.006	130.037	138.55
6	189.882	147.155	145.978	95.2195	240.574	254.529	105.3	145.076	145.994	146.61
7	148.247	126.342	127.908	120	167.794	126.85	123.784	133.235	124.268	126.927
8	134.795	127.385	121.697	122.503	159.399	139.126	136.804	124.822	122.682	123.774
10	331.996	138.413	128.935	77.064	353.871	260.086	185.354	118.813	149.424	138.433
11	120	120	120	120.245	120	120.416	120.264	120.408	120.219	120
12	120	120	120	120	120	120	120	120	120	120
14	120	120	120	120	120	120	120	120	120	120
16	120	120	120	120	120	120	120	120	120	120

5.5 Conclusion

In this chapter, an optimization framework is proposed to solve a multi-OD freeway network design problem for optimal lane design under endogenous AV demand. Due to the presence of binary lane design variables and the endogenous demand model, the proposed formulation results in a non-convex MINLP. This challenging problem is tackled by introducing Benders' decomposition approach which iteratively explores possible lane designs in the master problem and at each iteration solves a sequence of SODTA problems which is shown to converge to fixed-points representative of logit-compatible demand splits. The proposed approach is implemented on two hypothetical freeway networks with single and multiple origins and destinations. On the single-OD network, the fixed-point algorithm is found to converge at multiple fixed points providing different proportions of AV demand. However, these multiple fixed-points are found to have no effect on the objective function of the problem. Here, it is proven that for a fixed lane design, there exists at least one fixed-point representing the proportion of AV demand in the network. It is also proven that the proposed solution method converges to a local optima of the nonconvex problem and identify under which conditions this local optima is a global solution.The numerical results on the multi-OD network show that it is not beneficial in terms of system performance to provide AV lanes for all the links in the network.

It is observed that the optimal lane design of freeway network is non-trivial and can inform on the value of accounting for endogenous demand in the proposed freeway network design. The proposed model may also be useful for designing ramp metering for a freeway network with AV-exclusive lanes.

Chapter 6

Conclusion

This thesis began with three aims:

1. Provide a mathematical foundation for developing a framework for network-level analysis of traffic flow.

2. Explore the usefulness of the proposed model for various network-level design problems with advanced congestion mitigation strategies.

3. Explore the practicality of the proposed model for futuristic transport scenarios in an automation heavy network.

These aims are pursued in the three core chapters of this thesis. This chapter summarises the findings of this study while achieving these aims along with identifying the contributions to the existing literature, future research directions and final remarks.

6.1 Summary

This thesis is dedicated to explore the development, analysis and implementation of a novel system optimum dynamic traffic assignment (SODTA) framework. The framework embeds link transmission model (LTM) for dynamic network loading and traffic flow propagation, hence the name LTM-SODTA. After introducing the research problem with relevant background and gaps in the existing literature in Chapters 1 and 2, this formulation is developed in Chapter 3, providing the foundation to the entire thesis. In this chapter, the proposed model is implemented on single-OD and multi-OD networks, to illustrate its applicability. The subsequent Chapters 4 and 5 present four distinct applications of LTM-SODTA which illustrates the versatility of the model. These four distinct applications are: network design with capacity enhancement, travel demand management with departure time incentives and shared mobility services, optimal dedicated lane allocation for vehicles with automation under endogenous demand. In this section, a chapter-wise summary of the three core chapters of this thesis is presented.

In Chapter 3, an LTM based LP formulation is proposed and implemented on a single-OD and multi-OD network to solve a DTA model under SO traffic conditions. Compared to the original LTM, the proposed LTM-SODTA approach optimizes the turning ratios at each node of the network at each time step for an optimal system performance. The optimal solution flows from the two formulations are found to be consistent for describing free-flow as well as congested states of traffic accurately in terms of propagation of backward shock-waves, queuing of vehicles and optimal TSTT value. The objective function of the model also accounts for waiting time of the vehicles queued up at the entrance of a network. The proposed LP relaxes the strict minimum constraints on sending and receiving flows described in the original LTM theory using linear inequalities. The objective function handles this relaxation by penalizing vehicles' travel time in the network. During implementation of the LTM-SODTA on the example networks, the set of linear inequalities are found to be a suitable alternative to the strict minimum constraints, thus providing a valid LTM flow pattern. The proposed LTM-SODTA model in this chapter provides the foundation of this thesis. The subsequent chapters are built on this foundation to analyse various network design strategies such as capacity enhancement, departure-time incentive design, shared mobility service design and optimal lane allocation for autonomous vehicles in a mixed traffic network.

Chapter 4 encapsulates three out of the four applications mentioned above. The first application involves solving a network design problem (NDP) with potential capacity enhancement and compared with its cell transmission model (CTM) based NDP on an example network. Compared to the single-destination SODTA based on the CTM, the LTM-NDP formulation is found to involve considerably less decision variables, thus potentially providing a more scalable approach. Further, the model output of the CTM-NDP showed that there is no incentive to allocate non-uniform budget (leading to non-uniform capacity improvement) to the cells of the same link as the cell transfer flow is limited by the cell with the smallest capacity. This finding further advocates the use of LTM-NDP over CTM-NDP in terms of optimum budget allocation for a network design problem.

The LTM-SODTA model is further developed to design departure-time incentives for congestion mitigation and implemented on a multi-OD test network. A novel method of endogenous estimation of travel time is developed and adopted for incentivising the departure-time shift of commuters for an overall improvement in system performance. Only 50% of the demand is incentivised for leaving early or late from their respective origins. The model output shows that incentives have significant effect in reducing OD travel times. The OD-demand profiles are found to have a flattening effect as the total budget for incentive increases. The departure-time shift variable is found to shift most of the demand to the last departure-time in case of maximum budget, indicating an even spread of incentivised and non-incentivised demand throughout the demand loading period.

Chapter 4 concludes with another development of the LTM-SODTA model where a shared mobility service is designed where commuters are incentivised to share their rides based on the impact on their arrival times. These arrival times are calculated based on the endogenously estimated travel times. The model is implemented on a multi-OD test network. The model output shows that with increasing budget for incentivisation, more number of commuters share their rides leading to less number of cars in the network with better system performance.

Finally in Chapter 5, an optimization framework is proposed to solve a multi-OD freeway network design problem for optimal lane design under endogenous demand for vehicles with automation. Due to the presence of binary lane design variables and the endogenous demand model, the proposed formulation results in a nonconvex MINLP. This challenging problem is tackled by introducing Benders' decomposition approach which iteratively explores possible lane designs in the master problem and at each iteration solves a sequence of SODTA problems which is shown to converge to fixed-points representative of logit-compatible demand splits. The proposed approach is implemented on two hypothetical freeway networks with single and multiple origins and destinations. On the single-OD network, the fixed-point algorithm is found to converge at multiple fixed points providing different proportions of vehicles with automation. However, these multiple fixed-points are found to have no effect on the objective function of the problem. Here, it is proven that for a fixed lane design, there exists at least one fixed-point representing the proportion of vehicles with automation in the network. It is also proven that the proposed solution method converges to a local optima of the non-convex problem and identify under which conditions this local optima is a global solution. The numerical results on the multi-OD network show that it is not beneficial in terms of system performance to provide dedicated lanes for vehicles with automation for all the links in the network.

6.2 Contributions

This thesis makes the following contributions, mainly methodological, to the existing literature. A fully-functional, versatile and adaptable optimisation framework for SODTA is developed and explored on four different applications. Three out these four applications: network design with capacity enhancement and travel demand management with departure time incentives and shared mobility service, are relevant to the existing traffic conditions whereas the fourth one is applicable to a futuristic traffic scenario with automation technologies. This thesis records all the necessary attempts to maintain linearity throughout all the formulations by exploiting the mathematical structure of the constraint sets.

Although, the formulation for network design with capacity enhancement might be a classical example of solving network design problems, both departure-time incentives and shared mobility service embed a novel method of endogenous travel time estimation which dictates the incentives in a sensible manner to either shift departure times or stimulate ride-sharing behaviours in commuters.

Further, an optimisation framework is proposed, involving an optimal design of dedicated lanes for automated vehicles on a freeway network based on the demand of regular and automated vehicles which are endogenously calculated with a demand model embedded in the formulation. With an extensive review of existing literature, it is observed that an integrated framework of this kind has not been explored before. The computational complexity of such a non-convex formulation is dealt with a Benders' decomposition approach in which the sub-problem embeds a fixed-point procedure to account for the endogenous demand model, retaining the linear structure of the underlying traffic flow model. The existence of a fixed-point is proven and the proposed algorithm always converges to this fixed-point. Along this line, conditions are identified under which the proposed algorithm is globally optimal. This algorithm is implemented on a freeway network which reveals that deploying a maximum of exclusive autonomous vehicles does not necessarily minimize network travel time.

It is observed that the optimal lane design of freeway network is non-trivial and can inform on the value of accounting for endogenous demand in the proposed freeway network design. The proposed model may also be useful for designing ramp metering for a freeway network with exclusive lanes for vehicles with automation.

6.3 Extensions and future research directions

This thesis opens door to various interesting problems. A few of these extensions are listed as follows:

- One of the major limitations of SODTA models is the computational complexity hindering their application on large-scale networks. Any innovative algorithm to speed up the computation time of these models would be immensely helpful. These novel algorithms coupled with the upcoming automation features in vehicles may lead to real-time city-wide applications of SODTA models.
- As SO route choice incites questions related to realistic representation of travel behaviour, one of the most immediate directions for future research would be an investigation of user equilibrium (UE) vs SO route choice in general networks with mixed vehicular interactions.
- Future models may attempt to model lane-changing behaviour of vehicles as well leading to a more practical traffic representation.
- In this thesis, it is assumed that the first-in-first-out (FIFO) characteristic of vehicles are maintained without any imposed constraints. This may be explored with more details by comparing with a formulation with strict FIFO constraints.

- Vehicle holding is another well-known issue in SODTA models where vehicles wait at the nodes of a network even with available capacities at downstream links. With additional penalty terms in the model, this problem can be addressed. However, the resulting model may not retain its linear structure.

- In Chapter 3, LTM is claimed to demonstrate better computational efficiency and scalability over CTM with reduced number of variables and constraints on a small example network. It would be interesting to compare the performance of both models on large-scale networks in future.

- In this thesis, the departure-time incentive model, presented in Chapter 4, assumes that a certain percentage of the entire demand is compliant to shift their departure times. However, one can analyse the effect of this compliance rate with different percentages of the demand willing to shift their departure times.

- The shared mobility service model presented in this thesis, assumes that the entire travel demand is willing to share their rides. In future, this assumption can be relaxed by providing bounds on the vehicle occupancy variable or by allowing a fixed portion of demand to share their rides.

- In this thesis, the shared mobility service model is also simplified by allowing only passengers with same origin, destination, and departure time to share their rides. A future extension of this model might include complex ride-sharing services involving picking up and dropping off passengers as well.

- In Chapter 5, the total demand for the AV-exclusive lane design problem is considered fixed with variable class-wise demand. Future research may include the analysis of the impact of such AV-exclusive lane design on total demand in the network.

The possibilities of extending the applications of SODTA models are wide-ranging as these models are highly flexible tools for modelling network-level applications of innovative transport strategies. With the arrival of innovative transport concepts such as Mobility as a Service (MaaS), these models will hold great potential in near future in terms of mitigating traffic congestion with informed investment on network infrastructures driving the current traffic scenario towards a better future.

6.4 Final remarks

The motivation of this thesis was to devise a transportation network optimisation framework for social welfare with an SODTA model. It is evident that the system optimum routes generated by SODTA models have their limitations while representing realistic traffic behaviour. From the existing literature, it is realised that these limitations might have restricted extensive application of SODTA models. However, with the gradual advent of automation technologies in the transport industry, these models might become more and more relevant due to enhanced coordination and cooperation between supply (network infrastructure) and travel demand in the near future. Hence, it has never been timely enough to explore such traffic flow models with network-level objectives in a comprehensive manner to prepare the ground for the current transport system to reap maximum benefit from such advanced technologies. The main purpose of this thesis has been to contribute to this expedition towards future mobility.

It is hoped that the SODTA formulations presented in this thesis will kindle further interest in system-level network analysis in future researchers and practitioners to develop their own models, and that the four developments formulated in this thesis can be used to improve the efficiency of transport network operations along with informing infrastructure investments for social welfare.

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