Essays on concentration, size, performance, and benefits of the active fund management industry

Author:
Xu, Jingrui

Publication Date:
2017

DOI:
https://doi.org/10.26190/unsworks/3153

PLEASE TYPE

THE UNIVERSITY OF NEW SOUTH WALES
Thesis/Dissertation Sheet

Surname or Family name: **Xu**

First name: **Jingrui** Other name/s:

Abbreviation for degree as given in the University calendar: **PhD**

School: **School of Banking and Finance** Faculty: **UNSW Business School**

Title: **Essays on Concentration, Size, Performance, and Benefits of the Active Fund Management Industry**

Abstract 350 words maximum: (PLEASE TYPE)

We introduce a model of the active fund management industry (AFMI), and study the effect of a continuum of exogenous market concentration levels on the AFMI's performance (net alphas), AFMI size, and AFMI direct benefits (differences between performance benefits and costs of efforts exerted to produce them). Risk-neutral managers compete for many risk-averse investors' investments by optimally choosing costly effort and fees to deliver net alphas. Our model predicts that AFMI performance, size, and direct benefits increase (decrease) with market concentration if and only if gains from better investment opportunities due to higher concentration exceed (fall behind) the consequences of higher managerial costs. We then model endogenous concentration levels to fit empirical concentration measures. We empirically study the U.S. equity AFMI and find that, on average, AFMI net alphas and size increase with market concentration. Under current U.S. equity AFMI's low market concentration, and with no change in managerial productivity/effort opportunity costs, increasing market concentration is likely to increase the AFMI's net alphas, size, and direct benefits.

We extend our model to global settings. Higher foreign market concentration, implying more unexplored investment opportunities, makes managerial effort more productive, attracting efforts. Consequently, unexplored investment opportunities in local markets rise, increasing local effort productivity. However, higher foreign market concentration allows foreign managers to demand higher compensation, driving up local managers' reservation salaries, thus, effort costs. In equilibrium, higher foreign market concentration levels induce higher local net alphas, AFMI size, and the sum of direct benefits of managerial efforts spent in local and foreign markets if and only if gains from higher gross alpha production exceed consequences of higher managerial costs. Again, to fit data, we specialize our international model to an endogenous concentration framework and empirically study the effect of U.S. equity AFMI concentration on 30 global equity AFMIs. We find that 17 (5) markets' AFMI net alphas and 9 (2) markets' AFMI size, on average, are significantly negatively (positively) associated with U.S. equity AFMI concentration. Under no change in managerial productivity/effort costs, the current low, and probably decreasing, U.S. equity AFMI concentration would benefit a large proportion of global equity AFMIs.

Declaration relating to disposition of project thesis/dissertation

FOR OFFICE USE ONLY Date of completion of requirements for Award:

Essays on Concentration, Size, Performance, and Benefits of the Active Fund Management Industry

Jingrui Xu

A thesis in fulfilment of the requirements for the degree of Doctor of Philosophy

UNSW AUSTRALIA

School of Banking and Finance

UNSW Business School

March 2017

ORIGINALITY STATEMENT

'I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or substantial proportions of material which have been accepted for the award of any other degree or diploma at UNSW or any other educational institution, except where due acknowledgement is made in the thesis. Any contribution made to the research by others, with whom I have worked at UNSW or elsewhere, is explicitly acknowledged in the thesis. I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation and linguistic expression is acknowledged.'

COPYRIGHT STATEMENT

AUTHENTICITY STATEMENT

Abstract

We introduce a model of the active fund management industry (AFMI), and study the effect of a continuum of exogenous market concentration levels on the AFMI's performance (net alphas), AFMI size, and AFMI direct benefits (differences between performance benefits and costs of efforts exerted to produce them). Risk-neutral managers compete for many risk-averse investors' investments by optimally choosing costly effort and fees to deliver net alphas. Our model predicts that AFMI performance, size, and direct benefits increase (decrease) with market concentration if and only if gains from better investment opportunities due to higher concentration exceed (fall behind) the consequences of higher managerial costs. We then model endogenous concentration levels to fit empirical concentration measures. We empirically study the U.S. equity AFMI and find that, on average, AFMI net alphas and size increase with market concentration. Under current U.S. equity AFMI's low market concentration, and with no change in managerial productivity/effort opportunity costs, increasing market concentration is likely to increase the AFMI's net alphas, size, and direct benefits.

We extend our model to global settings. Higher foreign market concentration, implying more unexplored investment opportunities, makes managerial effort more productive, attracting efforts. Consequently, unexplored investment opportunities in local markets rise, increasing local effort productivity. However, higher foreign market concentration allows foreign managers to demand higher compensation, driving up local managers' reservation salaries, thus, effort costs. In equilibrium, higher foreign market concentration levels induce higher local net alphas, AFMI size, and the sum of direct benefits of managerial efforts spent in local and foreign markets if and only if gains from higher gross alpha production exceed consequences of higher managerial costs. Again, to fit data, we specialize our international model to an endogenous concentration framework and empirically study the effect of U.S. equity AFMI concentration on 30 global equity AFMIs. We find that 17 (5) markets' AFMI net alphas and 9 (2) markets' AFMI size, on average, are significantly negatively (positively) associated with U.S. equity AFMI concentration. Under no change in managerial productivity/effort costs, the current low, and probably decreasing, U.S. equity AFMI concentration would benefit a large proportion of global equity AFMIs.

Acknowledgements

I want to express my sincere appreciation to Professor David Feldman, my supervisor, who spent numerous hours in checking my work, discussing it with me, and offering me guidance. I also want to thank Dr. Konark Saxena, my co-supervisor, for providing advice on my research. Moreover, I thank participants of my Brown Bag seminars in the School of Banking and Finance, who allocated their valuable time to discussing my thesis topics, methodologies, and presentations.

I appreciate the courses offered by the School of Mathematics and Statistics, the School of Economics, and the School of Banking and Finance, which helped me to develop a solid academic background so that I could complete this thesis and can do good research in the future. I thank my teachers for offering me high-quality education.

The government of New South Wales offered me a scholarship so that I could focus on my research. The School of Banking and Finance of University of New South Wales provided facilities, databases, funding, and other relevant support for me to complete my research. I appreciate all of these.

I thank Linda, who provided proof-reading service for my thesis. I also thank my thesis examiners for checking my thesis and offering helpful comments.

The chapters of this thesis have been modified into two working papers, "Is the Active Fund Management Industry Concentrated Enough?" and "Concentration Effects and Cross-Effects in the International Active Fund Management Industry (Tentative Title)." The former was submitted to the *Journal of Financial Economics* and earned a "revise and resubmit" invitation. It was also presented in the following conferences: Australasian Finance and Banking Conference, Midwest Finance Association Annual Meetings, International Conference of the French Finance Association, International Conference of the Financial Engineering and Banking Society, Portuguese Finance Network International Conference, Spanish Finance Association Finance Forum, Centre for Analytical Finance, ISB, Hyderabad, Summer Research Conference in Finance, World Finance Conference, Behavioral Finance and Capital Market Conference (Best Paper Award), Annual Paul Woolley Centre Conference, and FIRN Annual Conference. The second paper was presented in Australasian Finance and Banking Conference. I thank the *Journal of Financial Economics* referee and the conference participants for helpful comments and suggestions.

I solely completed the majority of this thesis, following the requirements of the

University of New South Wales Conditions for Award of Doctor of Philosophy Policy. My supervisors, Professor David Feldman and Dr. Konark Saxena contributed to the research by offering discussions of ideas, directions, and editorial work.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

AFMI: active fund management industry

IAFMI: international active fund management industry

PS: Pastor and Stambaugh (2012)

PST: Pastor, Stambaugh, and Taylor (2015)

RA: risk-averse

RD: recursive demeaning

RN: risk-neutral

VAR: vector auto-regression

Chapter 1: Introduction

The global active fund management industry (AFMI) has controlled a large proportion of social wealth and is becoming more and more important in the global economy. Economic theories tell us that the level of competition (or concentration[1]) of an industry affects its efficiency. Thus, we want to ask how, and by which mechanism, does an AFMI' market competition (concentration) level affect its characteristics, such as performance, size, managerial effort, and fees. Extensive literature on the AFMI has focused on studying the economic forces that can explain fund manager compensation, their ability to generate value, and the exponential growth of an industry where identifying economic value added seems elusive.[2] However, the study of the relations between AFMI market concentration and its other characteristics is limited, and to our best knowledge, our topic is a new to the literature.

In Chapter 2, we define market concentration to reflect the market competitiveness level and introduce a model of AFMI concentration, effort[3], fees, size, and performance. Two hypotheses about the effect of market concentration prevail in the banking literature: the efficient-structure hypothesis, which suggests a positive relation between market concentration and firm efficiency, and the structure-conduct-performance hypothesis, which asserts a positive relation between market concentration and firms' performance due to extractions of monopolistic rents. We note that these two hypotheses are not necessarily mutually exclusive.[4] Following these hypotheses, we expect higher market concentration to

1) leave more unexplored investment opportunities[5] and allow fund managers to more efficiently use industry resources, such as human capital, inducing higher marginal effort efficiency; and

[1] We use *concentration* and *competition* as opposites.
[2] See, for example, Jensen (1968), Daniel, Grinblatt, Titman and Wermers (1997), Wermers (2000), Berk and Green (2004), Chan, Covrig and Ng (2005), Khorana, Servaes and Tufano (2005), Khorana, Servaes and Tufano (2008), Pastor and Stambaugh (2012), Ferreira, Keswani, Miguel and Ramos (2012a) and (2012b), and Berk and Binsbergen (2015).
[3] The *effort* in our model may be regarded as an effort-skill combination.
[4] These hypotheses and some related hypotheses, such as relative market power hypothesis, are discussed and tested, for example, in Berger and Hannan (1989), Berger (1995), and Goldberg and Rai (1996).
[5] An example for increasing productivity with concentration is as follows. Suppose that gold has been found and a handful of diggers yield high returns with little effort. As the number of diggers increases, the area available to each digger decreases, and with it diggers' "productivity", even under optimal effort. Fund managers are like the gold diggers; they seek net alpha, but as their number increases, unexplored opportunities decline concomitant with their productivity.

2) facilitate an increase in the opportunity cost of effort, inducing fund managers to require more compensation for effort.

Pastor and Stambaugh (2012), (PS), studied a framework with decreasing returns to scale (i.e., decreasing active managers' marginal ability to outperform passive benchmarks), where interactions between active fund managers and investors determine expected net alphas (net of management fees). Within their world, we model a continuum of market concentration levels and allow active fund managers to (optimally) exert costly effort when competing over investment funds by producing net alphas. We study equilibria with four types of investors: a single risk-neutral investor, infinitely many risk-neutral investors, a single risk-averse investor, and infinitely many risk-averse investors.[6]

We define the AFMI's direct benefit (function) as the (equilibrium) increase in gross alpha induced by (optimal) efforts level minus the cost of these efforts, for given market concentration levels.[7,8] We show that equilibrium fund expected net alphas and AFMI size increase (decrease) with market concentration if and only if the AFMI's direct benefits increase (decrease) with market concentration. Also, we find that if equilibrium fund expected net alphas are concave in market concentration, then the AFMI's direct benefits are concave in market concentration. Consequently, equilibrium AFMI size is also concave in market concentration. On the other hand,[9] if equilibrium AFMI size is convex in market concentration, then the AFMI's direct benefits are convex in market concentration and, consequently, equilibrium expected fund net alphas are convex in market concentration.

We specialize our model to allow endogenous concentration levels. We argue that this endogenous framework befits empirical market concentration measures

[6] Each of these cases fits particular economies. The cases where investors are risk-neutral are consistent with the situation where households employ private banks or other financial institutions to manage their wealth. These institutions then invest in active funds. Acting as intermediaries, they mainly care about funds' expected returns so may be regarded as risk-neutral investors. The cases where there is a single investor in the market is consistent with the situation where the societal wealth is centrally managed by the government, which determines the allocation of investments to active funds and other securities.

[7] For brevity and simplicity, we use the term *benefits* in the general sense allowing for negative benefits.

[8] The AFMI, however, may have indirect benefits that we do not model here. For example, monitoring, studying, and analyzing firms might incentivize managements to improve governance and productivity, and to reduce agency costs. Also, active fund management may induce transfer of wealth from less productive firms/investors to more productive ones and, even within endowment economies, may increase investors' derived utilities by improving information processing and/or risk sharing.

[9] Please note that the order of statements of this argument is different from that in the previous one, for reasons explained below.

because empirically we usually use market concentration measures that are calculated based on endogenous fund sizes.

In Chapter 3, we empirically test the implications of our model developed in Chapter 2, by using U.S. data. In particular, we define the U.S. active equity mutual fund industry as the AFMI that we focus on, and study how its fund net alphas and size change with market concentration. We analyze the AFMI market concentration-net alpha relation, using Pastor, Stambaugh, and Taylor's (2015) (PST) recursive-demeaning estimator to address endogeneity and omitted-variable-related issues. We study the AFMI market concentration-AFMI size relation using vector auto-regression (VAR) techniques to account for simultaneity in the determination of AFMI size and market concentration. We control for survival bias by using the Morningstar U.S. mutual fund database, which contains both surviving and terminated funds.

We find that both fund net alphas and AFMI size, on average, increase with market concentration. Moreover, both fund net alphas and AFMI size are concave in concentration. Our empirical results are robust to the use of alternative methods and measures. Our empirical findings are consistent with our model's theoretical implications under plausible parameter values, and have policy implications. Given the low market concentration in the current AFMI, and assuming no change in the tradeoff of managerial productivity and effort cost, increasing market concentration is likely to increase both fund net alphas and AFMI size; under plausible parameter values, the AFMI's direct benefits also increase. The literature has shown multiple ways to increase fund market concentration. For example, Massa (2003) suggested that mutual fund families allow investors to move money across family funds of different categories at low costs, lowering effective fees and reducing competition among funds. Under our findings, the formation of fund families in such markets might generate benefits in terms of higher net alphas and larger AFMI size.

In Chapter 4, we extend our study, and pose questions under an international context: how, and by which mechanism, does a foreign fund market's concentration level affect a local fund market's characteristics, such as performance, size, managerial efforts and fees. To answer, we introduce a model of international active fund management industries (IAFMI) equilibria where performance, size, managerial efforts and fees, are endogenously determined under continua of exogenous local and foreign market concentration levels.

For simplicity, we consider a two-country international model. In each country, there is an AFMI with competing fund managers who are risk-neutral and who invest their portfolios in both local and foreign stocks, and with infinitely many investors who are mean-variance risk-averse and who allocate their wealth across a passive international benchmark portfolio (which includes both domestic and foreign stocks) and local active funds. We deem this framework realistic because, in reality, due to high transaction costs in foreign countries, investors prefer local funds to foreign funds, whereas fund managers, facing low transaction costs, choose securities across countries. As before, we assume decreasing returns to scale in producing gross alphas at the fund and industry levels.

Our model allows fund managers, competing in net alpha productions, to spend two types of efforts: exploring investment opportunities in the local stock market and exploring investment opportunities in the foreign stock market. We expect gross alpha production and costs of managerial efforts to depend on concentration levels. In particular, we expect that higher local AFMI market concentration implies more unexplored investment opportunities in the local stock market, making effort spent in the local stock market more productive; at the same time, it allows local fund managers to ask for higher compensation for efforts spent in both the local and the foreign stock markets, increasing foreign managers' reservation prices of efforts spent in both stock markets, and thus increasing effort costs. Moreover, although higher local AFMI market concentration does not directly affect the productivity of effort spent in the foreign stock market, in equilibrium, it attracts managerial efforts to the local stock market, which has more unexplored investment opportunities. As a result, it leaves more unexplored opportunities in the foreign stock market, making effort spent in the foreign stock market more productive. Similarly, higher foreign AFMI market concentration implies more unexplored investment opportunities in the foreign stock market, making effort spent in the foreign stock market more productive. At the same time, it allows foreign fund managers to ask for higher compensation for efforts, increasing local managers' reservation prices of efforts and making efforts spent in both the local stock and the foreign stock markets more costly. Although higher foreign AFMI market concentration does not directly affect the productivity of effort spent in the local market, in equilibrium, it attracts managerial efforts to the foreign stock market, which has more unexplored investment opportunities. Thus it leaves more unexplored opportunities in the local stock market, making effort spent in the local stock market more productive.

We call the improvements in gross alpha production due to efforts minus the costs of the efforts, as AFMI direct benefits of efforts. We show that, in equilibrium, if and only if higher local AFMI market concentration exerts stronger (weaker) effects on gross alpha production due to efforts spent in the local and the foreign stock markets than on costs of these two types of efforts, i.e., if the sum of changes in direct benefits of these two types of efforts is positive (negative), then it induces higher (lower) local AFMI fund expected net alphas and AFMI size. Similarly, in equilibrium, if and only if higher foreign AFMI market concentration exerts stronger (weaker) effects on gross alpha production of these two types of efforts than on costs of them, i.e., if the sum of changes in direct benefits of these two types of efforts is positive (negative), it induces higher (lower) local AFMI fund expected net alphas and AFMI size.

To befit empirical market concentration measures, we specialize our model to allow endogenous concentration levels. We show that in equilibrium, although the relation between local AFMI market concentration and local fund expected net alphas, and the relation between local AFMI market concentration and local AFMI size are more complex, we still conclude that local AFMI fund expected net alphas and AFMI size move in the same direction with foreign AFMI market concentration.

Using similar methodologies as those in Chapter 3, we study 30 active equity mutual fund markets (which we define as AFMIs) and analyze how these markets' fund net alphas and AFMI size change with the local and the U.S. AFMI market concentrations. We find that, 17 (5) markets' fund net alphas, on average, are significantly negatively (positively) associated with the U.S. AFMI market concentration. while 9 (13) markets' fund net alphas, on average, are significantly negatively (positively) associated with the local AFMI market concentration. Also, we find that only 9 (2) markets' AFMI size, on average, are significantly negatively (positively) associated with the U.S. AFMI market concentration while only 7 (7) markets' AFMI size, on average, are significantly negatively (positively) associated with the local one. More importantly, we find that 15 (5) markets' fund net alphas and AFMI size are both, on average, negatively (positively) associated with the U.S. AFMI market concentration, and among them, 7 (1) markets' fund net alphas and AFMI size are both significantly negatively (positively) associated with it. When pooling all the markets' data together, we find that, on average, fund net alphas and AFMI size are both significantly negatively associated with the U.S. AFMI market concentration, but are insignificantly associated with the local AFMI market concentration. The fact that global fund markets' fund net alphas and AFMI size tend to move in the same direction with the U.S. AFMI market concentration is consistent with our theoretical implications.

Our findings in this chapter provide relevant implications for fund managers, investors, and regulators: the current low and probably decreasing market concentration in the U.S. AFMI, given the trade-off of higher U.S. AFMI market concentration is not changed, would benefit (harm) the global AFMI markets whose fund net alphas and AFMI size are, on average, negatively (positively) associated with the U.S. AFMI market concentration. Our results show that a large proportion of the global AFMI markets in our sample would benefit from that.

Overall, this thesis theoretically and empirically studies how local and foreign AFMI market concentrations affect relevant AFMI characteristics, such as fund net alphas and AFMI size. The rest of this thesis is organized as follows. Chapter 2 introduces a model of AFMI. Chapter 3 studies the implications of this model using U.S. active mutual fund data. Chapter 4 extends our study and develops a model of IAFMI, and uses data of 30 global active mutual fund markets to empirically test the model's implications. Chapter 5 concludes and discusses future research.

Chapter 2: A Model of Market Concentration and the Active Fund Management Industry

2.1. Introduction

Two central underpinnings of free market economics are 1) competition leads to better outcomes and 2) agents earn economic rents if and only if they have a competitive advantage. Because the incentives of earning future economic rents are crucial in motivating people to act and because people need an environment where a competitive advantage can be created so they can earn future economic rents, it is natural to try to understand whether the level of competition (or concentration[10]) in a given industry is optimal. This question is at the core of a central financial economic issue: the efficiency of the active fund management industry (AFMI) equilibrium. Extensive literature on the AFMI has focused on trying to understand the economic forces that can explain fund manager compensation, their ability to generate value, and the exponential growth of an industry where identifying economic value added seems elusive.[11]

We introduce a model of AFMI competition, effort[12], size, and performance. Specifically, we note that competition among asset management firms has grown dramatically over the past few decades with advancements in financial products and technology (see, for example, Gruber (1996) and Philippon and Reshef, (2012)). Worldwide, vast numbers of active fund managers are estimating the value of assets each day. These highly trained experts act to exploit any perceived differential—however small—between price and estimated asset value, hoping to be compensated for their efforts. This phenomenon raises important questions. Clearly, one needs some active management to ensure that security prices properly reflect relevant information, but do market concentration levels in the AFMI optimally balance opportunities and costs of gross alpha production? Our model provides economic insights regarding two opposing forces that influence economic outcomes when the concentration level of AFMI changes: available gross alpha-production opportunities and the corresponding

[10] We use *concentration* and *competition* as opposites.
[11] See, for example, Jensen (1968), Daniel, Grinblatt, Titman and Wermers (1997), Wermers (2000), Berk and Green (2004), Chan, Covrig and Ng (2005), Khorana, Servaes and Tufano (2005), Khorana, Servaes and Tufano (2008), Pastor and Stambaugh (2012), Ferreira, Keswani, Miguel and Ramos (2012a) and (2012b), and Berk and Binsbergen (2015).
[12] The "effort" in our model may be regarded as an effort-skill combination.

effort costs.[13]

We define market concentration to reflect the market competitiveness level.[14] Two hypotheses about the effects of market concentration prevail in the banking literature: the efficient-structure hypothesis, which suggests a positive relation between market concentration and firm efficiency, and the structure-conduct-performance hypothesis, which asserts a positive relation between market concentration and firms' performance due to extractions of monopolistic rents. We note that these two hypotheses are not necessarily mutually exclusive.[15] Following these hypotheses, we expect higher market concentration to

1) leave more unexplored investment opportunities[16] and allow fund managers to more efficiently use industry resources, such as human capital, inducing higher marginal effort efficiency; and
2) facilitate an increase in the opportunity cost of effort, inducing fund managers to require more compensation for effort.

Our model allows incorporating these effects of market concentration, calibrating parameters with real-world data, and ascertaining which implications are consistent with market equilibrium.

Pastor and Stambaugh (2012), (PS), studied a framework with decreasing returns to scale (i.e., decreasing active managers' marginal ability to outperform passive benchmarks), where interactions between active fund managers and investors determine expected net alphas (net of management fees). Within their world, we model a continuum of market concentration levels and allow active fund managers to (optimally) exert costly effort when competing over investment funds by producing net alphas.[17] We study equilibria with four types of investors: a single risk-neutral investor, infinitely many risk-neutral investors, a single risk-averse investor, and infinitely many

[13] Managers' efforts' cost levels may be viewed, of course, as reflections of their skills.
[14] Our results hold for any $M,\ M > 1$. Determining M endogenously would not change our results.
[15] These hypotheses and some related hypotheses, such as relative market power hypothesis, are discussed and tested, for example, in Berger and Hannan (1989), Berger (1995), and Goldberg and Rai (1996).
[16] An example for increasing productivity with concentration is as follows. Suppose that gold has been found and a handful of diggers yield high returns with little effort. As the number of diggers increases, the area available to each digger, and with it diggers' "productivity," decreases, even under optimal effort. Fund managers are like gold diggers; they seek net alpha, but as their number increases, unexplored opportunities decline concomitant with their productivity.
[17] In the literature, market concentration levels are exogenous, or endogenous (see, for example, Aguerrevere (2009) and Ambrose, Diop, and Yoshida (2014)); but, here, assuming exogenous concentration helps put a focus on what we study.

risk-averse investors.[18]

Within an AFMI equilibrium, we study the impact of changes in market concentration on endogenous managerial costly effort, endogenous net alpha production, and endogenous AFMI size. As in PS, and for simplicity, we fix the number of funds and define AFMI size as the ratio of assets under active management to total wealth. We show that with infinitely many mean-variance risk-averse investors, the effects of higher market concentration depend on a tradeoff. Higher market concentration not only increases opportunities in gross alpha production (the marginal efficiency of managerial effort in producing gross alpha) but also increases managerial effort costs due to increase in opportunity costs. We further show that if the former effect dominates (is dominated by) the latter one, higher market concentration induces higher (lower) equilibrium fund expected net alphas and larger (smaller) AFMI size.

We define the AFMI's direct benefit (function) as the (equilibrium) increase in gross alpha induced by (optimal) efforts level minus the cost of these efforts, for given market concentration levels.[19,20] We show that equilibrium fund expected net alphas and AFMI size increase (decrease) with market concentration if and only if the AFMI's direct benefits increase (decrease) with market concentration. Also, we find that if equilibrium fund expected net alphas are concave in market concentration, then the AFMI's direct benefits are concave in market concentration. Consequently, equilibrium AFMI size is also concave in market concentration. On the other hand,[21] if equilibrium AFMI size is convex in market concentration, then the AFMI's direct benefits are convex in market concentration and, consequently, equilibrium expected fund net alphas are convex in market concentration. Here, as in the literature, by construction, aggregate net alphas are zero-sum as they shift wealth between subsets of investors; see,

[18] Each of these cases fits particular economies. The cases where investors are risk-neutral are consistent with the situation where households employ private banks or other financial institutions to manage their wealth. These institutions then invest in active funds. Acting as intermediaries, they mainly care about funds' expected returns so may be regarded as risk-neutral investors. The cases where there is a single investor in the market is consistent with the situation where the societal wealth is centrally managed by the government, which determines the allocation of investments to active funds and other securities.

[19] For brevity and simplicity, we use the term *benefits* in the general sense allowing for negative benefits.

[20] The AFMI, however, may have indirect benefits which we do not model here. For example, monitoring, studying, and analyzing firms might incentivize managements to improve governance and productivity, and to reduce agency costs. Also, active fund management may induce transfer of wealth from less productive firms/investors to more productive ones, and even within endowment economies, may increase investors' derived utilities by improving information processing and/or risk sharing.

[21] Please note that the order of statements of this argument is different from that in the previous one, for reasons explained below.

for example, discussion in PS.[22,23]

Then, we specialize our model to allow endogenous concentration levels. This framework befits empirical market concentration measures which are calculated based on endogenous fund sizes. We test the implications of our model using empirical data in the next chapter.

In addition to the above topics we study the agency benefits due to market concentration, and look at equilibria with colluding fund managers.

Section 2.2 develops a model of AFMI concentration, performance, size, managerial efforts and fees. Section 2.3 presents a numerical example. Section 2.4 studies the agency benefits due to market concentration. Section 2.5 analyzes a framework with endogenous market concentration levels, and Section 2.6 concludes.

2.2. Theoretical Framework

We first develop a theoretical framework for modeling the effect of market concentration on fund managers' effort, fund fees, fund performance, AFMI size, and potential benefits.

For brevity and parsimonious notation, we assume that variables and functions are real, continuous, and twice differentiable. Within a one-period market, there are two types of agents: fund managers of M, $M > 1$ funds and N, $N \geq 1$, investors. Acting competitively, each manager, conditional on fund size and market concentration level, sets a proportional management fee and chooses an effort level to maximize the fund expected net alpha to attract investments.[24] In Case I, risk-neutral investors allocate investments among the M actively managed funds and a passive benchmark index fund to maximize their portfolios' expected returns. In Case II, mean-variance risk-averse investors allocate their investments to maximize their portfolios' Sharpe ratios.

Following PS, $\mathbf{r_F}$, a vector of M funds' returns in excess of the riskless rate that investors receive, follows the regression model

$$\mathbf{r_F} = \boldsymbol{\alpha} + \boldsymbol{\beta} r_p + \mathbf{u}, \tag{2.1}$$

[22] Pages 748-750, including footnote 6, and references therein. As in PS, we do not model the "other" investors, who facilitate the zero-sum.

[23] We note that this is also true at non-zero AFMI's direct and/or indirect benefits, as we measure net alphas ex-post under the equilibrium active fund management level.

[24] In our framework, the competition among managers is Bertrand competition, where the number of competitors is M, and the "prices" offered by managers in competition are fund net alphas net of management fees.

where $\mathbf{r_F}$ is an $M \times 1$ vector with elements $r_{F,i}$, $i = 1, \ldots, M$.

The benchmark-adjusted returns on the M funds that investors receive is

$$\mathbf{r} \triangleq \boldsymbol{\alpha} + \mathbf{u}. \tag{2.2}$$

The variables $\boldsymbol{\beta}$, $\mathbf{r}$, $\boldsymbol{\alpha}$, and $\mathbf{u}$ are $M \times 1$ vectors. $\boldsymbol{\alpha}$ is the vector of fund net alphas received by investors, and $\boldsymbol{\beta}$ is the vector of fund betas. Funds hold fully diversified portfolios and, thus, have unit betas; therefore, $\boldsymbol{\beta}$ is a unit vector. The scalar r_p is the excess return on the passive benchmark portfolio, with mean μ_p and variance σ_p^2; $\mathbf{u}$ is the residual vector, with elements that follow

$$u_i = x + \varepsilon_i, \quad i = 1, \ldots, M, \tag{2.3}$$

where ε_i's are mean zero and variance σ_ε^2 idiosyncratic risks, and are uncorrelated with each other, with x, and with r_p. The common factor x has mean zero, variance of σ_x^2, and is uncorrelated with r_p. The values of μ_p, σ_p^2, σ_ε^2, and σ_x^2 are constants that are common knowledge of both investors and managers.

Each element in $\boldsymbol{\alpha}$ has the following structure:

$$\alpha_i = a - b\frac{S}{W} + A(e_i; H) - f_i, \tag{2.4}$$

where S is the aggregate size of the active management industry and is equal to the sum of all the funds' sizes (i.e., $S = \sum_{i=1}^{M} s_i$); W is equal to S plus the amount invested in the passive benchmark; a and b are positive, unknown scalar parameters; a is the expected return on an initial small fraction of wealth invested in active management, net of any costs; and b is the absolute magnitude of the decreasing returns to scale at industry level. The first and second conditional moments of a and b are

$$\mathrm{E}\left(\begin{bmatrix} a \\ b \end{bmatrix} \middle| D\right) \triangleq \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \tag{2.5}$$

$$\mathrm{var}\left(\begin{bmatrix} a \\ b \end{bmatrix} \middle| D\right) \triangleq \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix}, \tag{2.6}$$

where D is investors' information set. As we do not focus on σ_{ab}'s effects on the equilibrium, we assume that $\sigma_{ab} = 0$. In other words, conditional on current information, we assume that how $\hat{a}$ deviates from a is unrelated to how $\hat{b}$ deviates from b. Our reasoning is that a and b are parameters requiring different estimation methods and σ_{ab} tends to be small in comparison to σ_a^2 and σ_b^2; thus, the assumption of $\sigma_{ab} = 0$ is reasonable. Finally, with f_i being a proportional management fee charged by manager i, manager i's fund expected net alpha is[25]

$$\mathrm{E}(\alpha_i \mid D) = \hat{a} - \hat{b}\frac{S}{W} + \hat{A}(e_i; H) - f_i. \tag{2.7}$$

Our model follows and builds on that of PS. In this partial equilibrium, the passive benchmark portfolio's returns are exogenously given and are unaffected by interactions between investors and managers. Managers' outperformance of the passive benchmark portfolio (i.e., net alphas), may come at the expense of "other investors," who may be noise traders, liquidity seekers, misinformed, or irrational.[26]

We allow manager i to spend a non-negative amount of proportional effort e_i (i.e., $e_i \in [0, \infty)$) to increase gross alpha (i.e., the alpha before subtracting the fee) by $A(e_i; H)$ under market concentration level H. Industrial organization theory suggests that market concentration not only depends on the number of incumbents, but also on threats of entry activity-limiting regulation and the competitiveness of related industries.[27] We assume that H is an exogenous constant because it depends mainly on some exogenous factors mentioned above.[28] It belongs to $[0,1)$. If $H = 0$, there are infinitely many small funds in the market, and the market is fully competitive. If $H = 1$, the market is monopolistic. If the fund managers are competing with each other, H belongs to $[0,1)$, and this is H's domain in our framework. $A(e_i; H)$ is the same across funds and has the following functional characteristics:

[25] Investors observe the passive benchmark and the AFMI funds' returns. The difference between these returns comes from three components: net alphas, the common risk factor, and idiosyncratic risks. As the distributions of the common risk and idiosyncratic risk are common knowledge, investors know the likelihood function of the net alphas. Given prior beliefs of net alphas, they form posteriors and update their beliefs. In our one-period model, there is no dynamic Bayesian updating, but we suggest that investors reached a fixed-point equilibrium. Further, because investors observe f_i, H, S and W, they can also infer $A(e_i, H)$. Here, where equilibrium optimal effort levels of all managers are same, the estimate $\hat{A}(e_i, H)$ could be subsumed in $\hat{a}$; and in an equilibria where managers' optimal effort levels differ, the estimates $\hat{A}_i(e_i, H)$, could be subsumed in f_i. For simplicity and brevity, we depress the notation of $\hat{A}(e_i, H)$ in favor of $A(e_i, H)$ and follow PS formulation, Equations (2.5) and (2.6).

[26] Please see the detailed discussion in PS, pp. 748–750.

[27] Please see the discussion by Claessens and Laeven (2003).

[28] In Section 2.5, we examine endogenous concentration.

- non-negative, i.e., $A(e_i;H) > 0,\ \forall e_i > 0, H$, $A(0;H) = 0,\ \forall H$,
- increasing and concave in effort, i.e., $A_{e_i}(e_i;H) \triangleq \partial A(e_i;H)/\partial e_i > 0,\ \forall e_i, H$, and $A_{e_i,e_i}(e_i;H) \triangleq \partial^2 A(e_i;H)/\partial e_i^2 < 0,\ \forall e_i, H$,
- increasing in market concentration, i.e., $A_H(e_i;H) \triangleq \partial A(e_i;H)/\partial H > 0,\ \forall e_i > 0, H$,
- positive cross-partial derivatives with respect to effort and market concentration, i.e., $A_{e_i,H}(e_i;H) = A_{H,e_i}(e_i;H) \triangleq \partial^2 A(e_i;H)/\partial H \partial e_i > 0,\ \forall e_i, H$.

The economic sense of the structure of $A(e_i;H)$ is as follows. A particular positive level of effort has a positive impact on gross alpha production. If managers spend no effort, the increment in gross alpha due to their effort is zero (i.e., $A(e_i;H) \geq 0,\ \forall e_i, H$ and $A(0;H) \geq 0,\ \forall H$). Under a particular market concentration level H , an increase in effort e_i may increase gross alpha, but the marginal increment is decreasing (i.e., $A_{e_i}(e_i;H) > 0$ and $A_{e_i,e_i}(e_i;H) < 0,\ \forall e_i, H$). The more concentrated the AFMI is, the relatively more investment opportunities there are, and the more marginally efficient is the use of industry resources.[29] Thus, managers can generate a higher increment in gross alpha for a given effort level e_i (i.e., $A_H(e_i;H) > 0$), and the marginal impact of e_i on gross alpha is also larger (i.e., $A_{e_i,H}(e_i;H) = A_{H,e_i}(e_i;H) > 0,\ \forall e_i, H$). These two assumptions of the partial derivatives are consistent with Hoberg, Kumar and Prabhala (2015), who found that a higher competition level limits managers' skills to create gross alpha persistently.

Managers' Cost

For simplicity and to focus on modelling decreasing returns to scale in gross alpha production, we assume that funds' fixed costs are zero.[30] This assumption is realistic because fixed costs to develop funds, such as registration fees and equipment

[29] In a more concentrated market, if a fund manager controls most of the industry resources and develops advanced strategies to produce gross alphas, other funds can mimic this fund's strategy and produce higher gross alphas given a particular effort level, so this assumption is still valid when a dominant fund in the market controls the majority of the resources.

[30] A non-zero fixed cost and decreasing returns to scale in gross alpha production (i.e., a cost component that is increasing and convex in fund size) would induce an average cost function that is U-shape in fund size (thus under some cases there is increasing returns to scale in gross alpha production). To focus on the decreasing returns to scale in gross alpha production, we ignore the fixed cost component in this model.

expenditure, are usually small (although not trivial) compared to variable costs related to employees' salaries and managers' compensation. We assume that average cost functions, $C^i(e_i, s_i; H)$, contain three independent positive components: $c_{0,i}$, the average cost for fund i to operate in the market before receiving investment and before manager i spends effort; $c_{1,i}s_i$, the average cost related to fund size; and $c_{2,i}(e_i;H)$, the average cost of managerial effort under a particular market concentration.[31] That is,

$$C^i(e_i, s_i; H) = c_{0,i} + c_{1,i}s_i + c_{2,i}(e_i; H). \tag{2.8}$$

To simplify our model, we let $c_{0,i}$, and the function $c_{2,i}(e_i;H)$ be the same across funds (we, thus, drop the subscript i from now on), but let $c_{1,i}$ be different across funds. We discuss the effects of similarities and differences in these parameters across funds later. The function $c_2(e_i;H)$ has the following characteristics:

- non-negative, i.e., $c_2(e_i;H) > 0,\ \forall e_i > 0, H$ and $c_2(0;H) = 0,\ \forall H$,
- increasing and convex in effort, i.e., $c_{2e_i}(e_i;H) \triangleq \partial c_2(e_i;H)/\partial e_i > 0,\ \forall e_i, H$ and $c_{2e_i,e_i}(e_i;H) \triangleq \partial^2 c_2(e_i;H)/\partial e_i^2 > 0,\ \forall e_i, H$,
- increasing in market concentration, i.e., $c_{2H}(e_i;H) \triangleq \partial c_2(e_i;H)/\partial H > 0,\ \forall e_i > 0, H$,
- positive cross-partial derivatives with respect to effort and market concentration, i.e., $c_{2e_i,H}(e_i;H) = c_{2H,e_i}(e_i;H) \triangleq \partial^2 c_2(e_i;H)/\partial H \partial e_i > 0,\ \forall e_i, H$.

The average cost function implies that if fund size s_i increases, manger i's average cost increases because the larger trades are associated with larger price impacts and higher execution costs and because of other factors that create diseconomies of scale in operation. $c_{1,i}$ is the average cost sensitivity to fund size s_i. Also, the cost of the fund is related to the incentive scheme that offers increasing and convex bonuses for employees' performance. Thus, the average cost function is increasing and convex in effort e_i ($c_{2e_i}(e_i;H) > 0$ and $c_{2e_i,e_i}(e_i;H) > 0,\ \forall e_i, H$). In addition, in markets that are

[31] To simplify our model, we assume there is no interaction between effort and size in the average cost function because it is unlikely that fund size affects managers' per dollar effort. We also assume that there is no interaction between concentration and size in the average cost function because it is unlikely that concentration affects managers' average cost sensitivities to fund sizes. Nevertheless, even if these interacting effects exist, they tend to be small in comparison to effects of other terms in the average cost function.

more concentrated, it is more costly to incentivize managerial efforts because market compensation levels for effort are higher, and/or because managers are less industrious. Thus, if markets are more concentrated, average costs due to effort are higher ($c_{2H}(e_i;H) > 0, \forall e_i, H$), and marginal costs due to effort are higher as well ($c_{2e_i,H}(e_i;H) = c_{2H,e_i}(e_i;H) > 0, \forall e_i, H$).

The total cost function of manager i is Equation (2.8) times fund size s_i, so manager i's total cost function is convex in s_i. Therefore, our fund cost model is consistent with that of Berk and Green (2004), who assumed decreasing returns to scale at fund level. Empirically, PST also reported evidence consistent with fund-level decreasing returns to scale.

We define AFMI's direct benefit function of manager i's net alpha production as

$$B(e_i;H) \triangleq A(e_i;H) - c_2(e_i;H). \tag{2.9}$$

$B(e_i;H)$ captures the direct benefit from effort exerted in active fund management, in terms of increase in gross alpha production minus the effort cost. We note that AFMI's active search for net alphas might have indirect effects that we do not model here. It might drive security prices toward their true values; it might induce firms to improve governance and performance, and reduce agency costs. It might induce transfer of wealth from less productive firms/investors to more productive ones. We note that we should interpret *benefits* generally, allowing them to be positive or negative.

Whether the AFMI's direct and or indirect benefits are non-zero or zero, here, as in the literature, gross alphas are zero-sum, because we measure gross alphas ex-post in the AFMI's equilibrium,. (See for example PS, pp. 748-750, including footnote 6, and references therein.)

Fund Managers' Problem

Manager i's economic profit is

$$s_i\left(f_i - C^i(e_i, s_i;H)\right), \tag{2.10}$$

and for the fund i to survive,

$$f_i - C^i(e_i, s_i;H) \geq 0. \tag{2.11}$$

Manager i's problem can be written as

$$\underset{e_i, f_i}{\text{Max}}\, s_i \left(f_i - C^i(e_i, s_i; H) \right) \tag{2.12}$$

subject to

$$e_i \geq 0,$$

$$f_i \geq 0.$$[32]

Risk-neutral investors would invest only in funds that generate the highest expected net alphas. For mean-variance risk-averse investors, we assume that the benefit of diversification across funds approaches zero because funds are well diversified and because the uncertainty of the parameters x, a and b are likely to be much larger than the uncertainty of ε_i. Also, real-world investors tend not to invest in many funds and do not diversify across funds. Thus, it is likely that not only are total diversification benefits zero, but also likely that marginal diversification benefits are trivial. Thus, our risk-averse investors invest only in funds that generate the highest expected net alphas and managers have to compete over net alphas. Manager i 's problem can be transformed as,

$$\underset{e_i, f_i}{\text{Max}}\, \text{E}(\alpha_i \mid D) \tag{2.13}$$

subject to $\quad f_i - C^i(e_i, s_i; H) \geq 0$, $e_i \geq 0$ and $f_i \geq 0$.

Proof. See the Appendix.

The proof intuition is as follows. Under competition, funds that offer higher expected net alphas draw (all) investments. Thus, in equilibrium funds offer similar expected net alphas. The possibility that other managers increase fund profits by improving expected net alphas, and their fund sizes, induces managers to maximize expected net alphas in order to "survive." We note that this aspect of the equilibrium is similar to that in PS, but in addition to their result, we show that it holds also in the case of finite number of managers, under Bertrand competition.

The following propositions provide results of fund managers' equilibrium optimal effort levels and fees.

PROPOSITION 1. For manager i, $i = 1, 2, ..., M$, if initial effort inputs generate non-

[32] For simplicity and brevity, we omit the condition in Equation (2.11) from the problem statement as it is implied by the optimization and, thus, is not binding.

positive AFMI's direct benefits of net alpha production (i.e., $B_{e_i}(0;H) \le 0, \forall H$), equilibrium optimal proportional effort levels e_i^* are zero (i.e., $e_i^* = 0$) and the optimal proportional fee f_i^* equals the average cost of operating funds $c_0 + c_{1,i}s_i$ (i.e., $f_i^* = c_{0,i} + c_{1,i}s_i$).

COROLLARY to PROPOSITION 1. Under the case of Proposition 1, the equilibrium is similar to that in PS. The managerial effort is not modeled, and managers optimally choose not to charge fees above opportunity costs.

PROPOSITION 2. For manager i, $i = 1, 2, ..., M$, if initial effort inputs generate positive AFMI's direct benefits of net alpha production (i.e., $B_{e_i}(0;H) > 0, \forall H$), equilibrium optimal effort-fee combinations (e_i^*, f_i^*) satisfy the following.

1) $f_i^* - C^i(e_i^*, s_i; H) = 0$ (optimal fees are set to be equal to costs).
2) $A_{e_i}(e_i^*; H) - c_{2e_i}(e_i^*; H) = B_{e_i}(e_i^*; H) = 0$ (the impact of marginal efforts on gross alpha is set to be equal to the marginal average costs of effort, thus marginal AFMI's direct benefits of net alpha production under the optimal effort level are zero).
3) $e_i^*{}'(H) \ge 0 (< 0)$ iff $A_{e_i,H}(e_i^*; H) - c_{2e_i,H}(e_i^*; H) \ge 0 (< 0)$, where $e_i^*{}'(H) \triangleq de_i^* / dH$ (where concentration is higher, equilibrium optimal efforts are higher (lower) if and only if higher concentration induces a larger (smaller) marginal effort impact on gross alphas than on costs).
4) The signs of $f_i^*{}'(H) \triangleq df_i^* / dH$ depend on the signs of $d(S/W)/dH$ and $e_i^*{}'(H)$ (whether higher concentrations induce higher equilibrium optimal fees depends on whether they induce an increase in equilibrium industry sizes and whether they induce an increase in equilibrium optimal efforts).
5) $B'(e_i^*; H) \ge 0 (< 0)$ iff $A_H(e_i^*; H) - c_{2H}(e_i^*; H) \ge 0 (< 0)$, where $B'(e_i^*; H) \triangleq dB(e_i^*; H) / dH$ (where concentrations are higher, equilibrium AFMI's direct benefits of net alpha production are higher (lower) if and only if higher concentrations induce a larger (smaller) impact on gross alphas than on costs).

6) $\mathrm{E}(\alpha_i \mid D) = \mathrm{E}(\alpha_j \mid D), \forall i, j$ (in equilibrium, managers offer market competitive net alphas).

7) $\mathrm{E}(r_{F,i} \mid D) / \sqrt{\mathrm{Var}(r_{F,i} \mid D)} = \mathrm{E}(r_{F,j} \mid D) / \sqrt{\mathrm{Var}(r_{F,j} \mid D)}, \forall i, j$ (in equilibrium, managers offer market competitive Sharpe ratios).

Proofs of Proposition 1 and 2, and corollary. See the Appendix.

The proof intuition is as follows. While competing for investments, managers maximize fund expected net alphas by choosing optimal efforts and fees, earning zero economic profits (break-even fees) in equilibrium. If higher concentration levels induce a higher (lower) marginal effort impact on gross alphas than a marginal effort impact on costs, managers optimally choose higher (lower) effort levels in producing fund net alphas. Also, concentration affects managers' costs by increasing levels of cost due to effort and by influencing levels of optimal (costly) efforts. If higher concentrations induce higher equilibrium optimal efforts, managers' costs are driven higher, resulting in higher break-even fees. Otherwise, where concentrations are higher, increases in cost levels due to effort are cancelled out by decreases in optimal effort levels. In this case, negative relations between break-even fees, which are equal to costs, and concentration indicate negative relations between equilibrium optimal effort levels and concentration. In addition, higher concentrations have two effects on the AFMI's direct benefits of net alpha production: directly increasing the levels of gross alphas and costs due to effort levels, and changing equilibrium optimal effort levels, consequently changing gross alphas and costs. In equilibrium, the latter effect is zero because the marginal effort impact on gross alphas is equal to the marginal effort impact on costs and the effect of higher concentration through effort on gross alphas is cancelled out by its effects through effort on costs. Therefore, if higher concentrations induce a higher direct impact on gross alphas than on costs, AFMI's direct benefits of net alpha production are higher.

Also, as there are no diversification benefits across funds, managers who provide higher expected net alphas dominate, attracting investments. Consequently, their fund costs increase, inducing higher (break-even) fees and lowering expected net alphas. Thus, in equilibrium, allocation of investments, or fund sizes, set expected net alphas to be equal across funds. If fund managers cannot produce the AFMI highest expected net alpha, even for an infinitesimal fund size, their funds go out of the market.

In addition, as funds have the same expected net alphas, they have the same expected returns. As the source of fund returns' variance is the same across funds, the fund return variance is the same across funds. Therefore, managers offer the same competitive Sharpe ratio.

The following proposition identifies the relation of different managers' equilibrium optimal effort levels, fees, benefits of effort, and AFMI share.

PROPOSITION 3. Under the same c_0 and the same functional form of $c_2(e_i;H)$ but different $c_{1,i}$'s across funds,

1) $e_i^* = e_j^*$ and $f_i^* = f_j^*$, $\forall i, j$ (equilibrium efforts and fees are the same across funds).
2) Therefore, $B(e_i^*;H) = B(e_j^*;H)$, $\forall i, j$ (equilibrium AFMI's direct benefits of net alpha production are the same across funds).
3) Fund sizes relate as $s_i / s_j = c_{1,j} / c_{1,i}$, $\forall i, j$.
4) AFMI shares, s_i / S's are

$$\frac{s_i}{S} = \left(c_{1,i} \sum_{j=1}^{M} \left(c_{1,j}{}^{-1} \right) \right)^{-1}, \ \forall i .$$

Proof of Proposition 3. See the Appendix.

The third point of Proposition 3 shows that managers' different costs of producing gross alphas, induce different fund sizes in equilibrium. The fourth point of Proposition 3 implies that funds' market shares are deterministic functions of $c_{1,i}$'s and are, thus, unaffected by the AFMI weight in total wealth, S/W. In other words, how investors weight the funds inside the AFMI is unaffected by how investors weight the AFMI as a whole relative to the passive benchmark. This property facilitates later results.

Proposition 3 is driven by the fact that c_0 and the functional form of $c_2(e_i;H)$ are the same across funds but $c_{1,i}$'s are different across funds. In contrast, Proposition 1 and 2 are valid even without this assumption. In fact, from Proposition 2, we can see that if $c_{0,i}$, $c_{1,i}$ and the functional form of $c_{2,i}(e_i;H)$ are different across funds, managers end up with different levels of effort, different fees, and different fund sizes in equilibrium. If all fund managers have the same c_0, c_1, and functional form of $c_2(e_i;H)$, they end up with the same equilibrium levels of effort, fees, and fund sizes.

We define the equilibrium optimal expected net alphas of an initial marginal investment in the AFMI as $X(e_i^*, H)$. Quantitatively,

$$X(e_i^*, H) \triangleq \hat{a} + A(e_i^*;H) - \left[c_0 + c_2(e_i^*;H)\right]. \tag{2.14}$$

For the AFMI to exist, we must have positive net alphas for initial infinitesimal investments into the AFMI:

$$X(e_i^*, H) > 0. \tag{2.15}$$

If Inequality (2.15) is violated, investors receive negative or zero net alpha from the AFMI and invest all their wealth in the passive benchmark. Also, to offer meaningful results, we assume that initial marginal allocations of effort generate positive AFMI's direct benefits, that is,

$$B_{e_i}(0, H) > 0,\ \forall i, \forall H, \tag{2.16}$$

such that the optimal effort e_i^* is positive, finite, and attainable, i.e., $B_{e_i}(e_i^*, H) = 0,\ e_i^* < K,\ \forall i, \forall H$ for some constant K. We focus on the case under Proposition 2 in the following analyses.

Investors' Problem

Let $\boldsymbol{\delta}_\mathbf{j}$ denote the $M \times 1$ vector of weights that investor j places on the M funds, with elements $\delta_{j,i}$, $i = 1, \ldots, M$. Thus, investor j's excess return is

$$r_j = \boldsymbol{\delta}_\mathbf{j}^\mathbf{T}\mathbf{r}_\mathbf{F} + (1 - \boldsymbol{\delta}_\mathbf{j}^\mathbf{T}\boldsymbol{\iota}_\mathbf{M})r_p, \tag{2.17}$$

where $\boldsymbol{\iota}_\mathbf{M}$ is an $M \times 1$ vector with elements equal to 1. Assuming all funds have beta loadings on the benchmark equal to 1 (i.e., $\boldsymbol{\beta} = \boldsymbol{\iota}_\mathbf{M}$), based on (2.1) and (2.17), we have

$$r_j = r_p + \boldsymbol{\delta}_\mathbf{j}^\mathbf{T}(\boldsymbol{\alpha} + \mathbf{u}). \tag{2.18}$$

Further, we have

$$\mathrm{E}(r_j \mid D) = \mu_p + \boldsymbol{\delta}_\mathbf{j}^\mathbf{T}\mathrm{E}(\boldsymbol{\alpha} \mid D) = \mu_p + \boldsymbol{\delta}_\mathbf{j}^\mathbf{T}\left[\hat{a} - \hat{b}\frac{S}{W} + A(e_i^*;H) - f_i^*\right]\boldsymbol{\iota}_\mathbf{M},\ \forall j. \tag{2.19}$$

Equation (2.19) is valid because the fund expected net alphas are the same across funds in equilibrium as implied by Proposition 2. Also,

$$\mathrm{Var}(r_j \mid D) = \sigma_p^2 + \left[\sigma_a^2 + \sigma_x^2 + \sigma_b^2 \left(\frac{S}{W} \right)^2 \right] \left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} \right)^2 + \sigma_\varepsilon^2 \left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}} \right), \ \forall j . \tag{2.20}$$

We characterize equilibria in the cases where 1) there are infinitely many small risk-neutral investors, 2) there is a single large risk-neutral investor, 3) there are infinitely many small mean-variance risk-averse investors, and 4) there is a single large mean-variance risk-averse investor. Here, *infinitely many small investors* means $N \to \infty$ investors have finite wealth, and their choices cannot affect fund sizes. *A single large* investor means $N = 1$, a single investor who controls all market wealth, and his or her choices determine fund sizes. In the following analyses, we focus on cases 3 and 4, whereas cases 1 and 2 are discussed in the Appendix.

Mean-Variance Risk-Averse (RA) Investors

If there are infinitely many (i.e., $N \to \infty$) small mean-variance risk-averse investors, none of them can affect fund sizes. Also, investors' investment in the AFMI dilutes funds' expected returns due to decreasing returns to scale in funds. In addition, mean-variance risk-averse investors face risk-return tradeoffs in marginal allocations. Investor j's objective is to maximize the portfolio's Sharpe ratio by choosing portfolio weights, $\boldsymbol{\delta}_{\mathbf{j}}$, $j = 1, \ldots M$. This investor's problem is

$$\underset{\boldsymbol{\delta}_{\mathbf{j}}}{\mathrm{Max}} \frac{\mathrm{E}(r_j \mid D)}{\sqrt{\mathrm{Var}(r_j \mid D)}} = \underset{\boldsymbol{\delta}_{\mathbf{j}}}{\mathrm{Max}} \left\{ \frac{\mu_p + \boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}} (\hat{a} - \hat{b} \frac{S}{W} + A(e_i^*; H) - f_i^*) \boldsymbol{\iota}_{\mathbf{M}}}{\sqrt{\sigma_p^2 + \left[\sigma_a^2 + \sigma_x^2 + \sigma_b^2 \left(\frac{S}{W} \right)^2 \right] \left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} \right)^2 + \sigma_\varepsilon^2 \left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}} \right)}} \right\}, \tag{2.21}$$

subject to

$$\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} \leq 1, \tag{2.22}$$

$$\delta_{j,i} \geq 0, \ \forall i , \tag{2.23}$$

$$f_i^* - C^i(e_i^*, s_i; H) = 0, \ \forall i , \tag{2.24}$$

$$A_{e_i}(e_i^*; H) - c_{2e_i}(e_i^*; H) = B_{e_i}(e_i^*; H) = 0, \ \forall i . \tag{2.25}$$

Condition (2.22) is a form of wealth constraint, saying that investors cannot borrow from the passive benchmark to invest in the AFMI. Condition (2.23) says that there is no short sale of funds. Conditions (2.24) and (2.25) reflect managers' equilibrium optimal choices. Also, as we assume that there are no marginal diversification benefits across funds, we set the term $\sigma_{\varepsilon}^{2}\left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}}\boldsymbol{\delta}_{\mathbf{j}}\right)$ to zero when solving the optimization problem (2.21). Because the equilibrium is symmetric, we have

$$\boldsymbol{\delta}_{\mathbf{j}}^{*\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}} = S / W \,. \tag{2.26}$$

Where there is a single (i.e., $N=1$) large investor, he or she determines s_i, $i=1,\ldots,M$, thus, S/W, to maximize the portfolio's Sharpe ratio. The investor not only faces a tradeoff between allocating additional dollars to funds, taking advantage of fund net alphas, and diluting returns on wealth already in funds, but also faces a risk-return tradeoff. The problem is

$$\begin{aligned}
&\underset{\boldsymbol{\delta}_1}{\mathrm{Max}} \frac{\mathrm{E}(r_1 \mid D)}{\sqrt{\mathrm{Var}(r_1 \mid D)}} \\
&= \underset{\boldsymbol{\delta}_{\mathbf{j}}}{\mathrm{Max}} \left\{ \frac{\mu_p + \boldsymbol{\delta}_1{}^{\mathbf{T}}(\hat{a} - \hat{b}\boldsymbol{\delta}_1{}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}} + A(e_i^*;H) - f_i^*)\boldsymbol{\iota}_{\mathbf{M}}}{\sqrt{\sigma_p^2 + \left[\sigma_a^2 + \sigma_x^2 + \sigma_b^2\left(\boldsymbol{\delta}_1{}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}}\right)^2\right]\left(\boldsymbol{\delta}_1{}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}}\right)^2 + \sigma_{\varepsilon}^2\left(\boldsymbol{\delta}_1{}^{\mathbf{T}}\boldsymbol{\delta}_1\right)}} \right\}
\end{aligned} \tag{2.27}$$

subject to Conditions (2.22) – (2.25). We also set the term $\sigma_{\varepsilon}^{2}\left(\boldsymbol{\delta}_{\mathbf{1}}{}^{\mathbf{T}}\boldsymbol{\delta}_{\mathbf{1}}\right)$ to zero and also have $\boldsymbol{\delta}_{\mathbf{1}}^{*\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}} = S / W$. Because of the analytical complexity of this case, $N=1$, we rely on a numerical solution.

The next proposition describes the equilibrium in the $N \to \infty$ case.

PROPOSITION RA1, Unique Nash Equilibrium.

Where $N \to \infty$,

1) there exists a unique Nash equilibrium, $\left\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\right\}$, where

   $\mathbf{e}^*$ is an $M \times 1$ vector with managers' optimal effort allocations, e_i^*,

   $\mathbf{f}^*$ is an $M \times 1$ vector with managers' optimal fee allocations, f_i^*, and

   $\boldsymbol{\delta}^*$ is an $M \times N$ matrix with vectors of investors' optimal wealth weights allocations to funds, $\boldsymbol{\delta}_{\mathbf{j}}^*$;

2) in this equilibrium, managers produce the same expected net alpha, which drives their economic profits to zero, by charging only break-even fees; and investors allocate the same wealth proportions to each of the funds.

Proof of Proposition RA1. See the Appendix.

The following proposition identifies equilibrium properties.

PROPOSITION RA2, Equilibrium by Optimal Allocations.

For $i = 1, 2, ..., M$, we have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} > 0$; and where $N \to \infty$, the equilibrium optimal S/W is either 1 or a real positive solution of the (constrained embedded) first-order condition of the investors' problem (a cubic equation) substituting $\boldsymbol{\delta}_{\mathbf{j}}^{*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} = S/W$,

$$-\gamma\sigma_b^2 \left(\frac{S}{W}\right)^3 - \left[\gamma\sigma_a^2 + \gamma\sigma_x^2 + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{S}{W} + X(e_i^*, H) = 0,$$

where $\gamma \triangleq \mu_p / \sigma_p^2$.

Where $N = 1$, numerical solutions are required.

The intuition of Proposition RA2 is as follows. Whether $N \to \infty$ or $N = 1$, investors allocate investments to funds based on their risk-return tradeoffs. Investing too much wealth in the AFMI increases portfolio risk, so they choose to limit those investments, leaving $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} > 0$. Where $N \to \infty$, the properties of the cubic equation guarantee at least one real positive root. The solution of S/W is the largest or the smallest real positive root of the cubic equation or 1, depending on which maximizes (2.21).

COROLLARY to PROPOSITION RA2. Where $N \to \infty$, for large enough W, such that $S/W < 1$, we have

$$\frac{d(S/W)}{dX(e_i^*; H)} = \frac{1}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} > 0, \text{ and}$$

$$\frac{d(S/W)}{d\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} = \frac{-(S/W)}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} < 0.$$

That is, higher initial marginal fund expected net alphas induce a larger equilibrium AFMI size relative to total wealth, whereas a stronger decreasing returns to scale effect in the AFMI induces a smaller equilibrium AFMI size relative to total wealth.

The intuition of this corollary is as follows. Where $S/W < 1$, an increase (decrease) in $X(e_i^*, H)$ shifts up (down) the cubic function in Proposition RA2, inducing a larger (smaller) S/W as the maximizer of investors' objective function. The economic sense is that a higher level of equilibrium optimal expected net alpha of an initial marginal investment, $X(e_i^*;H)$, attracts more investments to the AFMI. Also, we can see that $\hat{b}$ is the expected decreasing returns to scale at the industry level, based on current information, whereas $\left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W$ may be regarded as the equilibrium decreasing returns to scale factor at the fund level because it is calculated by all the fund average cost sensitivities to size, $c_{1,i}$. Thus, the factor $\hat{b}+\left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W$ may be regarded as the combined decreasing returns to scale factor. Investors invest less in funds if the effect of decreasing returns to scale is stronger in the AFMI. The following proposition offers the comparative statics.

PROPOSITION RA3 AFMI Size Sensitivity to Concentration.

1) Where $N \to \infty$ and $S/W < 1$, we have

   a. $$\frac{d\left(S/W\right)}{dH} = \frac{d\left(S/W\right)}{dX(e_i^*;H)}\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]$$

   $d\left(S/W\right)/dH \geq 0(<0)$ iff $A_H(e_i^*;H) - c_{2H}(e_i^*;H) \geq 0(<0)$ (where concentration is higher, equilibrium industry size is larger (smaller) if and only if the higher concentration induces a larger (smaller) impact on gross alphas than on costs).

   b. $$\frac{d^2\left(S/W\right)}{dH^2} = \frac{d\left(S/W\right)}{dX(e_i^*;H)}\frac{d^2 B(e_i^*;H)}{dH^2} - 6\gamma\sigma_b^2 \frac{S}{W}\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]^2\left[\frac{d\left(S/W\right)}{dX(e_i^*;H)}\right]^3.$$

   If $d^2B(e_i^*;H)/dH^2 \leq 0$, then $d^2\left(S/W\right)/dH^2 \leq 0$ (the fact that $B(e_i^*;H)$ is concave in H indicates that S/W is concave in H), and if $d^2\left(S/W\right)/dH^2 \geq 0$, then $d^2B(e_i^*;H)/dH^2 \geq 0$ (the fact that S/W is convex in H indicates that $B(e_i^*;H)$ is convex in H).

2) Where $N \to \infty$ and $S/W = 1$, S/W is unrelated to market concentration.
3) Where $N = 1$ and $S/W < 1$, numerical solutions are required to analyze the signs of $d\left(S/W\right)/dH$ and $d^2\left(S/W\right)/dH^2$.
4) Where $N = 1$ and $S/W = 1$, S/W is unrelated to market concentration.

The intuition is as follows. Where $N \to \infty$, a higher H affects industry size

S/W through the equilibrium optimal expected net alpha of an initial marginal investment, $X(e_i^*;H)$. If a higher H induces a larger (smaller) impact on gross alphas than on costs, then it creates a larger (smaller) $X(e_i^*;H)$ and, consequently, attracts more (less) investments in the AFMI—if investors have additional wealth to allocate to funds (i.e., $S/W<1$). From Proposition 2, we see that $B'(e_i^*;H)=A_H(e_i^*;H)-c_{2H}(e_i^*;H)$; thus, in this case, a higher H induces a larger S/W if and only if it induces a higher $B(e_i^*;H)$. Regarding the second-order derivative $d^2\left(S/W\right)/dH^2$ where $N\to\infty$ and $S/W<1$, if investors are risk-neutral (see Proposition RN3 in the Appendix), $d^2\left(S/W\right)/dH^2$ is positively proportional to $d^2B(e_i^*;H)/dH^2$. That is, as H changes, the change of marginal S/W depends on the change of marginal $B(e_i^*;H)$ because investors share all AFMI's direct benefits of net alpha production in a market with competing managers. However, mean-variance risk-averse investors face, in addition, a risk-return tradeoff. Holding other parameters the same, if investors' marginal portfolio risks are higher, they optimally invest less in funds, so higher H induces smaller marginal S/W in equilibrium. Thus, $d^2\left(S/W\right)/dH^2$ in the risk-averse case is reduced by an adjustment term for risk (the second component in the expression of $d^2\left(S/W\right)/dH^2$ in the first point of Proposition RA3). We can see that if $B(e_i^*,H)$ is concave in H, S/W must be concave in H; if S/W is convex in H, $B(e_i^*;H)$ must be convex in H. Where $N=1$, the situation is more complex because the single investor internalizes the whole market and, further, faces an additional tradeoff between allocating additional wealth to funds to increase returns and diluting returns on wealth already in funds. If investors have no additional wealth to allocate to funds (i.e., $S/W=1$), the market is at a corner solution and H has no effect on S/W.

PROPOSITION RA4, Net Alpha Sensitivity to Concentration.

1) Where $N\to\infty$ and $S/W<1$, we have

a. $$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right] \left\{1 - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{d\left(S/W\right)}{dX(e_i^*;H)}\right\}.$$

$d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $A_H(e_i^*;H) - c_{2H}(e_i^*;H) \geq 0(<0)$ (where concentration is higher, the equilibrium optimal expected net alphas are larger (smaller) if and only if this higher concentration induces a larger (smaller) impact on gross alphas than on costs).

b. $$\left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{d^2B(e_i^*;H)}{dH^2}\left\{1 - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{d(S/W)}{dX(e_i^*;H)}\right\} + 6\sigma_b^2\gamma\frac{S}{W}\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \times\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]^2\left[\frac{d\left(S/W\right)}{dX(e_i^*;H)}\right]^3.$$

If $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \leq 0$, then $d^2B(e_i^*;H)/dH^2 \leq 0$, (the fact that $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is concave in H indicates that $B(e_i^*;H)$ is concave in H). If $d^2B(e_i^*;H)/dH^2 > 0$, then $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} > 0$ (the fact that $B(e_i^*;H)$ is convex in H indicates that $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is convex in H).

2) Where $N \to \infty$ and $S/W = 1$, we have

a. $d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = A_H(e_i^*;H) - c_{2H}(e_i^*;H)$.

$d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $A_H(e_i^*;H) - c_{2H}(e_i^*;H) \geq 0(<0)$ (where concentration is higher, the equilibrium optimal expected net alphas are larger (smaller) if and only if this higher concentration induces a larger (smaller) impact on gross alphas than on costs).

b. $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = d^2B(e_i^*;H)/dH^2$.

$d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $d^2B(e_i^*;H)/dH^2 \geq 0(<0)$

($\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is convex (concave) in H if and only if $B(e_i^*;H)$ is convex (concave) in H).

3) Where $N=1$ and $S/W<1$, numerical solutions are required to analyze the signs of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and $d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$.

4) Where $N=1$ and $S/W=1$, we have

a. $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = A_H(e_i^*;H) - c_{2H}(e_i^*;H)$.

$d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $A_H(e_i^*;H) - c_{2H}(e_i^*;H) \geq 0(<0)$ (where concentration is higher, the equilibrium optimal expected net alphas are larger (smaller) if and only if this higher concentration induces a larger (smaller) impact on gross alphas than on costs).

b. $d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = d^2B(e_i^*;H)/dH^2$.

$d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $d^2B(e_i^*;H)/dH^2 \geq 0(<0)$

($\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is convex (concave) in H if and only if $B(e_i^*;H)$ is convex (concave) in H .

The intuition of Proposition RA4 is as follows. Where $N \to \infty$ and S/W, a higher H influences $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ at two stages. At the first stage, it changes managers' ability to produce expected net alphas, which is represented by the first component of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$. At the second stage, investors react to the changes in fund expected net alphas by adjusting the investment level to the funds, consequently affecting $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ under a decreasing returns to scale framework. This effect is represented by the second component of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$. If investors are risk-neutral, they adjust their investment level merely based on the changes in fund expected net alphas, driving $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ to zero, so the second component of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is zero (see Proposition RN4 in Appendix). However, if investors are risk-averse, their risk-return tradeoff makes their reaction to changes in fund expected net alphas less intense. That is, they subdue their additional investments to funds when inferring higher fund expected net alphas and limit their reduction in investments to funds when observing lower fund expected net alphas, inducing a positive value in the second component of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ (i.e., $1-\left[\hat{b}+\left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] d(S/W)/dX(e_i^*;H)>0$). Therefore, whether a higher H increases $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ depends on whether it has a larger impact on gross alphas than on costs (i.e., the sign of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ depends only on the sign of $A_H(e_i^*;H)-c_{2H}(e_i^*;H)$). As in equilibrium, $B'(e_i^*;H)=A_H(e_i^*;H)-c_{2H}(e_i^*;H)$, so whether a higher H increases $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ depends on whether it increases $B(e_i^*;H)$. Also, as H changes, the change of marginal $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ (i.e., $d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$) is positively proportional to the change of marginal $B(e_i^*;H)$, i.e., $d^2B(e_i^*;H)/dH^2$ plus a positive adjustment term that captures the effects of risk. This adjustment term is positive because, holding all other parameters the same, if investors' marginal portfolio risks of investing in funds are higher, investors optimally invest less in funds. In doing so, they exert a smaller impact on net alphas; thus, a higher H induces a higher marginal $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$. We can see that if $d^2B(e_i^*;H)/dH^2$ is positive, $d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ must be positive, whereas if $d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is negative, $d^2B(e_i^*;H)/dH^2$ must be negative. Where $S/W=1$, whether $N\to\infty$ or $N=1$, $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is equal to $A_H(e_i^*;H)-c_{2H}(e_i^*;H)$, and $d^2\mathrm{E}(\alpha_i \mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is equal to $d^2B(e_i^*;H)/dH^2$. This is because investors have no additional wealth to allocate to funds, so they exert no impact on marginal $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$, making the marginal equilibrium optimal expected net alphas depend only on the effect of H on managers' ability to produce net alphas. Where $N=1$ and $S/W<1$, the single investor faces an additional tradeoff between allocating additional dollars to funds to take advantage of fund net alphas and diluting returns on wealth already in funds. In this case, numerical solutions are required to analyze the signs of $d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$.

PROPOSITION RA5, Relation between Net Alpha and Market Share.

Whether $N \to \infty$ or $N = 1$, an increase (decrease) in $c_{1,i}$, while $c_{1,j}, \forall j \neq i$ are unchanged, induces a decrease (increase) in s_i / S, and an increase (decrease) in $s_j / S, \forall j \neq i$. Also,

1. Where $N \to \infty$ and $S/W < 1$, or $S/W = 1$, an increase (decrease) in $c_{1,i}$, while $c_{1,j}, \forall j \neq i$ are unchanged, induces a decrease (increase) in $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$; thus, $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i / S are positively related—internality effect; it induces a decrease (increase) in $\mathrm{E}(\alpha_j \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}, \forall j \neq i$; thus, $\mathrm{E}(\alpha_j \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_j / S are negatively related $\forall j \neq i$—externality effect.
2. Where $N = 1$ and $S/W < 1$, numerical solutions are required to analyze the relation between $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i / S.
3. Where $N = 1$ and $S/W = 1$, an increase (decrease) in $c_{1,i}$, while $c_{1,j}, \forall j \neq i$ are unchanged, induces a decrease (increase) in $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$; thus, $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i / S are positively related—internality effect; it induces a decrease (increase) in $\mathrm{E}(\alpha_j \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}, \forall j \neq i$, thus $\mathrm{E}(\alpha_j \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_j / S are negatively related $\forall j \neq i$—externality effect.

The intuition of Proposition RA5 is as follows. Based on Proposition 3, we can see that any change in $c_{1,i}$, keeping $c_{1,j}, \forall j \neq i$ unchanged, results in a change in s_i / S in the opposite direction and a change in $s_j / S, \forall j \neq i$ in the same direction. Also, a higher $c_{1,i}$, affects $\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ at the two stages. At the first stage, it decreases manager i's average cost and, thus, induces higher fund expected net alphas. As manager i offers a higher fund expected net alpha, investments shift into fund i from other funds, making all those funds' fund expected net alphas higher due to decreasing returns to scale at fund level. At the second stage, an increase in fund expected net alphas attracts investments in the AFMI, which in turn drives down fund expected net alphas due to decreasing returns to scale at industry level. Where $N \to \infty$ and $S/W < 1$, as investors' portfolio risks increase (decrease) when they invest more (less) in the funds. They subdue investments to funds when observing an increase in fund expected net alphas and limit investment reduction when observing a decrease in fund expected net alphas. Thus investors' risk-aversion mitigates the countered effect at the second stage and makes the first stage's effect dominant. Where $S/W = 1$, whether $N = 1$ or $N \to \infty$, investors have no additional wealth to allocate to funds, so their investments have no impact on marginal equilibrium optimal expected net alphas, causing the first stage's effect to dominate. In all these cases, we find $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$'s are driven down by an increase in $c_{1,i}$, keeping $c_{1,j}$, $\forall j \neq i$ unchanged; consequently, we have a positive relation between $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ and s_i / S (internality effect) and a negative relation between $\mathrm{E}(\alpha_j \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ and s_j / S, $\forall j \neq i$ (externality effect). Where $N = 1$ and $S/W < 1$, the single investor faces an additional tradeoff between allocating additional dollars to funds to take advantage of fund expected net alphas and diluting returns on wealth already in funds. This situation is more complex, and we rely on a numerical solution to solve it.

Proof of Proposition RA2, RA3, RA4, RA5 and the corresponding corollary. See the Appendix.

2.3. Numerical Example

We provide a numerical analysis of the AFMI under our framework and set the parameter values as follows: $W = 100$, $M = 100$, $\mu_p = 0.05$, $\sigma_p = 0.1$, $\sigma_x = 0.05$, $\hat{a} = 0.15$, $\hat{b} = 0.3$, $\sigma_a = 0.4$, $\sigma_b = 0.4$, and $\sigma_{ab} = 0$.

To simplify the case, we assume the parameters in the average cost functions are the same across funds (thus we can drop the indicator i); so, in equilibrium, funds have same levels of effort, fees, and sizes. We assume the functional form of $A(e_i; H)$, for numerical analysis, is $A(e_i; H) = (H + A_0)\ln(1 + e_i)$, where A_0 is a positive parameter. Also, the functional form of $c_2(e_i; H)$ is $c_2(e_i; H) = c_2^1 (H + c_2^2) e_i^2$. The parameters of $A(\bullet; H)$ and $C^i(\bullet, \bullet; H)$ are set as follows: $c_0 = 0.005$, $c_1 = 0.1$, $c_2^1 = 2.5$, $c_2^2 = 0.01$,

$A_0 = 0.5$.

We choose 100 points evenly spread on [0, 0.999] to be the value of the market concentration level H and study how equilibrium values change with H.

Figure 2.1 illustrates the numerical results for the case of infinitely many mean-variance risk-averse investors. We can see that e_i^* decreases with market concentration because our numerical calibration makes H's impact on marginal effort impact on costs larger than marginal effort impact on gross alphas, across the domain of H. Also, f_i^* decreases with H when H is small and increase with H when H becomes large because managers' costs decrease with H first and then slightly increase with H. We also have S/W increase with $X(e_i^*, H)$, as we expect in our model. Moreover, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ is always positive, consistent with our model. Also S/W, $B(e_i^*; H)$ and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ first decrease with H and then slightly increase with H because the difference of H's impact on gross alphas and its impact on costs first decreases with H and then slightly increases with H. In addition, S/W, $B(e_i^*; H)$, and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ are convex at the same time, in our calibration.

Figure 2.2 demonstrates the numerical results where there is a single mean-variance risk-averse investor in the market. The results are similar except that S/W is much smaller and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ is much larger. The reason is that the single investor internalizes the AFMI, limiting the investments in the funds and maximizing his or her portfolio Sharpe ratio.

Figure 2.1. Infinitely Many Small Mean-Variance Risk-Averse Investors—Comparative Statistics with Respect to Market Concentration

This figure presents the numerical results for the case of infinitely many small mean-variance risk-averse investors. The two subplots on the top illustrate the equilibrium optimal management effort and management fees at each market concentration level. The two subplots in the middle report $B(e_i^*, H)$ at each market concentration level and the equilibrium *S/W* ratio at each $X(e_i^*, H)$ level. The two subplots at the bottom show the equilibrium *S/W* ratio and the equilibrium fund expected net alphas at each market concentration level.

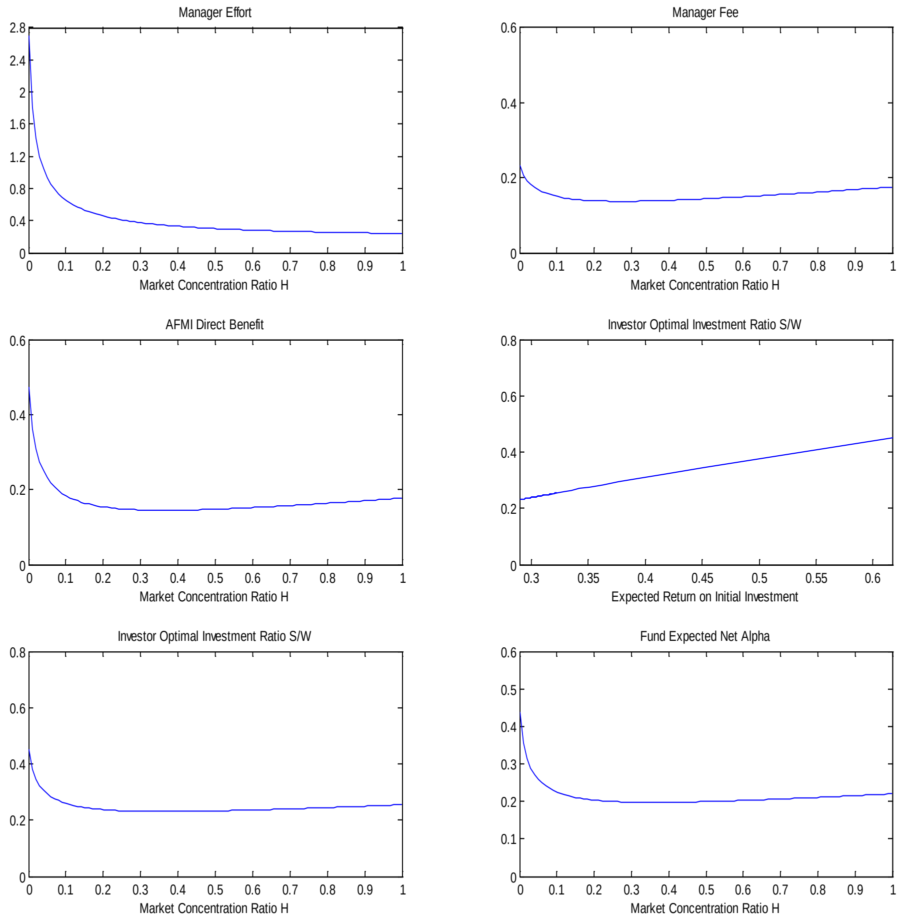
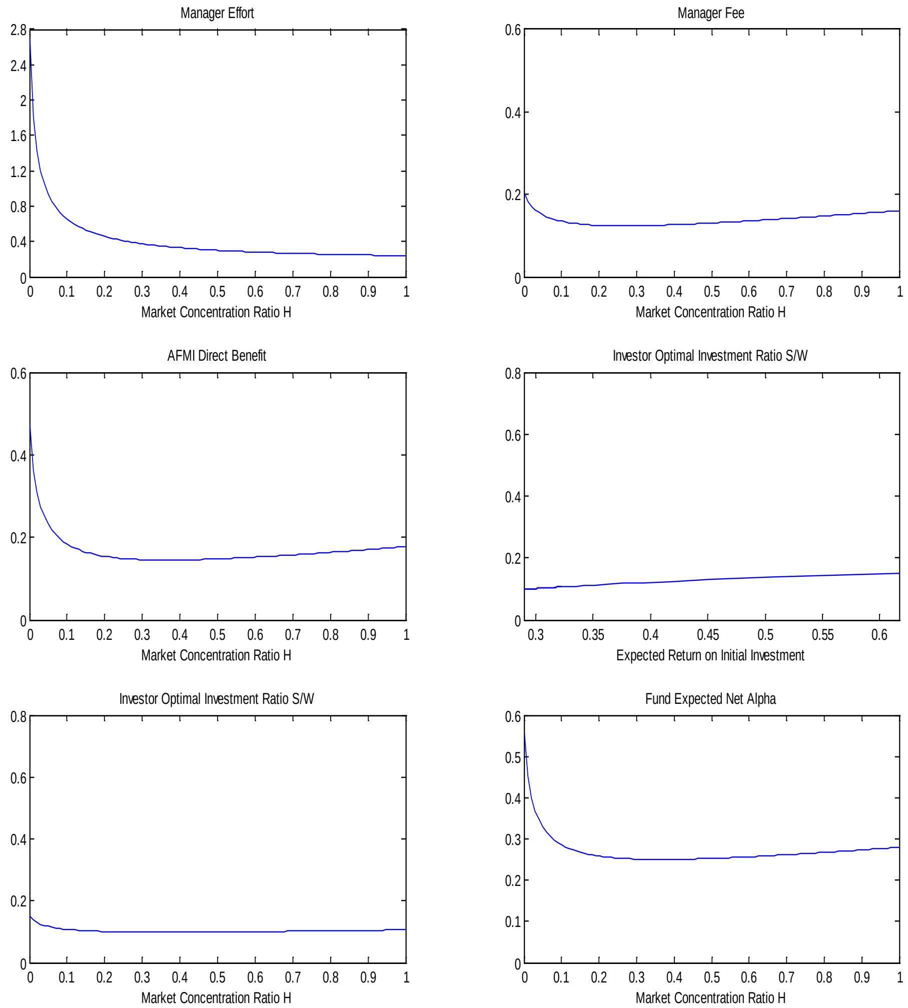


Figure 2.2. A Single Large Mean-Variance Risk-Averse Investor—Comparative Statistics with Respect to Market Concentration

This figure presents the numerical results for the case of a single large mean-variance risk-averse investor. The two subplots on the top illustrate the equilibrium optimal management effort and management fees at each market concentration level. The two subplots in the middle report $B(e_i^*, H)$ at each market concentration level and the equilibrium *S/W* ratio at each $X(e_i^*, H)$ level. The two subplots at the bottom show the equilibrium *S/W* ratio and the equilibrium fund expected net alphas at each market concentration level.



2.4. Agency Benefits Due to Market Concentration

A higher fund market concentration implies that fund managers may earn higher agency benefits and charge investors higher fees. Modeling that, we can decompose the management fee into two parts:

$$f_i = fa(H) + fe_i, \ \forall i, \tag{2.28}$$

where $fa(H)$ represents agency benefits, in terms of percentage fees, that managers (can) charge under a particular level of industry concentration H, and fe_i is the endogenous component chosen by manager i. We assume that $fa(H)$ is the same for all managers, and

$$fa(H) \geq 0, \forall H, \tag{2.29}$$

$$fa'(H) \triangleq dfa(H)/dH > 0, \forall H, \tag{2.30}$$

$$fa(1) < \infty \text{, and} \tag{2.31}$$

$$fa(0) = 0. \tag{2.32}$$

The rationale for these assumptions is as follows. Agency benefits are non-negative; the higher the market concentration is, the higher agency benefits are. Agency benefits are highest under monopolistic markets and are bounded from above; the agency costs on investors are zero if the AFMI is perfectly competitive with infinitely many small fund managers.

Competing Managers

In a market with managers competing for investments, fe_i can be positive or negative. If it is positive, manager i charges an additional fee on top of agency benefits; if it is negative, manager i is subsidizes investors in order to increase investments in the fund. Let managers' optimal fee be

$$f_i^* = fa(H) + fe_i^*, \ \forall i. \tag{2.33}$$

Our previous analysis demonstrated that in equilibrium, fund managers charge break-even fees in order to compete for investments; so regardless of agency cost levels $fa(H)$, managers choose fe_i's such that f_i^*'s are break-even fees. As managers charge break-even fees in equilibrium, $B(e_i^*;H)$ is transferred to investors in its entirety. We call the dollar amount of agency costs Ψfa; thus,

$$\Psi fa = fa(H)\frac{S}{W}W \ . \tag{2.34}$$

As $fa(H)$ increases with H, we know that if S/W increases with H, then Ψfa increases with H. If S/W decreases with H, then whether Ψfa increases with H depends on the rate that $fa(H)$ increases with H relative to the rate that S/W decreases with H. The curvature of Ψfa in H is inconclusive.

We call the dollar amount of AFMI's direct benefits from managers' optimal effort ΨB; thus,

$$\Psi B = B(e_i^*;H)\frac{S}{W}W \ . \tag{2.35}$$

Then, if $A_H(e_i^*;H) - c_{2H}(e_i^*;H)$ is positive (negative), $B(e_i^*;H)$ and S/W both increase (decrease) with H, inducing ΨB to increase (decrease) with H. It is possible that $fa(H) > B(e_i^*;H)$ and $\Psi fa > \Psi B$. That is, under particular levels of market concentration, managers' agency costs surpass AFMI's direct benefit from managers' effort. Still, because of managers' competition for investments in their funds, they would set fe_i^* to be negative to subsidize investors' investments such that managers are earning break-even fees. Investors, however, only care about and only observe f_i^*; they do not observe f_i^*'s components and cannot base their decisions on either $fa(H)$ or fe_i^*.

Colluding Managers

Managers, may use market power to collude in charging fees higher than those that endogenously arise in the previous non-collusive equilibrium.[33] We assume that their (market) power to charge higher fees is increasing with industry's concentration level and that managers agree on a single collusive fee, $f(H)$. Thus, we now have

$$f_i = fa(H) = f(H), \ \forall i .^{34} \tag{2.36}$$

We consider the case where $N \to \infty$. Conditional on the industry collusive fee rate, managers maximize their corresponding funds' profits by exerting optimal effort levels, incorporating investors' optimal reactions. We can concisely write the total

[33] Under our functional assumptions, it is irrational for managers to collude in lowering fees.
[34] Please note that Equations (2.29) to (2.32) still hold.

industry profit function, Π, as a sum of the industry funds profit functions:

$$\Pi^*(e_1^*, e_2^*, \ldots, e_M^*; H) \triangleq \underset{\{e_i\}_{i=1}^M}{\operatorname{Max}} \Pi(e_1, e_2, \ldots, e_M; H) = \underset{\{e_i\}_{i=1}^M}{\operatorname{Max}} \sum_{i=1}^M s_i \left(f(H) - C^i(e_i, s_i; H) \right), \quad (2.37)$$

subject to

$$f(H) - C^i(e_i, s_i; H) \geq 0, \;\; \forall i, H \,. \quad (2.38)$$

Optimal effort levels e_i^* are given by the first-order-condition

$$\sum_{i=1}^M \frac{A_{e_i}(e_i^*; H)\left[f(H) - C^i(e_i^*, s_i; H)\right] - \frac{S}{W}\left[c_{1,i}\left(\sum_{i=1}^M c_{1,i}{}^{-1}\right)^{-1} WA_{e_i}(e_i^*; H) + c_{2e_i}(e_i^*; H)\right]}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b}} = 0$$

$$, \quad (2.39)$$

where S/W is given by

$$-\gamma\sigma_b^2\left(\frac{S}{W}\right)^3 - \left[\gamma\sigma_a^2 + \gamma\sigma_x^2 + \hat{b}\right]\frac{S}{W} + A(e_i^*, H) - f(H) = 0\,, \quad (2.40)$$

if $S/W < 1$, and $S/W = 1$ otherwise. The second-order-condition is satisfied, so e_i^* is a maximum point. Also, in equilibrium, where $S/W < 1$, we have

$$\frac{d\left(S/W\right)}{dH} = \frac{A_{e_i}(e_i^*; H)e_i^*{}'(H) + A_H(e_i^*; H) - f'(H)}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b}}\,, \quad (2.41)$$

where $f'(H) \triangleq df(H)/dH$, and

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} =$$

$$\frac{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right]\left[A_{e_i}(e_i^*; H)e_i^*{}'(H) + A_H(e_i^*; H) - f'(H)\right]}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b}}\,. \quad (2.42)$$

Therefore, both $d\left(S/W\right)/dH \geq 0(<0)$ and $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} \geq 0(<0)$ if and only if $A_{e_i}(e_i^*; H)e_i^*{}'(H) + A_H(e_i^*; H) - f'(H) \geq 0(<0)$. The economic sense is that if in equilibrium, a higher concentration increases fund gross alphas more than the exogenous fees, managers optimally choose efforts to produce higher fund expected net alphas to attract investments that increase industry profits. Where $S/W=1$, managers choose the minimum e_i^* to make $S/W=1$. The reason is that if managers can optimally induce investors to invest all their wealth in funds, they choose the minimum effort to do so. In this case, both S/W and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ are unaffected by H, so

$$\frac{d\left(S/W\right)}{dH}=0, \tag{2.43}$$

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=0\cdot \tag{2.44}$$

The results of $d^2\left(S/W\right)/dH^2$ and $d^2\mathrm{E}(\alpha_i \mid D)/dH^2|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ require numerical analysis.

To choose the optimal collusion fee, a "collusion planner" would write a mapping from H to $f(H)$, where each point in the range is the one for which industry profits, Π^*, are the highest.

Discussion of Covariance between $\hat{a}$ and $\hat{b}$

We assume that $\sigma_{ab}=0$, but we note that the value of σ_{ab} affects the equilibrium results because it affects portfolio risks. If σ_{ab} (in absolute value) is large relative to other risk sources, such as σ_a^2, σ_b^2, and σ_x^2, changes in investors' wealth allocations to funds, would induce changes in their portfolio risks, affecting in turn their optimal demands. This would make our theoretical results in propositions RA3, RA4, RA5 and results in Section 2.5 more complex. We believe that consequences of such an analysis would not be directly material to the issues that we explore here and would obfuscate the analysis. We, thus, assume that precisions of estimates of a and b, conditional on current information, are not closely related, making $\sigma_{ab}\to 0$.

2.5. Endogenous Market Concentration Level and Empirical Analysis

Our model allows for an endogenous market concentration level. If we define H as the Herfindahl-Hirschman index (HHI), which is the sum of market shares squared, then for an M firms' market $H \in [\frac{1}{M}, 1)$. [35] Using funds' equilibrium market share, as identified in Proposition 3, we can write the equilibrium market concentration H^* as

$$H^* \triangleq \sum_{i=1}^{M}\left(\frac{S_i}{S}\right)^2 = \sum_{i=1}^{M}\left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}{}^{-1}\right)\right)^{-2}. \tag{2.45}$$

We can see that H^* is determined by $c_{1,i}$'s. Specifically, depending on the size of $c_{1,i}$ relative to that $c_{1,j}$, $\forall j \neq i$, an increase in $c_{1,i}$, holding $c_{1,j}$, $\forall j \neq i$ constant, increases or decreases H^*.

In the case where there are infinitely many risk-averse investors, an increase in $c_{1,i}$ affects the equilibrium fund expected net alphas in two ways: 1) its direct impact leads to lower equilibrium fund expected net alphas (Proposition RA5), and 2) depending on fund i's size relative to rivals, it increases or decreases H^*, which consequently increases (decreases) equilibrium fund expected net alphas if and only if $A_H(e_i^*;H^*) - c_{2H}(e_i^*;H^*) \geq (<) 0$ (Proposition RA4). Similarly, an increase in $c_{1,i}$ affects the equilibrium AFMI size in two ways: 1) its direct impact leads to an (inverse direction) AFMI size change, and 2) it increases or decreases H^*, which consequently increases (decreases) the equilibrium AFMI size if and only if $A_H(e_i^*;H^*) - c_{2H}(e_i^*;H^*) \geq (<) 0$. Thus, in the endogenous market concentration case, the relation between the market concentration and the equilibrium fund expected net alphas and AFMI size is more complex.[36]

In the next chapter, we proceed with an empirical analysis of the benefits and costs of changing market concentration levels of the AFMI using the version of our model with endogenous concentration. In this sense, this version of our model befits

[35] In an M AFMI, for example, the Herfindahl-Hirschman index could have values between the highest concentration, 1, where one of the funds captures practically all the market share, and the lowest concentration, $1/M$, where market shares are evenly divided.

[36] We believe that our cost function, Equation (2.8), is a concise one that essential effects within our model. To assure that all our functional form restrictions of the non-specialized model (exogenous concentration levels), which we deem basic and simple, hold in the specialized one (endogenous concentration levels); however, we need to impose additional, technical, "second order," parameter restrictions. For brevity and simplicity, we do not impose these restrictions. We call the parameter values that make the specialized model abide by these restrictions *plausible*. We, later, confirm that the said technical restrictions are not empirically binding. That is, imposing these restrictions would not have changed our empirical results. In other words, the empirically estimated parameters fall within the plausible parameters range.

available data of empirical market concentration levels, such as the HHI. Popular empirical market concentration measures, such as HHI, are functions of rivals' relative sizes. We expect that market characteristics, such as regulation, transaction costs, tax rates, and barriers to entry, affect funds' cost sensitivity to size (i.e., $c_{1,i}$'s). As a result, they affect relative fund sizes and, thus, the level of empirical market concentration measures. We use empirical techniques to control potential endogeneity of market concentration measures.

Whether fund net alphas and AFMI size move in the same direction with market concentration and whether both are concave (convex) in it become empirical questions. Further, in cases where active fund management creates value, if fund net alphas and AFMI size increase (decrease) with market concentration, our model predicts positive (negative) benefits of marginal managerial effort, for plausible parameter values. We note that both signs of the benefits of changing concentration levels are plausible alternatives to a null hypothesis of no benefit of active fund manager effort.

2.6. Conclusion

We develop a theoretical model to analyze an AFMI equilibrium where we investigate performance, size, and managers' costly (optimal) effort under a continuum of exogenous market concentration levels. We use Pastor and Stambaugh's (2012) framework, where gross alpha production is of decreasing returns to scale at the industry level, and we similarly model the decreasing returns to scale effect at the fund level. Higher market concentration levels imply better utilization of industry resources and the existence of more unexplored investment opportunities, making managers' efforts more productive. At the same time, however, higher market concentration levels allow managers to require higher compensation for effort, making effort costs higher.

Our model's comparative statistics characterize the association between fund net alphas and a continuum of exogenous market concentration levels, and that between AFMI size and market concentration. In particular, we consider the case of infinitely many mean-variance risk-averse investors whose portfolio risks increase with investments in funds. The funds' expected net alphas increase with market concentration if and only if higher concentration induces a larger impact on gross alpha production than on the costs of effort (i.e., higher concentration induces higher AFMI's direct benefits of net alpha production). Observing an increase in fund expected net alphas, due to higher market concentration, mean-variance risk-averse investors increase their mutual fund holdings but reach optimum investment levels at higher expected net alphas than before. Thus, the equilibrium fund expected net alphas become positively associated with market concentration. In addition, the concavity of fund expected net alphas in market concentration indicates that the AFMI's direct benefits of net alpha production are concave in market concentration. This further induces concavity of AFMI size in market concentration.

We also study the consequences of increased agency costs under higher market concentration levels in a market where managers can collude to pursue agency benefits. In addition, we specialize our model to allow for endogenous market concentration levels, which befits empirical market concentration measures and enables us to study the model empirically in the next chapter.

Chapter 3: U.S. Active Fund Management Industry Market Concentration, Fund Net Alphas, and Industry Size

3.1. Introduction

We use the Herfindahl-Hirschman index (HHI) and other indices as market concentration measures, to empirically study the implications of our model developed in the last chapter. In particular, we study how the fund net alphas and size of the U.S. active equity mutual fund industry change with market concentration.

Our empirical methodology uses Pastor, Stambaugh, and Taylor’s (2015) (PST) recursive-demeaning estimator to address endogeneity and omitted-variable-related issues, and uses vector auto-regression (VAR) techniques to account for simultaneity in determination of AFMI size and market concentration. We control for survival bias by using the Morningstar U.S. mutual fund database, which contains both surviving and terminated funds.

We find that both fund net alphas and AFMI size, on average, increase with market concentration. Moreover, both fund net alphas and AFMI size are concave in concentration. Our empirical results are robust to the use of alternative methods and measures.

Our empirical findings are consistent with our model’s theoretical implications under plausible parameter values, and have policy implications. Given the low market concentration in the current AFMI, and assuming no change in the tradeoff of managerial productivity and effort cost, increased market concentration is likely to increase both fund net alphas and AFMI size; under plausible parameter values, AFMI’s direct benefits also increase. The literature has shown multiple ways to increase fund market concentration. For example, Massa (2003) suggested that mutual fund families allow investors to move money across family funds of different categories at low costs, lowering effective fees and reducing competition among funds. Under our findings, formation of fund families in such markets might generate benefits in terms of higher net alphas and larger AFMI size.

Section 3.2 presents our empirical methodology. Section 3.3 describes our data. Section 3.4 reports our empirical results. Section 3.5 concludes.

3.2. Methodology

Our goal is to analyze how the AFMI size and fund net alphas change with market concentration of an AFMI. Next, we describe how we measure concentration, fund net alpha, and our econometric strategy to estimate the effect of changing concentration on net alpha and AFMI size, controlling for endogeneity and omitted-variable bias-related issues.

Measures of AFMI Concentration

Following the literature, we measure competitiveness of an AFMI using three indices (see, for example, Berger and Hannan (1989), Geroski (1990), Berger (1995), Goldberg and Rai (1996), Nickell (1996), Berger, Bonime, Covitz and Hancock (1999), Cremers, Nair, and Peyer (2008), and Giroud and Mueller (2011)):

1) the HHI:

$$H_t = \sum_{i}^{m_t} MS_{i,t}^2 \ ; \tag{3.1}$$

2) the normalized Herfindahl-Hirschman index (NHHI):

$$NHHI_t = \frac{m_t \times H_t - 1}{m_t - 1} \ ; \tag{3.2}$$

3) the sum of the first five largest funds' market shares (5-Fund-Index):

$$5\_Fund\_Index = \sum_{i=1}^{5} MS_{i,t} \ , \tag{3.3}$$

where $MS_{i,t}$ is the market share of fund i at time t, measured by the fund's assets under management at time *t* over the total assets under management in its market at time *t*, and m_t is the number of funds at time t. As the HHI is related to the number of funds, for a robustness check, we also use two measures not related to the number of funds to measure market concentration, the NHHI used by Cremers, Nair and Peyer (2008) and the 5-Fund-Index, one of the common measures of market concentration.

Choice of Benchmarks and Net Alpha Estimation

Berk and Binsbergen (2015) argue that to measure the value added by a fund, its performance should be compared to the next-best investment opportunity available to investors. In our model, we assume that a single passive benchmark exists and is common knowledge to investors and managers. We make these assumptions because they keep our theoretical analysis parsimonious and relaxing them does not alter the key insights from our model. However, in our empirical section, we allow for multiple benchmarks and match each active equity mutual fund to a set of tradable index funds that reasonably replicate passive alternatives available to an average mutual fund investor. Specifically, we assume the following return-generating process:

$$R_{i,t} = \alpha_{i,t} + b_{i,t}^1 F_t^1 + b_{i,t}^2 F_t^2 + \ldots + b_{i,t}^n F_t^n , \tag{3.4}$$

where the indices *i* and *t* represent the fund and time indices, whereas *n* indicates the number of tradable index funds in the market. $R_{i,t}$ is the return net of management fee of a fund, and F_t^1 through F_t^n are the returns net of management fees of tradable index funds in different asset classes. We use net alpha (instead of gross alpha) in our empirical analyses because we are interested in aspects of market competition and investor benefits predicted by our model, where net alpha is the relevant quantity. We treat the index funds F_t^1 through F_t^n as a basis fund set that may be used to replicate the returns on any passive benchmarks used by mutual fund investors.

To perform our analysis, we first need to calculate fund net alphas ($\alpha_{i,t}$). For each active fund in our sample, we calculate a set of weights on our basis fund set that sum to one and minimize the tracking error between the active fund return and a corresponding passive benchmark portfolio return (Sharpe (1992)). We note that our empirical design of identifying passive benchmarks, using matching tradable index funds, fits our theoretical structure, which assumes the appropriate passive benchmarks for each fund.

We perform this analysis on a rolling basis, using returns from months (*t* - 60) to (*t* - 1) to avoid look-ahead bias. That is, we identify coefficients $b_{i,t}^1$ to $b_{i,t}^n$ to minimize the variance of the residual. These coefficients are constrained to be between zero and one (we do not allow short selling), and their sum is constrained to be one. These coefficients identify the portfolio weights in our basis index fund set that provides the estimated minimum "tracking error" passive benchmark of a fund.

Next, to calculate a fund's net alphas in month *t*, we subtract the returns on the identified passive portfolio (the style benchmark) for month *t* from the active equity fund's returns in month *t* and that of the style benchmark in month *t*. This provides us with fund net alphas in each month for each fund.

To ensure the robustness of our results, we also use an alternative method to fund net alphas. Specifically, it is possible that traded index funds do not capture unobservable risk factors that drive excess returns. Errors in our set of passive benchmarks or our matching strategy may result in net alphas that measure exposure to such unobservable risk factors instead of fund manager performance. Using the method developed by Connor and Korajczyk (1988), we estimate unobserved common factors in our estimated fund net alphas using the principal components of our estimated fund net alphas series. We use these estimated principal components to control for unobserved common factors in the fund net alphas. Specifically, we regress each fund's fund net alphas on the first two principal components without a constant term. We refer to the residuals of these regressions as PC-adjusted fund net alphas and use them as the dependent variable for robustness checks.

Controlling for Endogeneity and Omitted-Variable Bias

Estimating the effect of market concentration on performance is a challenge because market concentration is determined endogenously. PST explain why a simple regression is likely to deliver biased estimates and introduce a recursive demeaning (RD) estimator to avoid the biases. In analyzing the relation between fund net alphas and market concentration, we use the RD estimation procedure of PST. We estimate the effects of size (β_1) and market concentration (β_2 and β_3) on fund net alphas using the following panel regression:

$$\overline{\alpha_{i,t}} = \beta_1 \overline{MS_{i,t-1}} + \beta_2 \overline{H_{t-1}} + \beta_3 \overline{H_{t-1}^2} + \overline{\varepsilon_{i,t}} \,. \tag{3.5}$$

The bar above the variables denotes forward-demeaned variables, defined below:

$$\overline{\alpha_{i,t}} = \alpha_{i,t} - \frac{1}{T_i - t + 1} \sum_{s=t}^{T_i} \alpha_{i,s} \,, \tag{3.6}$$

$$\overline{MS_{i,t}} = MS_{i,t} - \frac{1}{T_i - t + 1} \sum_{s=t}^{T_i} MS_{i,s} \,, \tag{3.7}$$

$$\overline{H_t} = H_t - \frac{1}{T_i - t + 1} \sum_{s=t}^{T_i} H_s \,, \tag{3.8}$$

$$\overline{H_t^2} = H_t^2 - \frac{1}{T_i - t + 1} \sum_{s=t}^{T_i} H_t^2 \,, \tag{3.9}$$

where T_i is the number of time-series observations of fund i. We run robustness checks by replacing H_t (HHI) with the NHHI and with the 5-Fund-Index.

The RD method in Equation (3.5) can control for the fund fixed effect. We include market share as a control, not only because the equilibrium market share provides information on a fund's cost sensitivity to fund size (Proposition 3), but also because current empirical studies show a linear relation between changes in market share and fund performance (Spiegel and Zhang (2013)) and use it as a firm-level market power measure (e.g., Berger, Bonime, Covitz and Hancock (1999) and Nickell (1996)). There may be potential endogeneity (reverse causality) between AFMI shares and fund net alphas because when fund net alphas are higher, corresponding asset values increase and funds attract investments, both leading to a higher market share. This endogeneity issue may bias our results. We address this endogeneity issue using an instrumental variable method. In the first stage, we regress $\overline{MS_{i,t-1}}$ (recursively forward-demeaned market share) on $\underline{MS_{i,t-1}}$ (recursively backward-demeaned market share) without a constant term. In the second stage, we use the fitted value from the first stage to run Equation (3.5),[37] where

$$\underline{MS_{i,t}} = MS_{i,t} - \frac{1}{t-1}\sum_{s=1}^{t-1} MS_{i,s} \,. \tag{3.10}$$

To be a valid instrument of $\overline{MS_{i,t-1}}$, $\underline{MS_{i,t-1}}$ must satisfy the relevance and exclusion conditions. The relevance condition is likely to hold because both $\overline{MS_{i,t-1}}$ and $\underline{MS_{i,t-1}}$ are derived from $MS_{i,t-1}$ and are, thus likely to be closely related. The exclusion condition is also likely to hold because the backward-looking information in $\underline{MS_{i,t-1}}$ is unlikely to be helpful in predicting the forward-looking net alpha information in $\overline{\varepsilon_{i,t}}$, where $\overline{\varepsilon_{i,t}}$ is the residual in the RD method.

On the other hand, there is no reason to believe that fund net alphas, as individual fund performance, are endogenous with the market concentration ratios as industry-level measures. In particular, there is no reason to believe that innovations in market concentration are correlated with the error term in the regression. Thus, we directly use the recursive forward-demeaned market concentration ratios in the model.

In analyzing the relation between the AFMI size and market concentration, we use the vector auto-regression (VAR) method. Although theoretically we assume

[37] We correct the second-stage standard error estimates of β_1 by incorporating the estimation errors from the first-stage regression.

market concentration is exogenous, empirically this industry-level variable may be endogenous with another industry-level variable, the industry size. The VAR method can address this potential endogeneity issue, and our model is

$$\begin{bmatrix} IS_t \\ H_t \\ H_t^2 \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix} + \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_1 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \times \begin{bmatrix} IS_{t-1} \\ H_{t-1} \\ H_{t-1}^2 \end{bmatrix} + \begin{bmatrix} e_{a,t} \\ e_{b,t} \\ e_{c,t} \end{bmatrix}, \tag{3.11}$$

where IS_t is AFMI size at time t and $e_{a,t}$, $e_{b,t}$ and $e_{c,t}$ are the residuals.

3.3. Data

We obtain our data from Morningstar Direct. Our sample contains 1,374 actively managed U.S. equity mutual funds from January 1979 to December 2014. The Appendix supplements the data description below.

We use keywords in Morningstar to identify U.S. mutual funds (both open-end and closed-end) and exclude index funds, enhanced index funds, funds of funds, and in-house funds of funds. Also, we require funds to be classified as Equity in the Global Broad Category Group, and we further exclude international funds, real estate funds, and sector funds. Next, we use the Fund ID provided by Morningstar to aggregate fund share class-level information to fund-level information. Since we use a 5-year rolling window to estimate fund net alphas, we require each of our active equity mutual funds to have at least 10 years' return observations.[38] Using these filters, we obtain our sample of 1,374 actively managed U.S. equity mutual funds.

The index funds used in the style-matching model are also from Morningstar. We require index funds to have no missing observations in our sample period so that the style-matching model is consistent and stable. The factors used in the style-matching model include index funds with the Morningstar Institutional Categories of Small Core, Large Core and S&P 500 Tracking, and the CRSP Fama-French risk-free rate. All the fund returns are net of administrative and management fees and other costs taken out of fund assets.

Table 3.1 reports the summary statistics. It shows that monthly fund returns are positive on average but vary a great deal, from smaller than -14% to more than 13%, with a standard deviation of more than 5%. Monthly fund net alphas are positive on

[38] We also omit some rare cases where there is a gap with more than 5 years' return observations missing.

average, also with a wide variation. We also report summary statistics of the fit of our passive benchmark-matching method using *R*-squared, which is measured as

$$Rsqr_{i,t} = 1 - \frac{Var(\alpha_{i,t})}{Var(R_{i,t})}, \tag{3.12}$$

where $Var(\alpha_{i,t})$ is the variance of the residuals of the regression, and $Var(R_{i,t})$ is the variance of $R_{i,t}$. The average R-squared in our style-matching model are quite high at 0.86, and the variation around the mean is relatively small.

To analyze robustness, we redo our analyses using fund net alphas adjusted by the first two principal components. The values of the monthly relative industry size (total funds' net assets divided by stock market capitalization) and the monthly fund sizes in December 2014 dollars (funds' net assets divided by stock market capitalization in the same month, multiplied by the stock market capitalization in December 2014) are similar to the sample in PST.

The number of active equity mutual funds in our sample increases over time. The market concentration measures, such as the HHI, NHHI, and 5-Fund Index, with fluctuations, tend to decrease over time. Also, all three market concentration measures do not seem to perform with skewness.

Table 3.1. Summary Statistics

Our sample period is from January 1979 to December 2014, and monthly data is used. Panel A reports the summary statistics for fund-level data, and Panel B reports those for industry-level data. Fund Net Return and Fund Net Alpha are in percentages, and both are net of administrative and management fees and other costs taken out of fund assets. The Style-Matching Model R-sqr, AFMI Share, HHI, NHHI, and 5-Fund-Index are in decimals. Fund Size is measured in $100 millions and is equal to the fund's total net assets under management, divided by the stock market capitalization in the same month, and multiplied by the stock market capitalization in December 2014. Industry Size is the sum of funds' net assets under management divided by the stock market capitalization in the same month. Number of Funds is in units.

				Percentile				
Variable	Obs.	Mean	Std.	1st	25th	50th	75th	99th
Panel A: Fund-Level Data								
Fund Net Return (%)	321456	0.8736	5.1508	-14.4922	-1.7976	1.2998	3.8907	13.0053
Fund Net Apha (%)	246553	0.0349	1.9499	-5.4465	-0.8570	0.0215	0.9156	5.5982
Style-Matching Model R-sqr (decimal)	246557	0.8607	0.1175	0.4223	0.8178	0.8953	0.9408	0.9894
Fund Size (in 100 Million of 2014 Dec Dollars)	314083	28.7796	95.3306	0.0399	1.3833	5.5718	20.1835	416.9203
Fund Market Share (decimal)	314083	0.0012	0.0041	0.0000	0.0000	0.0002	0.0007	0.0185
Panel B: Industry-Level Data								
Industry Size (decimal)	432	0.0982	0.0591	0.0200	0.0389	0.1035	0.1638	0.1801
Number of Funds (No.)	432	850.2	659.5	86.0	249.0	677.5	1468.5	2126.0
HHI (decimal)	432	0.0191	0.0230	0.0061	0.0101	0.0157	0.0243	0.0382
NHHI (decimal)	432	0.0157	0.0139	0.0057	0.0094	0.0141	0.0201	0.0269
5-Fund-Index (decimal)	432	0.2166	0.0765	0.1240	0.1640	0.1986	0.2650	0.3438

Because our sample differs from PST, we check for any alarming systematic differences by evaluating the returns to scale relation in our sample. Table 3.2 reports the estimated relation of fund net alpha and fund size and fund industry size, using PST's RD model. We find that fund net alpha is significantly negatively associated with lagged industry size and is insignificantly negatively associated with lagged fund size. In an unreported robustness check, we replace lagged fund size by lagged log of fund size and find consistent results. Thus, we find evidence of decreasing returns to scale at the industry level. These findings are consistent with PST's.

Table 3.2. Sample Check for Decreasing Returns to Scale Assumption

This table reports the results of RD panel regression model, following PST. Fund Net Alpha is the dependent variable. Fund Size is measured in $100 millions, and is equal to the fund's total net assets under management, divided by the stock market capitalization in the same month, multiplied by the stock market capitalization in December 2014. Industry Size is the sum of AFMI's net assets under management divided by the stock market capitalization in the same month. The unit of coefficients is percentage. Standard errors are clustered by fund and presented in parentheses. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level in a two-tail t-test, respectively.

Dependent Variable	Fund Net Alpha		
	(1)	(2)	(3)
Lagged Fund Size	-0.0005		-0.0007
	(0.0010)		(0.0010)
Lagged Industry Size		-1.0211***	-1.0388*
		(0.1302)	(0.5524)
Observations	239,537	245,178	239,537
R-Sqr	0.0000	0.0003	0.0003
Adjusted R-Sqr	0.0000	0.0003	0.0003

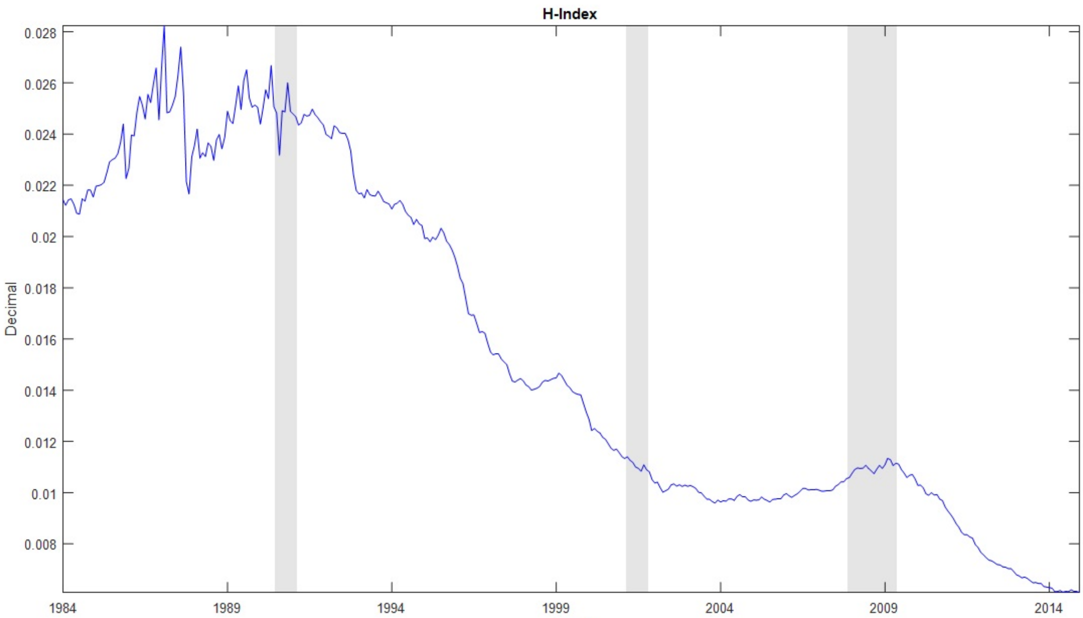
3.4. Empirical Results

In this section, we implement our theoretical model to empirically evaluate effects of market concentration levels on the U.S. mutual fund industry.

Figure 3.1 shows the HHI value from January 1984 to December 2014. We can see that before 1990, the HHI value was relatively high, fluctuating from 0.02 to 0.03. After that, it continued decreasing; and in the current years, it has reached 0.006, which is around a quarter of the values before 1990. This figure shows that the concentration of the U.S. active equity mutual fund market decreased substantially. Alternative market concentration measures, such as NHHI and 5-Fund Index, show similar trends.

Figure 3.1. HHI of the U.S. Active Equity Mutual Fund Market

The HHI value is in decimals. The gray bars represent the recession periods. The sample period is from January 1984 to December 2014



We first evaluate the relation between fund net alphas and market concentration. The results using the RD method are shown in Table 3.3. Panel A reports the results using fund net alpha as the dependent variable. In the first two columns, we find that the coefficient of the first-order term of lagged HHI is significantly positive, whereas the coefficient of the second-order term is significantly negative. This result is robust to including lagged market share and lagged industry size as controls. This suggests that the effect of concentration is distinct from the effect of decreasing returns to scale at the fund and industry level. To control for the possibility of unaccounted common factors in the estimated net alphas, we also use principal component (PC)-adjusted fund net alphas as the dependent variable (Panel B) and find similar results.

The main result of this table is that fund net alphas, on average, are increasing concave in fund market concentration. Our theoretical results, then, indicate that for plausible parameter values, higher levels of market concentration induce increases in gross alpha production opportunities that are higher than those in managers' effort costs.

Table 3.3. Market Concentration and Fund Net Alpha

This table reports the results of our RD panel regression model. Panel A reports the results using the Fund Net Alpha as the dependent variable, whereas Panel B reports the results using the PC-Adjusted Fund Net Alpha (adjusted by the first two principal components of fund net alphas) as the dependent variable. Market Share is equal to a fund's net assets under management divided by the sum of all funds' net assets under management in the same month. Industry Size is the sum of AFMI's net assets under management divided by the stock market capitalization in the same month. HHI is calculated as the sum of squares of each fund's market share, and HHI^2 is the square of HHI. The unit of coefficients is percentage. Standard errors are clustered by fund and presented in parentheses. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level in a two-tail t-test, respectively.

	Panel A					*Panel B*		
Dependent Variable	Fund Net Alpha					PC-Adjusted Fund Net Alpha		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Lagged HHI	6.5277***	40.0796***	39.1271**	35.3591***	34.8497**	2.5033***	11.8816***	9.4626**
	(1.0362)	(4.8085)	(17.7560)	(4.6913)	(17.4796)	(0.8485)	(4.3336)	(4.1665)
Lagged HHI^2		-1,110.4260***	-1,081.2070*	-1,402.0231***	-1,394.8592**		-310.3817**	-459.8112***
		(156.2080)	(589.6182)	(184.4615)	(700.8128)		(142.7130)	(161.0837)
Lagged Market Share			-12.0701		-15.1453			
			(23.6434)		(24.8670)			
Lagged Industry Size				-1.8946***	-1.9440			-0.9709***
				(0.3977)	(1.4847)			(0.2800)
Observations	245,178	245,178	239,537	245,178	239,537	245,179	245,179	245,179
R-Sqr	0.0002	0.0004	0.0004	0.0006	0.0006	0.0000	0.0001	0.0002
Adjusted R-Sqr	0.0002	0.0004	0.0004	0.0006	0.0006	0.0000	0.0001	0.0002

We use a VAR to evaluate the relation between industry size and market concentration. The results are shown in Table 3.4. The first column of each model specification shows how AFMI size is associated with HHI. The result of interest in this table is that AFMI size is significantly positively associated with lagged HHI (model specification 1) and is significantly negatively associated with the second order of lagged HHI (model specification 2). If we further include a time trend or year dummies into the model, we find consistent results (model specifications 3 and 4). Thus, AFMI size is increasing concave in market concentration. The positive relation between industry size and market concentration indicates that, consistent with our previous tests, higher market concentration levels, on average, for plausible parameter values, increase gross alphas more than they increase managers' effort costs. Also, we find concavity of both industry size and fund net alphas in concentration levels, again consistent with our model's theoretical prediction. As an aside, we also note that in the second column, AFMI size has little effect on HHI: the magnitude of the coefficient of the lagged AFMI size is almost zero.

Table 3.4. Market Concentration and AFMI Size

This table reports the results of our VAR model. Industry Size is the sum of funds' net assets under management divided by the stock market capitalization in the same month. HHI is calculated as the sum of the squares of each fund's market share, and HHI^2 is the square of HHI. Time Trend is set to be one for January 1984 and to increase by one each month. Robust standard errors are used and presented in parentheses. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level in a two-tail t-test, respectively.

	(1)		(2)			(3)			(4)		
	Industry Size	HHI	Industry Size	HHI	HHI^2	Industry Size	HHI	HHI^2	Industry Size	HHI	HHI^2
Lagged Industry Size	1.0033***	-0.0001***	1.0052***	-0.0000***		0.9952***	-0.0000		0.8772***	-0.0000*	
	(0.0014)	(0.0000)	(0.0035)	(0.0000)		(0.0063)	(0.0000)		(0.0293)	(0.0000)	
Lagged HHI	28.7108***	0.5227***	49.1392*	0.9419***		75.3866**	0.9305***		934.1272***	0.5642***	
	(3.7047)	(0.0234)	(27.8090)	(0.0084)		(31.0463)	(0.0093)		(112.5959)	(0.0231)	
Lagged HHI^2			-136.0583*		0.9290***	-204.4277**		0.9177***	-2,460.1055***		0.5593***
			(76.1298)		(0.0085)	(84.6050)		(0.0094)	(300.2898)		(0.0227)
Time Trend						0.0058*	0.0000	0.0000			
						(0.0033)	(0.0000)	(0.0000)			
Constant	-0.4798**	0.0145***	-0.9431	0.0010	-0.0002	-1.3195	-0.0000	-0.0006*	15.4874***	0.0056	0.0000
	(0.1997)	(0.0013)	(0.8024)	(0.0007)	(0.0002)	(0.8414)	(0.0011)	(0.0004)	(5.4163)	(0.0034)	(0.0011)
Year Dummies	No	No	No	No	No	No	No	No	Yes	Yes	Yes
Observations	431	431	431	431	431	431	431	431	431	431	431
R-Sqr	0.999	0.663	0.999	0.406	0.156	0.999	0.423	0.180	0.999	0.664	0.518

Robustness

In addition to reported tables, we have examined the robustness of our main empirical results in various cases. We analyze the sensitivity of our results to various measures of market concentration such as the NHHI and 5-Fund Index. We find consistent results. We also analyze the sensitivity of the results in Table 3.2 and Table 3.3 by using fund fixed-effect regressions instead of the RD method. Most of the results are consistent, except when regressing the PC-adjusted fund net alpha on market concentration measures; we find that the significance of market concentration measure is reduced. Furthermore, we analyze whether our results are driven by small funds. We redo our main analyses using observations after restricting our sample to funds with a net asset value above $15 million in any month of our sample period. Again, we find consistent results.

To test whether our main results are stable across sub-samples, we redo our analyses in Table 3.3 for three sub-periods. We find a significantly positive relation between fund net alphas and lagged HHI in all three sub-periods.

3.5. Conclusion

In this chapter, we empirically study the implications of our model developed in the last chapter. We use Morningstar's U.S. active equity mutual fund data. First, we find that on average, fund net alphas are negatively associated with fund size and AFMI size, confirming decreasing returns to scale at both fund and industry levels. More importantly, we also find that, on average, both fund net alphas and AFMI size are increasing concave with market concentration.

Our findings have policy implications for the U.S. AFMI. Under the current, empirically identified, tradeoff between changes in managerial productivity and in effort costs due to changes in the AFMI concentration level, increases in concentration levels are likely to increase fund net alphas, AFMI size, and AFMI's direct benefits of net alpha production. These implications support the efficiency of the prevailing real-world AFMI structure of competing fund families, which is more concentrated than that of competing individual funds.

In the next chapter, we extend our study under an international context, and analyze how a foreign AFMI's market concentration as well as the local one, affects the local AFMI.

Chapter 4: A Model of the International Active Fund Management Industry, Theory and Empirical Tests

4.1. Introduction

Recent studies (see, for example, Pastor and Stambaugh (PS) (2012)) and the previous chapters of this thesis have shown that competition in the active fund management industry (AFMI) affects funds' performance, size, fees, and other relevant characteristics. Here, we pose questions under an international context: how, and by which mechanism, does a foreign fund market's competition (concentration) level affect a local fund market's characteristics, such as, performance, size, and fees. To answer, we introduce a model of international active fund management industries (IAFMI) equilibria where performance, size, fees, and managerial efforts are endogenously determined under a continuum of exogenous local and foreign market concentration levels. To fit empirical concentration measures, we then specialize the model to one where concentration levels are endogenously determined.

For simplicity, we consider a two-country international model. In each country, there is an active fund management industry with competing fund managers, who invest their portfolios in both local and foreign stocks, and with infinitely many investors who are mean-variance risk-averse and who allocate their wealth across a passive international benchmark portfolio (which includes both domestic and foreign stocks) and local active funds. We deem this framework realistic because, in reality, due to transaction costs, investors prefer local funds to foreign funds, whereas fund managers, facing lower transaction costs, choose securities across countries. As in Chapter 2, we assume decreasing returns to scale in producing gross alphas at fund and industry levels (i.e., the larger the fund size or industry size, the more difficult it is for managers to produce gross alphas).

Our model allows fund managers, competing in net alpha productions, to spend two types of efforts: exploring investment opportunities in the local stock market and exploring investment opportunities in the foreign stock market. Following Chapter 2, we expect gross alpha production and costs of managerial efforts to depend on concentration levels. In particular, we expect that a higher local AFMI market concentration implies more unexplored investment opportunities in the local stock market, making effort spent in the local stock market more productive; at the same time, it allows local fund managers to ask for higher compensation for effort spent in both the local and the foreign stock markets, increasing foreign managers' reservation prices of efforts spent in both stock markets, and thus increasing effort costs. Moreover, although higher local AFMI market concentration does not directly affect the productivity of effort spent in the foreign stock market, in equilibrium, it attracts managerial efforts to the local stock market, which has more unexplored investment opportunities. As a result, it leaves more unexplored opportunities in the foreign stock market, making effort spent in the foreign stock market more productive. Similarly, higher foreign AFMI market concentration implies more unexplored investment opportunities in the foreign stock market, making effort spent in the foreign stock market more productive. At the same time, it allows foreign fund managers to ask for higher compensation for efforts, increasing local managers' reservation prices of efforts and making efforts spent in both the local and the foreign stock markets more costly. Although higher foreign AFMI market concentration does not directly affect the productivity of effort spent in the local stock market, in equilibrium, it attracts managerial efforts to the foreign stock market which has more unexplored investment opportunities. Thus it leaves more unexplored opportunities in the local stock market, making effort spent in the local stock market more productive. Different from Chapter 2, our two-country international model allows us to incorporate the effects of both local and foreign AFMI market concentration levels on market equilibria.

As in Chapter 2, we set the number of funds in both the local and the foreign AFMI markets, and define a country's AFMI size as the ratio of its assets under active fund management to its total wealth. We also call the improvements in gross alpha production due to efforts minus the costs of these efforts, as AFMI's *direct benefits*. We show that, in equilibrium, if and only if higher local AFMI market concentration exerts stronger (weaker) effects on gross alpha production due to efforts spent in the local and the foreign stock markets than on costs of these two types of efforts, i.e., the sum of changes in direct benefits of these two types of efforts is positive (negative), then it induces higher (lower) local AFMI fund expected net alphas and AFMI size. Similarly, in equilibrium, if and only if higher foreign AFMI market concentration exerts stronger (weaker) effects on gross alpha production of these two types of efforts than on costs of them, i.e., the sum of changes in direct benefits of these two types of efforts is positive (negative), it induces higher (lower) local AFMI fund expected net alphas and AFMI size.

Besides the first-order relation between AFMI market concentrations and AFMI fund expected net alphas, and between AFMI market concentrations and AFMI size, we, also, provide their second-order relations. We show that in equilibrium, if local fund expected net alphas are concave in local (foreign) AFMI market concentration, then the sum of changes in direct benefits of these two types of efforts is concave in local (foreign) AFMI market concentration. Consequently, equilibrium local AFMI size is also concave in local (foreign) AFMI market concentration. On the other hand, if equilibrium local AFMI size is convex in local (foreign) AFMI market concentration, then the sum of changes in direct benefits is convex in local (foreign) AFMI market concentration and, consequently, local fund expected net alphas are convex in local (foreign) AFMI market concentration.

We specialize our model to allow endogenous concentration levels. We show that in equilibrium, although the relation between local AFMI market concentration and local fund expected net alphas, and the relation between local AFMI market concentration and local AFMI size are more complex, we still conclude that local AFMI fund expected net alphas and AFMI size, move in the same direction with foreign AFMI market concentration. We believe that this endogenous concentration framework befits empirical concentration measures, which measures the relative fund size distribution in an industry with a given number of funds.

Using the Normalized-Herfindahl-Hirschman index (NHHI) and other indices as concentration measures, we study our model empirically. We study 30 active equity mutual fund markets, and analyze how these markets' fund net alphas and AFMI size change with the local and the U.S. equity AFMI market concentration. We find that, 17 (5) markets' fund net alphas, on average, are significantly negatively (positively) associated with the U.S. NHHI. while 9 (13) markets' fund net alphas, on average, are significantly negatively (positively) associated with the local NHHI. Also, we find that only 9 (2) markets' AFMI size, on average, are significantly negatively (positively) associated with the U.S. NHHI, while only 7 (7) markets' AFMI size, on average, are significantly negatively (positively) associated with the local NHHI. More importantly, we find that 15 (5) markets' fund net alphas and AFMI size are both, on average, negatively (positively) associated with the U.S. NHHI; among them, 7 (1) markets' fund net alphas and AFMI size are both significantly negatively (positively) associated with the U.S. NHHI. When pooling all the markets' data together, we find that, on average, fund net alphas and AFMI size, are both significantly negatively associated with the U.S. NHHI, but are insignificantly associated with the local NHHI. The fact that global fund markets' fund net alphas and AFMI size tend to move in the same direction with the U.S. AFMI concentration is consistent with our theoretical implications.

We use Pastor, Stambaugh and Taylor's (2015) (PST) recursive-demeaning estimator to address endogeneity and omitted-variable-related issues when studying the AFMI market concentrations-net alpha relation, and we use vector auto-regression (VAR) techniques to account for simultaneity in determination of local AFMI size and local AFMI market concentration when studying the AFMI market concentrations-AFMI size relation. We control for survival bias by using Morningstar Direct's global database, which contains both surviving and terminated funds. Our empirical results are robust to the use of alternative methods and measures.

Our findings provide relevant implications for fund managers, investors, and regulators. The current low and probably decreasing market concentration in the U.S. AFMI, given the trade-off of higher U.S. AFMI market concentration is not changed, would benefit (harm) the global AFMIs whose fund net alphas and AFMI size are, on average, negatively (positively) associated with the U.S. AFMI concentration. Our results show that a large proportion of the global AFMIs in our sample would benefit from that.

Current international studies report how a fund market's size, managerial fees, fund performance, flow-performance relationship, and portfolio choice differ with the fund market's fundamental characteristics, such as regulation, transaction costs, stock market developments, and sophistication of investors. (See, for example, Khorana, Servaes and Tufano (2005), Khorana, Servaes and Tufano (2008), Ferreira, Keswani, Miguel and Ramos (2012a) and (2012b), and Chan, Covrig and Ng (2005).) Our paper complements the literature by showing that a foreign fund market's characteristics, such as market concentration, may also affect the local fund market.

Some international studies analyze how investment activities in one country facilitate the transmission of shocks to other countries and influence the portfolio returns there (See for example, Jotikasthira, Lundblad and Ramadorai (2012) and

Goldstein and Pauzner (2004)). Other international studies analyze how the regulation in one country affects funds' investments in other countries (see for example, Defond, Hu, Hung and Li (2011) and Yu and Wahid (2014)). Similar to these papers' rationale, our paper studies how foreign AFMI market concentration, which is affected by investment activities and regulations there, affects funds' returns in the local market. However, we provide another mechanism to analyze cross-market relations.

Section 4.2 develops the theoretical model, Section 4.3 presents the empirical methods and results, and Section 4.4 concludes.

4.2. Theoretical Framework

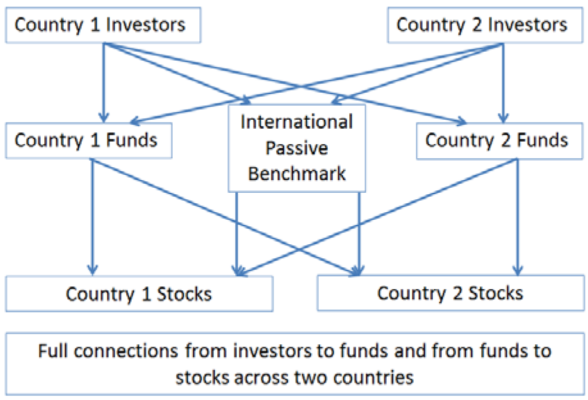
We begin by developing a theoretical framework for modelling the influence of local and foreign AFMI market concentration on local fund managers' efforts, fees, fund performance, fund industry size, and direct benefits of managerial efforts. For simplicity, we consider a two-country international model. Each country has an active fund industry with competing fund managers who invest their portfolios in stocks, and has infinitely many investors who are mean-variance risk-averse and who choose to allocate their wealth across a passive benchmark portfolio and active funds.

Let us consider three cases of how these two countries connect with each other. In Case One, investors in each country can invest in funds in both countries, and managers in each country can invest in stocks in both countries. In this case, these two countries' fund industries, in practice, can be regarded as one fund market, and we can use the framework in the previous chapters to study it. In Case Two, investors in each country can only invest in domestic funds, and fund managers can only invest in domestic stocks. In this case, these two countries' fund industries are, practically, treated as two independent fund markets, and we can use the framework in the previous chapters to study each market. In Case Three, due to transaction costs, investors in each country can only invest in domestic funds, whereas fund managers who face low transaction costs can invest in stocks in both countries. In this case, fund managers are competing for investments in the local country, but each country's AFMI market concentration level affects the gross alpha production and effort cost in both countries. This case is our paper's main focus, and we develop a model to study it. Figure 4.1 illustrates all three cases.

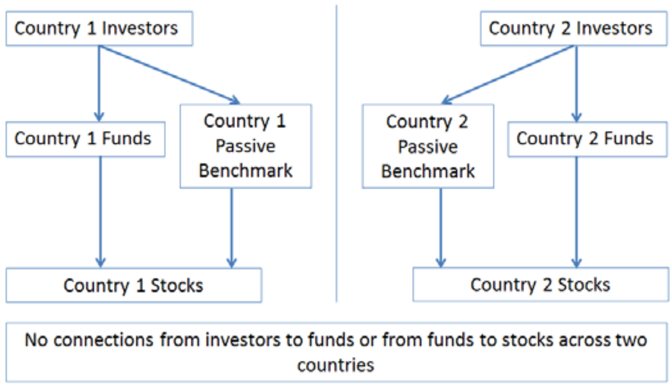
Figure 4.1. Three Cases of a Two-Country Model

This figure shows the three cases we consider in a two-country model. In Case One, investors can invest in funds of both countries, and managers can invest in stocks of both countries. In this case, these two countries' fund markets, in practice, can be regarded as one fund market. In Case Two, investors can only invest in local funds, and fund managers can only invest in local stocks. In this case, these two countries' fund markets are practically two independent fund markets. In Case Three, investors can only invest in local funds, whereas fund managers can invest in stocks of both countries. In this case, each country's fund managers are competing for local investments, and each country's AFMI market concentration level affects the gross alpha production and effort cost in both countries' AFMIs.

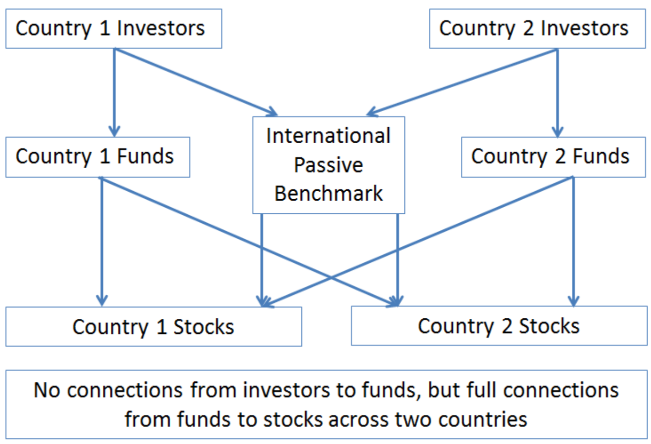
Case One



Case Two



Case Three



Setting

Our model is a one-period model, containing two countries, Country 1 and Country 2, and we use superscript $k, k = 1,2$, to denote the parameters in each country, respectively. For simplicity and without loss of generality, we assume the exchange rate of these two countries' currencies is one. In Country k, there are two types of agents: M^k ($M^k > 1$) active fund managers and N^k ($N^k \to \infty$) investors. Fund managers can invest in stocks in both countries. They are risk-neutral, and they choose proportional management fees and efforts to maximize fund profits. On the other hand, investors are mean-variance risk-averse, and choose their allocation weights on a passive international benchmark portfolio (which includes both domestic and foreign stocks) and local funds to maximize their portfolio Sharpe ratios. Each investor is small, and each investor's investment cannot affect fund sizes.

Due to the symmetry of Country 1 and 2, we can focus on Country 1 only.

Fund Managers' Problem

Manager i in Country 1 maximizes his or her economic profit

$$\max_{e_i^{11}, e_i^{12}, f_i^1} s_i^1 [f_i^1 - C_i^1(e_i^{11}, e_i^{12};\ s_i^1, H^1, H^2)] \tag{4.1}$$

with the constraints

$$f_i^1 - C_i^1(e_i^{11}, e_i^{12}; s_i^1, H^1, H^2) \geq 0, \tag{4.2}$$

$$e_i^{11} \geq 0, \tag{4.3}$$

$$e_i^{12} \geq 0, \tag{4.4}$$

$$f_i^1 \geq 0. \tag{4.5}$$

Here s_i^1, f_i^1, e_i^{11}, e_i^{12}, and $C_i^1(e_i^{11}, e_i^{12};\ s_i^1, H^1, H^2)$ represent manager i's fund size, (nonnegative) proportional management fee, (nonnegative) proportional effort spent in Country 1, (nonnegative) proportional effort spent in Country 2, and average cost function, where H^1 and H^2 represent Country 1's and Country 2's AFMI market concentration measures, respectively. We define the domain of H^1 and H^2 as $[0, 1)$, where 0 represents the fully competitive market situation, and 1 represents the monopoly market situation. This domain implies that managers are competing in the market. Also, inequality (4.2) shows that manager i's profit rate should be nonnegative in order to survive.

Here we assume that the marginal diversification benefits of investing in an additional fund are trivial, such that managers have to compete for investments over net alphas. Under competition, manager i has to maximize his or her fund expected net alpha given fund size and market concentration levels. Thus, manager i's problem can be transformed to

$$\max_{e_i^{11}, e_i^{12}, f_i^1} \mathrm{E}(\alpha_i^1 | D) \tag{4.6}$$

subject to constraints (4.2), (4.3), (4.4), and (4.5). *Proof.* See the Appendix.

The proof intuition is as follows. Under competition, funds that offer higher expected net alphas draw (all) investments. Thus, in equilibrium, funds offer similar expected net alphas. The possibility that other managers increase fund profits by improving expected net alpha and their fund sizes induces managers to maximize expected net alphas in order to "survive." We note that this aspect of the equilibrium is similar to that in PS; but in addition to their result, we show that it holds also in the case of finite number of managers under Bertrand competition.[39]

Manager i's average cost function has the following form:[40]

$$\begin{aligned} C_i^1(e_i^{11}, e_i^{12};\ s_i^1, H^1, H^2) = c_0^1 + c_{1,i}^1 s_i^1 + c_2^{11}(e_i^{11};\ H^1, H^2) \\ + c_2^{12}(e_i^{12};\ H^1, H^2), \end{aligned} \tag{4.7}$$

where c_0^1 and $c_{1,i}^1$ are constants and $c_2^{11}(e_i^{11};\ H^1, H^2)$ and $c_2^{12}(e_i^{12};\ H^1, H^2)$ are costs due to e_i^{11} and e_i^{12}, respectively. Each fund's operation cost is positive, so $c_0^1 > 0$. Also, we assume decreasing returns to scale at fund level, so fund average cost increases with fund size, i.e., $c_{1,i}^1 > 0$. Following Chapter 2, here we also assume the fixed costs of operating a fund is zero to simplify the model and focus on the decreasing returns to scale effect. $c_2^{11}(e_i^{11};\ H^1, H^2)$ and $c_2^{12}(e_i^{12};\ H^1, H^2)$ have the following functional characteristics:

- nonnegative, i.e., $c_2^{11}(0;\ H^1, H^2) = 0$, $c_2^{12}(0;\ H^1, H^2) = 0$, $\forall\, H^1, H^2$, $c_2^{11}(e_i^{11};\ H^1, H^2) > 0$, $\forall\ e_i^{11} > 0, H^1, H^2$, and $c_2^{12}(e_i^{12};\ H^1, H^2) > 0$, $\forall\ e_i^{12} > 0, H^1, H^2$;

[39] In our model, competition among managers is Bertrand Competition, where the "prices" offered by managers are fund expected net alphas.

[40] To simplify our model, we assume there is no interaction between efforts and size in the average cost function because it is unlikely that fund size affects managers' per dollar efforts. We also assume that there is no interaction between concentration and size in the average cost function because it is unlikely that concentration affects managers' average cost sensitivity to fund sizes. Nevertheless, even if these interacting effects exist, they tend to be small in comparison to the effect of other terms in the average cost function.

- increasing convex in effort, as we assume increasing marginal cost for each unit of effort, i.e., $c^{11}_{2\,e_i^{11}}(e_i^{11};\, H^1, H^2) > 0$, $c^{11}_{2\,e_i^{11},e_i^{11}}(e_i^{11};\, H^1, H^2) > 0, \forall\ e_i^{11}, H^1, H^2$, $c^{12}_{2\,e_i^{12}}(e_i^{12};\, H^1, H^2) > 0$, $c^{12}_{2\,e_i^{12},e_i^{12}}(e_i^{12};\, H^1, H^2) > 0,\ \forall\ e_i^{12}, H^1, H^2$;
- increasing with H^1, and positive cross partial derivatives with respect to effort and H^1, because higher local market concentration facilitates manager i to ask for higher compensation for efforts, increasing effort costs, i.e., $c^{11}_{2\,H^1}(e_i^{11};\, H^1, H^2) > 0$, $c^{11}_{2\,e_i^{11},H^1}(e_i^{11};\, H^1, H^2) > 0$, $\forall\ e_i^{11} > 0, H^1, H^2$, $c^{12}_{2\,H^1}(e_i^{12};\, H^1, H^2) > 0$, $c^{12}_{2\,e_i^{12},H^1}(e_i^{12};\, H^1, H^2) > 0,\ \forall\ e_i^{12} > 0, H^1, H^2$;
- increasing with H^2, and positive cross partial derivatives with respect to effort and H^2, because higher foreign market concentration facilitates managers in the foreign market to ask for higher compensation for efforts, increasing manager i's reservation price of efforts, thus increasing effort costs, i.e., $c^{11}_{2\,H^2}(e_i^{11};\, H^1, H^2) > 0$, $c^{11}_{2\,e_i^{11},H^2}(e_i^{11};\, H^1, H^2) > 0$, $\forall\ e_i^{11} > 0, H^1, H^2$, $c^{12}_{2\,H^2}(e_i^{12};\, H^1, H^2) > 0$, $c^{12}_{2\,e_i^{12},H^2}(e_i^{12};\, H^1, H^2) > 0,\ \forall\ e_i^{12} > 0, H^1, H^2$;
- no cross partial effects of two countries' concentration on costs of efforts, i.e., $c^{11}_{2\,H^1,H^2}(e_i^{11};\, H^1, H^2) = 0$, $\forall\ e_i^{11}, H^1, H^2$, $c^{12}_{2\,H^1,H^2}(e_i^{12};\, H^1, H^2) = 0$, $\forall\ e_i^{12}, H^1, H^2$.

By spending efforts, manager i improves his or her fund net alpha. Manager i's net alpha has the following form:

$$\alpha_i^1 = a^1 - b^1 \frac{S^1}{W^1} + A^{11}(e_i^{11};\, H^1, H^2) + A^{12}(e_i^{12};\, H^1, H^2) - f_i^1, \tag{4.8}$$

where a^1 and b^1 are positive constants, with conditional mean and variance

$$\mathrm{E}\begin{pmatrix} a^1 \\ b^1 \end{pmatrix} D \Big) \triangleq \begin{pmatrix} \widehat{a^1} \\ \widehat{b^1} \end{pmatrix}, \quad \mathrm{Var}\begin{pmatrix} a^1 \\ b^1 \end{pmatrix} D \Big) \triangleq \begin{pmatrix} \sigma_{a^1}^2 & \sigma_{a^1 b^1} \\ \sigma_{a^1 b^1} & \sigma_{b^1}^2 \end{pmatrix},$$

where D is the information set of investors. For simplicity, we assume $\sigma_{a^1 b^1} = 0$. S^1 and W^1 are the fund industry size and total wealth (controlled by investors), respectively, in Country 1. We define $A^{11}(e_i^{11};\, H^1, H^2)$ and $A^{12}(e_i^{12};\, H^1, H^2)$ as the impact of e_i^{11} and e_i^{12}, respectively, on fund i's gross alpha. $A^{11}(e_i^{11};\, H^1, H^2)$ and $A^{12}(e_i^{12};\, H^1, H^2)$ have the following functional characteristics:

- nonnegative, i.e., $A^{11}(0;\ H^1,H^2)=0$, $A^{12}(0;\ H^1,H^2)=0$, $\forall\ H^1,H^2$, $A^{11}(e_i^{11};\ H^1,H^2)>0$, $\forall\ e_i^{11}>0,H^1,H^2$, $A^{12}(e_i^{12};\ H^1,H^2)>0$, $\forall\ e_i^{12}>0,H^1,H^2$;
- increasing concave in effort, as we assume marginal productivity of efforts is decreasing, i.e., $A_{e_i^{11}}^{11}(e_i^{11};\ H^1,H^2)>0,\ A_{e_i^{11},e_i^{11}}^{11}(e_i^{11};\ H^1,H^2)<0, \forall\ e_i^{11},H^1,H^2$ $A_{e_i^{12}}^{12}(e_i^{12};\ H^1,H^2)>0,\ A_{e_i^{12},e_i^{12}}^{12}(e_i^{12};\ H^1,H^2)<0,\ \forall\ e_i^{12},H^1,H^2$;
- $A^{11}(e_i^{11};\ H^1,H^2)$ increases with H^1 and has positive cross partial derivatives with respect to H^1 and e_i^{11}, as higher H^1 implies more unexplored investment opportunities and higher efficiency in using fund industry resources in Country 1, i.e., $A_{H^1}^{11}(e_i^{11};\ H^1,H^2)>0,\ A_{e_i^{11},H^1}^{11}(e_i^{11};\ H^1,H^2)>0,\ \forall\ e_i^{11}>0,H^1,H^2$;
- $A^{11}(e_i^{11};\ H^1,H^2)$ is unaffected by H^2, but at equilibrium value $e_i^{11^*}$, it increases with H^2 because a higher H^2 implies more unexplored opportunities in Country 2, attracting managerial efforts, leaving more unexplored opportunities in Country 1 in equilibrium (substitution effect) and improving effort productivity, i.e., $A_{H^2}^{11}(e_i^{11};\ H^1,H^2)=0$, $\forall\ e_i^{11}\neq e_i^{11^*},H^1,H^2$, $A_{H^2}^{11}\left(e_i^{11^*};\ H^1,H^2\right)>0$, $A_{e_i^{11},H^2}^{11}\left(e_i^{11^*};\ H^1,H^2\right)>0,\ \forall\ e_i^{11^*}>0,H^1,H^2$;
- $A^{12}(e_i^{12};\ H^1,H^2)$ increases with H^2 and has positive cross partial derivatives with respect to H^2 and e_i^{12}, as higher H^2 implies more unexplored investment opportunities in Country 2, i.e., $A_{H^2}^{12}(e_i^{12};\ H^1,H^2)>0,\ A_{e_i^{12},H^2}^{12}(e_i^{12};\ H^1,H^2)>0$, $\forall\ e_i^{12}>0,H^1,H^2$;
- $A^{12}(e_i^{12};\ H^1,H^2)$ is unaffected by H^1, but at equilibrium value $e_i^{12^*}$, it increases with H^1 because a higher H^1 implies more unexplored opportunities in Country 1, attracting managerial efforts, leaving more unexplored opportunities in Country 2 in equilibrium (substitution effect) and improving effort productivity, i.e., $A_{H^1}^{12}(e_i^{12};\ H^1,H^2)=0$, $\forall\ e_i^{12}\neq e_i^{12^*},H^1,H^2$, $A_{H^1}^{12}\left(e_i^{12^*};\ H^1,H^2\right)>0$, $A_{e_i^{12},H^2}^{12}\left(e_i^{12^*};\ H^1,H^2\right)>0,\ \forall\ e_i^{12^*}>0,H^1,H^2$;
- No cross partial effects of two markets' concentration on effort impacts, i.e., $A_{H^1,H^2}^{11}(e_i^{11};\ H^1,H^2)=0,\ \forall\ e_i^{11},H^1,H^2,\ A_{H^1,H^2}^{12}(e_i^{12};\ H^1,H^2)=0,\ \forall\ e_i^{12},H^1,H^2$.

Thus, manager i's fund expected net alpha is[41]

$$\mathrm{E}(\alpha_i^1 | D) = \widehat{a^1} - \widehat{b^1}\frac{S^1}{W^1} + A^{11}(e_i^{11};\ H^1, H^2) + A^{12}(e_i^{12};\ H^1, H^2) - f_i^1. \tag{4.9}$$

Define the direct benefits of e_i^{11} and e_i^{12} as follows:

$$B^{11}(e_i^{11};\ H^1, H^2) \triangleq A^{11}(e_i^{11};\ H^1, H^2) - c_2^{11}(e_i^{11};\ H^1, H^2), \tag{4.10}$$

$$B^{12}(e_i^{12};\ H^1, H^2) \triangleq A^{12}(e_i^{12};\ H^1, H^2) - c_2^{12}(e_i^{12};\ H^1, H^2). \tag{4.11}$$

These two terms are important for social planners and policy makers, as they capture the direct benefits of e_i^{11} and e_i^{12}, respectively, in terms of increase in gross alpha production minus the corresponding effort costs.

Investors' Problem

There are infinitely many mean-variance risk-averse investors in Country 1. Define the M^1 funds' returns in excess of the risk-free rate earned by investor j, $j = 1,2$ ..., as $\mathbf{r_F^1}$, a $M^1 \times 1$ vector with elements $r_{F,i}^1$, $i = 1, \dots, M^1$. Returns follow a regression model:

$$\mathbf{r_F^1} = \boldsymbol{\alpha}^1 + \boldsymbol{\beta}^1 r_p + x^1 \boldsymbol{\iota}_\mathbf{M} + \boldsymbol{\varepsilon}^1, \tag{4.12}$$

where $\boldsymbol{\alpha}^1$ is a $M^1 \times 1$ vector of fund net alphas in Country 1, with each element as α_i^1, $i = 1, \dots, M^1$. $\boldsymbol{\beta}^1$ is the beta loading of each fund to an international benchmark portfolio. To simplify the framework, we assume each fund has beta loading equal to one to the international benchmark portfolio[42] so that $\boldsymbol{\beta}^1$ is the same as the $M^1 \times 1$ unit vector $\boldsymbol{\iota}_{\mathbf{M^1}}$. r_p is the international benchmark's return in excess of the risk-free rate, with mean $\mu_p, \mu_p > 0$ and variance $\sigma_p^2, \sigma_p^2 > 0$. x^1 is the common risk factor of fund returns in Country 1, with mean 0 and variance $\sigma_x^2, \sigma_x^2 > 0$. $\boldsymbol{\varepsilon}^1$ is a $M^1 \times 1$ vector of fund idiosyncratic risk factors in Country 1, and each of its elements is ε_i^1, $i =$

[41] Investors observe the passive benchmark and the AFMI funds' returns. The difference between these returns comes from three components: net alphas, the common risk factor, and idiosyncratic risks. As the distributions of the common risk and idiosyncratic risk are common knowledge, investors know the likelihood function of the net alphas. Given prior beliefs of net alphas, they form posteriors and update their beliefs. In our one-period model, there is no dynamic Bayesian updating, but we suggest that investors reached a fixed-point equilibrium. Further, because investors observe fees, fund sizes, and industry size, they can also infer $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$. For simplicity and brevity, we depress the notation of $\hat{A}^{11}(e_i^{11};\ H^1, H^2)$ and $\hat{A}^{12}(e_i^{12};\ H^1, H^2)$ in favor of $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$, as these two functions are deterministic.

[42] This is a common assumption, as active funds usually have diversified portfolios. See the discussion in Pastor and Stambaugh (2012).

$1, \ldots, M^1$, which has mean 0 and variance $\sigma_\varepsilon^2, \sigma_\varepsilon^2 > 0$. The parameters $\mu_p, \sigma_p^2, \sigma_x^2$, and σ_ε^2 are known to both investors and managers.

Investor j's portfolio return (in excess of risk-free rate) is

$$r_j^1 = \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} \mathbf{r}_\mathbf{F}^\mathbf{1} + (1 - \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}^1}) r_p = r_p + \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} (\boldsymbol{\alpha}^\mathbf{1} + x^1 \boldsymbol{\iota}_{\mathbf{M}^1} + \boldsymbol{\varepsilon}^\mathbf{1}), \tag{4.13}$$

where $\boldsymbol{\delta}_\mathbf{j}^\mathbf{1}$ is a $M^1 \times 1$ vector of weights that investor j allocates to the M^1 funds, with each element as $\delta_{j,i}^1$. Investor j's problem is

$$\max_{\boldsymbol{\delta}_\mathbf{j}^\mathbf{1}} \frac{\mathrm{E}\left(r_j^1 \middle| D\right)}{\sqrt{\mathrm{Var}\left(r_j^1 \middle| D\right)}} \tag{4.14}$$

subject to

$$\delta_{j,i}^1 \geq 0, \ \forall i, \tag{4.15}$$

$$\boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}^1} \leq 1. \tag{4.16}$$

Constraints (4.15) and (4.16) tell us that investors cannot short sell funds, or short sell the international benchmark portfolio. To simplify our analysis, we assume that in equilibrium, all investors have the same weights allocated to funds (i.e., an symmetric equilibrium), such that

$$\boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^*} = \boldsymbol{\delta}_\mathbf{k}^{\mathbf{1}^*}, \ \forall j \neq k. \tag{4.17}$$

In this case, in equilibrium, the fund industry size in Country 1 is

$$\frac{S^{1^*}}{W^1} = \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^{*\mathbf{T}}} \boldsymbol{\iota}_{\mathbf{M}^1}, \ \forall j. \tag{4.18}$$

Proposition I1, Unique Nash Equilibrium

i. There exists a unique Nash equilibrium, $\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\}$, where

$\mathbf{e}^{\mathbf{11}^*}$ is a $M^1 \times 1$ vector which aggregates Country 1 individual managers' optimal effort allocated to Country 1's stock market, $e_i^{11^*}$;

$\mathbf{e}^{\mathbf{12}^*}$ is a $M^1 \times 1$ vector which aggregates Country 1 individual managers' optimal effort allocated to Country 2's stock market, $e_i^{12^*}$;

$\mathbf{f}^{\mathbf{1}^*}$ is a $M^1 \times 1$ vector which aggregates Country 1 individual managers' optimal fee, $f_i^{1^*}$;

$\boldsymbol{\delta}^{\mathbf{1}^*}$ is a $M^1 \times N^1$ vector which aggregates Country 1 individual investors' wealth weights allocations to funds, $\boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^*}$.

ii. In this equilibrium, managers produce the same expected net alpha (thus the same Sharpe ratio) that drives their economic profits to zero, by charging only break-even fees, and investors allocate the same wealth proportions to each of the funds.

The intuition of the equilibrium is the same as that offered by Chapter 2. As there are no diversification benefits across funds, managers providing higher expected net alphas dominate, attracting investments. Consequently, their fund costs increase inducing higher (breakeven) fees and lowering expected net alphas. Given other managers' net alphas, if a fund manager cannot produce the AFMI highest expected net alpha, even for an infinitesimal fund size, investments continue to shift out of his or her fund, lowering the fund costs and allowing the manager to charge a lower fee to improve net alpha. Thus, in equilibrium, managers will choose efforts and fees to maximize fund net alphas, and the allocation of investments, or fund sizes, sets expected net alphas to be equal across funds. In addition, as funds have the same expected net alphas, they have the same expected returns. The source of fund returns' variance is the same across funds, so the fund return variance is the same across funds. Therefore, all managers offer the same competitive Sharpe ratio.

Proposition I2, Levels of Equilibrium Optimal Efforts, Fees and Fund Market Share

For $i, i = 1, \dots, M^1$ and $j, j = 1, \dots, M^1$, with $i \neq j$

i. $e_i^{11^*} = e_j^{11^*}$, $e_i^{12^*} = e_j^{12^*}$, and $f_i^{1^*} = f_j^{1^*}$, $\forall i, j$ (equilibrium optimal efforts and fees are the same across funds).

ii. Therefore, $B^{11}\left(e_i^{11^*};\, H^1, H^2\right) = B^{11}\left(e_j^{11^*};\, H^1, H^2\right)$ and $B^{12}\left(e_i^{12^*};\, H^1, H^2\right) = B^{12}\left(e_j^{12^*};\, H^1, H^2\right)$, $\forall i, j$ (equilibrium direct benefits of efforts are the same across funds).

iii. Fund sizes relate as $s_i^{1^*}/s_j^{1^*} = c_{1,j}^1/c_{1,i}^1$, $\forall i, j$, where $s_i^{1^*}$ is the fund size in equilibrium for fund i, $\forall i$.

iv. AFMI equilibrium market shares, $(s_i^1/S^1)^*$'s are, $(s_i^1/S^1)^* = \left[c_{1,i}^1 \sum_{j=1}^{M^1} \left(c_{1,i}^1\right)^{-1}\right]^{-1}$, $\forall i$.

The intuitions of Proposition I2 is similar to those of the Proposition 2 in Chapter 2. Because the functional form of $A^{11}(e_i^{11};\, H^1, H^2)$ and $c_2^{11}(e_i^{11};\, H^1, H^2)$ are the same across funds, based on the first-order condition $A_{e_i^{11}}^{11}\left(e_i^{11^*};\, H^1, H^2\right) -$

$c^{11}_{2\,e^{11}_i}\left(e^{11^*}_i;\ H^1, H^2\right) = 0$, $e^{11^*}_i$'s are the same across funds thus $B^{11}\left(e^{11^*}_i;\ H^1, H^2\right)$'s are the same across funds. Similarly, $e^{12^*}_i$'s and $B^{12}\left(e^{12^*}_i;\ H^1, H^2\right)$'s are the same across funds. In equilibrium, as fund expected net alphas are the same and efforts are the same, $f^{1^*}_i$'s are the same across funds. Consequently, we have $c^1_{1,i}s^{1^*}_i$'s the same across funds, and the other results follow. This proposition shows that managers' different costs, $c^1_{1,i}$'s, in producing gross alphas, induce different fund sizes in equilibrium. In particular, if fund i's decreasing return-to-scale effect at the fund level is stronger (i.e., $c^1_{1,i}$ is larger); then in equilibrium, it ends up with smaller size. Also, funds' market shares are deterministic functions of $c^1_{1,i}$'s and are, thus, unaffected by the AFMI weight in total wealth $(s^1_i/S^1)^*$. In other words, in equilibrium, how investors allocate weights across funds is unaffected by how they weight the whole fund industry relative to the passive international benchmark.

Proposition I3, Equilibrium Optimal Efforts, Fees, Direct Benefits of Efforts, and Concentrations

For manager $i, i = 1, \dots, M^1$,

i. if the initial effort input allocated to Country 1 (2) generates non-positive direct benefit, i.e., $B^{11}_{e^{11}_i}(0;\ H^1, H^2) \le 0$ $\left(B^{12}_{e^{12}_i}(0;\ H^1, H^2) \le 0\right)$, the equilibrium effort level $e^{11^*}_i = 0$ $(e^{12^*}_i = 0)$.

ii. If the initial effort input allocated to Country 1 (2) generates positive direct benefit, i.e., $B^{11}_{e^{11}_i}(0;\ H^1, H^2) > 0$ $\left(B^{12}_{e^{12}_i}(0;\ H^1, H^2) > 0\right)$, the equilibrium effort level $e^{11^*}_i > 0$ $(e^{12^*}_i > 0)$. Also, the equilibrium optimal efforts and fees satisfies the following:

a. $A^{11}_{e^{11}_i}\left(e^{11^*}_i;\ H^1, H^2\right) - c^{11}_{2\,e^{11}_i}\left(e^{11^*}_i;\ H^1, H^2\right) = B^{11}_{e^{11}_i}\left(e^{11^*}_i;\ H^1, H^2\right) = 0$ (impact of marginal effort spent in Country 1's stock market is equal to marginal cost of this effort, thus marginal direct benefit of $e^{11^*}_i$ is zero).

$A^{12}_{e^{12}_i}\left(e^{12^*}_i;\ H^1, H^2\right) - c^{12}_{2\,e^{12}_i}\left(e^{12^*}_i;\ H^1, H^2\right) = B^{12}_{e^{12}_i}\left(e^{12^*}_i;\ H^1, H^2\right) = 0$ (impact of marginal effort spent in Country 2's stock market is equal to marginal cost of this effort, thus marginal direct benefit of $e^{12^*}_i$ is zero).

b. $de^{11^*}_i/dH^1 \ge 0 (< 0)$ iff $A^{11}_{e^{11}_i,H^1}\left(e^{11^*}_i;\ H^1, H^2\right) - c^{11}_{2\,e^{11}_i,H^1}\left(e^{11^*}_i;\ H^1, H^2\right) \ge 0 (< 0)$, $de^{12^*}_i/dH^1 \ge 0 (< 0)$ iff

$A^{12}_{e_i^{12},H^1}\left(e_i^{12^*};\ H^1,H^2\right) - c^{12}_{2\ e_i^{12},H^1}\left(e_i^{12^*};\ H^1,H^2\right) \geq 0(<0)$ (equilibrium optimal effort spent in Country 1's (2's) stock market increases with H^1 if and only if higher H^1 induces larger marginal impact of $e_i^{11^*}$ ($e_i^{12^*}$) on gross alpha than on marginal cost of $e_i^{11^*}$ ($e_i^{12^*}$.).

c. $de_i^{11^*}/dH^2 \geq 0(<0)$ iff $A^{11}_{e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) \geq 0(<0)$, $de_i^{12^*}/dH^2 \geq 0(<0)$ iff $A^{12}_{e_i^{12},H^2}\left(e_i^{12^*};\ H^1,H^2\right) - c^{12}_{2\ e_i^{12},H^2}\left(e_i^{12^*};\ H^1,H^2\right) \geq 0(<0)$ (equilibrium optimal effort spent in Country 1's (2's) stock market increases with H^2 if and only if higher H^2 induces larger marginal impact of $e_i^{11^*}$ ($e_i^{12^*}$) on gross alpha than marginal cost of $e_i^{11^*}$ ($e_i^{12^*}$));

d. $\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^1} \geq 0\ (<0)$ iff $A^{11}_{H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ H^1}\left(e_i^{11^*};\ H^1,H^2\right) \geq 0\ (<0)$, $\frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^1} \geq 0\ (<0)$ iff $A^{12}_{H^1}\left(e_i^{12^*};\ H^1,H^2\right) - c^{12}_{2\ H^1}\left(e_i^{12^*};\ H^1,H^2\right) \geq 0\ (<0)$ (direct benefit of $e_i^{11^*}$ ($e_i^{12^*}$) increases with H^1 if and only if higher H^1 induces larger impact of $e_i^{11^*}$ ($e_i^{12^*}$) on gross alpha than cost of $e_i^{11^*}$ ($e_i^{12^*}$)).

e. $\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^2} \geq 0\ (<0)$ iff $A^{11}_{H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ H^2}\left(e_i^{11^*};\ H^1,H^2\right) \geq 0\ (<0)$, $\frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^2} \geq 0\ (<0)$ iff $A^{12}_{H^2}\left(e_i^{12^*};\ H^1,H^2\right) - c^{12}_{2\ H^2}\left(e_i^{12^*};\ H^1,H^2\right) \geq 0\ (<0)$ (direct benefit of $e_i^{11^*}$ ($e_i^{12^*}$) increases with H^2 if and only if higher H^2 induces larger impact of $e_i^{11^*}$ ($e_i^{12^*}$) on gross alpha than cost of $e_i^{11^*}$ ($e_i^{12^*}$).)

f. The sign of $df_i^{1^*}/dH^1$ depends on the signs of $d(S^1/W^1)^*/dH^1$, $de_i^{11^*}/dH^1$, and $de_i^{12^*}/dH^1$, and the sign of $df_i^{1^*}/dH^2$ depends on the signs of $d(S^1/W^1)^*/dH^2$, $de_i^{11^*}/dH^2$, and $de_i^{12^*}/dH^2$ (whether a higher H^1 (H^2) induces higher equilibrium optimal fees, depends on whether it increases equilibrium industry size, and whether it increases $e_i^{11^*}$ and $e_i^{12^*}$.)

The intuition of this proposition is as follows. Managers' efforts increase both the gross alpha productions and fund costs at the same time, and the net effect of these two decreases with effort amounts. As managers have to maximize fund net alphas to compete for investments, where the first unit of e_i^{11} (e_i^{12}) generates non-positive net alphas, they will optimally choose to spend no e_i^{11} (e_i^{12}) in equilibrium; otherwise, their equilibrium optimal effort $e_i^{11^*}$ ($e_i^{12^*}$) is positive. To provide economic insight, we focus on the cases where equilibrium optimal efforts are positive in the following analyses. In this case, managers will choose $e_i^{11^*}$ ($e_i^{12^*}$) such that its marginal impact on net alpha production is zero to maximize fund net alphas when competing for investments. Also, if a higher H^1 (H^2) induces higher marginal impact of e_i^{12} (e_i^{11}) on gross alpha than on costs, managers optimally choose a higher (lower) effort level in producing fund net alphas. In addition, higher H^1 (H^2) has two effects on $B^{12}\left(e_i^{12^*};\ H^1, H^2\right)$ ($B^{11}\left(e_i^{11^*};\ H^1, H^2\right)$): directly increasing the levels of gross alphas and costs due to $e_i^{12^*}$ ($e_i^{11^*}$), and changing the levels of $e_i^{12^*}$ ($e_i^{11^*}$), consequently changing the gross alphas and costs. In equilibrium, the latter effect is zero, because the marginal impact of $e_i^{12^*}$ ($e_i^{11^*}$) on gross alphas equal to its marginal impact on costs, and the effect of higher H^1 (H^2) through $e_i^{12^*}$ ($e_i^{11^*}$) on gross alphas is cancelled out by its effect through effort on costs. Therefore, if higher H^1 (H^2) induces a higher direct impact on gross alphas than on costs, direct benefits of equilibrium efforts, $B^{12}\left(e_i^{12^*};\ H^1, H^2\right)$ ($B^{11}\left(e_i^{11^*};\ H^1, H^2\right)$), measured as the net of gross alpha production due to efforts minus costs due to efforts, are higher. In addition, higher H^1 (H^2) influences managers' costs in equilibrium by changing the equilibrium Country 1 AFMI size and, thus, fund sizes, by increasing the level of costs due to efforts, and by changing the level of $e_i^{12^*}$ and $e_i^{11^*}$. Consequently, managers' equilibrium (break-even) fees are affected.

We define the equilibrium optimal expected net alpha of an initial marginal investment in AFMI as

$$\begin{aligned} X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) = \widehat{a^1} + A^{11}(e_i^{11};\ H^1, H^2) + A^{12}(e_i^{12};\ H^1, H^2) \\ -c_0^1 - c_2^{11}(e_i^{11};\ H^1, H^2) \\ - c_2^{12}(e_i^{12};\ H^1, H^2) \end{aligned} \tag{4.19}$$

For AFMI to exist, this variable should be positive to attract investments from investors. To provide economic insight, we assume

$$X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) > 0,\ \forall\, H^1, H^2. \tag{4.20}$$

Proposition I4, Equilibrium by Optimal Allocations

For manager i, $i = 1, \ldots, M^1$, in equilibrium, we have $\mathrm{E}(\alpha_i^1 | D)|_{\{\mathbf{e^{11}}^*, \mathbf{e^{12}}^*, \mathbf{f^1}^*, \boldsymbol{\delta^1}^*\}} > 0$, and the optimal AFMI size $\frac{S^1}{W^1}^*$ is either 1 or a real positive solution of the (constrained embedded) first-order condition of investors' problem, substituting $\frac{S^1}{W^1}^* = \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^{*\mathbf{T}}} \boldsymbol{\iota}_{\mathbf{M^1}}$,

$$-\gamma\sigma_{b^1}^2 \left(\frac{S^1}{W^1}^*\right)^3 - \left\{\gamma\sigma_{a^1}^2 + \gamma\sigma_x^2 + \widehat{b^1} + \left[\sum_{j=1}^{M^1} \left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\} \frac{S^1}{W^1}^* +$$

$X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) = 0$, where $\gamma \triangleq \mu_p / \sigma_p^2$. Also,

$$\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)} = \frac{1}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^1}{W^1}^*\right)^2 + \sigma_{a^1}^2 + \sigma_x^2\right] + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1} > 0, \text{ and}$$

$$\frac{d\left(S^1/W^1\right)^*}{d\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}} = \frac{-\left(S^1/W^1\right)^*}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^1}{W^1}^*\right)^2 + \sigma_{a^1}^2 + \sigma_x^2\right] + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1} < 0.$$

The intuition of Proposition I4 is the same as that of Proposition RA2.

Proposition I5, Equilibrium AFMI Size Sensitivity to Concentrations

i. Where $\frac{S^1}{W^1}^* < 1$, we have

a. $\frac{d\left(S^1/W^1\right)^*}{dH^1} = \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)} \left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1}\right]$,

so $\frac{d\left(S^1/W^1\right)^*}{dH^1} \geq 0\ (< 0)$ iff $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} \geq 0\ (< 0)$

(where H^1 is higher, the equilibrium AFMI size is larger (smaller) if and only if higher H^1 induces a larger (smaller) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$.)

b. $\frac{d^2\left(S^1/W^1\right)^*}{d{H^1}^2} = \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)} \left[\frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{d{H^1}^2} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{d{H^1}^2}\right] -$

$6\gamma^1\sigma_{b^1}^2 \frac{S^1}{W^1}^* \left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1}\right]^2 \left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)}\right]^3$,

so if $\frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{d{H^1}^2} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{d{H^1}^2} \leq 0$ then $\frac{d^2\left(S^1/W^1\right)^*}{d{H^1}^2} \leq 0$, and if $\frac{d^2\left(S^1/W^1\right)^*}{d{H^1}^2} \geq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{d{H^1}^2} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{d{H^1}^2} \geq 0$.

(The fact that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1, H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1, H^2\right)$ with respect to H^1 is negative indicates that equilibrium AFMI size is concave in H^1. The fact that equilibrium AFMI size is convex in H^1 indicates that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1,H^2\right)$ with respect to H^1 is positive.)

c. $\frac{d\left(S^1/W^1\right)^*}{dH^2} = \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};\,H^1,H^2\right)}\left[\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^2}\right]$,

so $\frac{d\left(S^1/W^1\right)^*}{dH^2} \geq 0\ (< 0)$ iff $\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^2} \geq 0\ (< 0)$.

(Where H^1 is higher, the equilibrium AFMI size is larger (smaller) if and only if higher H^2 induces a larger (smaller) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$.)

d. $\frac{d^2\left(S^1/W^1\right)^*}{d{H^2}^2} = \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};\,H^1,H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{d{H^2}^2} + \frac{d^2B^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{d{H^2}^2}\right] -$

$6\gamma^1\sigma_{b^1}^2\frac{S^1}{W^1}^*\left[\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^2}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};\,H^1,H^2\right)}\right]^3$,

so if $\frac{d^2B^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{d{H^2}^2} + \frac{d^2B^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{d{H^2}^2} \leq 0$ then $\frac{d^2\left(S^1/W^1\right)^*}{d{H^2}^2} \leq 0$, and if $\frac{d^2\left(S^1/W^1\right)^*}{d{H^2}^2} \geq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{d{H^2}^2} + \frac{d^2B^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{d{H^2}^2} \geq 0$.

(The fact that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1,H^2\right)$ with respect to H^2 is negative indicates that equilibrium AFMI size is concave in H^2. The fact that equilibrium AFMI size is convex in H^2 indicates that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1,H^2\right)$ with respect to H^2 is positive.)

e. $\frac{d^2\left(S^1/W^1\right)^*}{dH^1dH^2} = \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};\,H^1,H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^1dH^2}\right] -$

$6\gamma^1\sigma_{b^1}^2\frac{S^1}{W^1}^*\left[\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^1}\right]\left[\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^2} +\right.$

$\left.\frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^2}\right]\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};\,H^1,H^2\right)}\right]^3$,

so the sign of the cross partial derivative of $\left(S^1/W^1\right)^*$ with respect to H^1 and H^2 depends on the signs and magnitudes of $\frac{d^2B^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^1dH^2}$, $\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^1}$, and $\frac{dB^{11}\left(e_i^{11^*};\,H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};\,H^1,H^2\right)}{dH^2}$.

ii. Where $\frac{S^{1*}}{W^1} = 1, \frac{S^{1*}}{W^1}$ does not depend on H^1 or H^2.

Proposition I6, Equilibrium Expected Net Alpha Sensitivity to Concentrations

i. Where $\frac{S^{1*}}{W^1} < 1$, we have

a. $\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}*}\}} = \left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]\Bigg\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Bigg\}$,

so $\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}*}\}} \geq 0\ (<0)$ iff $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1} \geq 0\ (<0)$.

(Where H^1 is higher, the equilibrium fund expected net alpha is larger (smaller) if and only if higher H^1 induces a larger (smaller) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$.)

b. $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}*}\}} = \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\right]\Bigg\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Bigg\} + 6\gamma\sigma_{b^1}^2\frac{S^{1*}}{W^1}\Bigg\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\Bigg\}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3$,

so if $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}*}\}} \leq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}} \leq 0$, and if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}} \geq 0$, then $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}*}\}} \geq 0$.

(The fact that the equilibrium expected net alpha is concave in H^1 indicates that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};H^1,H^2\right)$ with respect to H^1 is negative. The fact that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};H^1,H^2\right)$ with respect to H^1 is positive indicates that the equilibrium expected net alpha is convex in H^1.)

c. $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^2}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\}} = \left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}\right]\Big\{1 -$

$\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Big\}$,

so $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^2}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\}} \geq 0\ (<0)$ iff $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2} \geq 0\ (<0)$.

(Where H^2 is higher, the equilibrium fund expected net alpha is larger (smaller) if and only if higher H^2 induces a larger (smaller) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$.)

d. $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\}} = \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}}\right]\Big\{1 -$

$\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Big\} + 6\gamma\sigma_{b^1}^2\frac{S^1}{W^1}^*\Big\{\widehat{b^1} +$

$\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\Big\}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3$,

so if $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\}} \leq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}} \leq 0$, and if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}} \geq 0$, then $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\}} \geq 0$.

(The fact that the equilibrium expected net alpha is concave in H^2 indicates that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1,H^2\right)$ with respect to H^2 is negative. The fact that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1,H^2\right)$ with respect to H^2 is positive indicates that equilibrium expected net alpha is convex in H^2.)

e. $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^1dH^2}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\}} = \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}\right]\Big\{1 -$

$\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Big\} + 6\gamma\sigma_{b^1}^2\frac{S^1}{W^1}^*\Big\{\widehat{b^1} +$

$\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\Big\} \left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1}\right] \left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2}\right] \left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)}\right]^3,$

so the sign of $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^1 dH^2}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$ depends on the signs and magnitudes of $\frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1 dH^2} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1 dH^2}$, $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1}$, and $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2}$.

ii. Where $\frac{S^{1^*}}{W^1} = 1$, we have

a. $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = \frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1}$,

so $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} \geq 0\ (<0)$ iff $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} \geq 0\ (<0)$.

(Where H^1 is higher, equilibrium fund expected net alpha is larger (smaller) if and only if higher H^1 induces a larger (smaller) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$.)

b. $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = \frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^{1^2}}$,

so $\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} \leq 0\ (<0)$, iff $\frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^{1^2}} \leq 0\ (<0)$.

(The equilibrium expected net alpha is concave (convex) in H^1 if and only if the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*}; H^1, H^2\right)$ and $B^{12}\left(e_i^{12^*}; H^1, H^2\right)$ with respect to H^1 is negative (positive).)

c. $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^2}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = \frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2}$,

so $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^2}\Big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} \geq 0\ (<0)$ iff $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2} \geq 0\ (<0)$.

(Where H^2 is higher, the equilibrium fund expected net alpha is larger (smaller) if and only if higher H^2 induces a larger (smaller) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$.)

d. $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}^*}\}} = \frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}}$,

so $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}^*}\}} \le 0\ (<0)$ iff $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}} \le 0\ (<0)$.

(The equilibrium expected net alpha is concave (convex) in H^2 if and only if the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\ H^1,H^2\right)$ with respect to H^2 is negative (positive).)

e. $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^1dH^2}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}^*}\}} = \frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}$,

so the sign of $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^1dH^2}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ depends on the sign of $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}$.

If we combine Proposition I6 and I7, we can see that $d(S^1/W^1)^*/dH^1$ and $d\mathrm{E}(\alpha_i^1|D)/dH^1|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ always have the same signs. The intuition is as follows. Higher H^1 influences $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ at two stages. At the first stage, it changes the effort impact on gross alpha and effort costs. If it increases effort impact on gross alpha more (less) than it increases effort costs, i.e., $A_{H^1}^{11}\left(e_i^{11^*};\ H^1,H^2\right) + A_{H^1}^{12}\left(e_i^{12^*};\ H^1,H^2\right) - c_{2\,H^1}^{11}\left(e_i^{11^*};\ H^1,H^2\right) - c_{2\,H^1}^{12}\left(e_i^{12^*};\ H^1,H^2\right) = \frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1} \ge 0\ (<0)$, then it increases (decreases) fund expected net alphas. In this case, the sum of changes in direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$ is positive (negative). At the second stage, higher (lower) fund expected net alphas attracts (discourage) investors to invest in funds, which consequently pushes down (up) fund expected net alphas, due to decreasing returns to scale effects. In our model investors are risk-averse, and their portfolio risks increase with wealth allocated to the funds. Thus, their risk consideration makes their reactions to changes in fund expected net alphas less intensive (i.e., they subdue their additional investments in AFMI when inferring higher fund expected net alphas, and limit their reduction in investments AFMI when inferring lower fund expected net alphas), mitigating the second-stage effect and allowing the first-stage effect to dominate. Therefore, in equilibrium, if higher H^1 increases effort impact on gross alpha more (less) than it increases effort costs (i.e., the sum of changes in direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$ is positive (negative)), then it increases (decreases) $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$, and $(S^1/W^1)^*$ also increases (decreases). The difference between our model's results and those of Chapter 2, is that in our model, higher H^1 changes the impacts of both $e_i^{11^*}$ and $e_i^{12^*}$ (manager efforts spent in local and foreign stock markets), whereas in Chapter 2 where there is only one market, higher market concentration affects only the impact of manager effort spent in local market. We can also see that $d(S^1/W^1)^*/dH^2$ and $d\mathrm{E}(\alpha_i^1|D)/dH^2|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ always have the same signs: in equilibrium, if higher H^2 increases effort impacts on gross alpha more (less) than it increases effort costs (i.e., the sum of changes in direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$ is positive (negative)), it increases (decreases) $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$, and $(S^1/W^1)^*$ also increases (decreases). The intuition is similar as above.

Proposition I7, Relation Between Equilibrium Expected Net Alpha and Market Share

An increase in $c_{1,i}^1$, while $c_{1,j}^1, \forall j \neq i$ are unchanged, induces a decrease (increase) in s_i^1/S^1 and an increase (decrease) in $s_j^1/S^1, \forall j \neq i$. Also, it induces a decrease (increase) in $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$; thus $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ and s_i^1/S^1 are positively related—internality effect. It induces a decrease (increase) in $\mathrm{E}\left(\alpha_j^1|D\right)|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$; thus $\mathrm{E}\left(\alpha_j^1|D\right)|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ and s_j^1/S^1 are negatively related—externality effect.

The proof intuition is the same as that of Proposition RA5.

Proofs of all the Propositions. See the Appendix.

Numerical Results

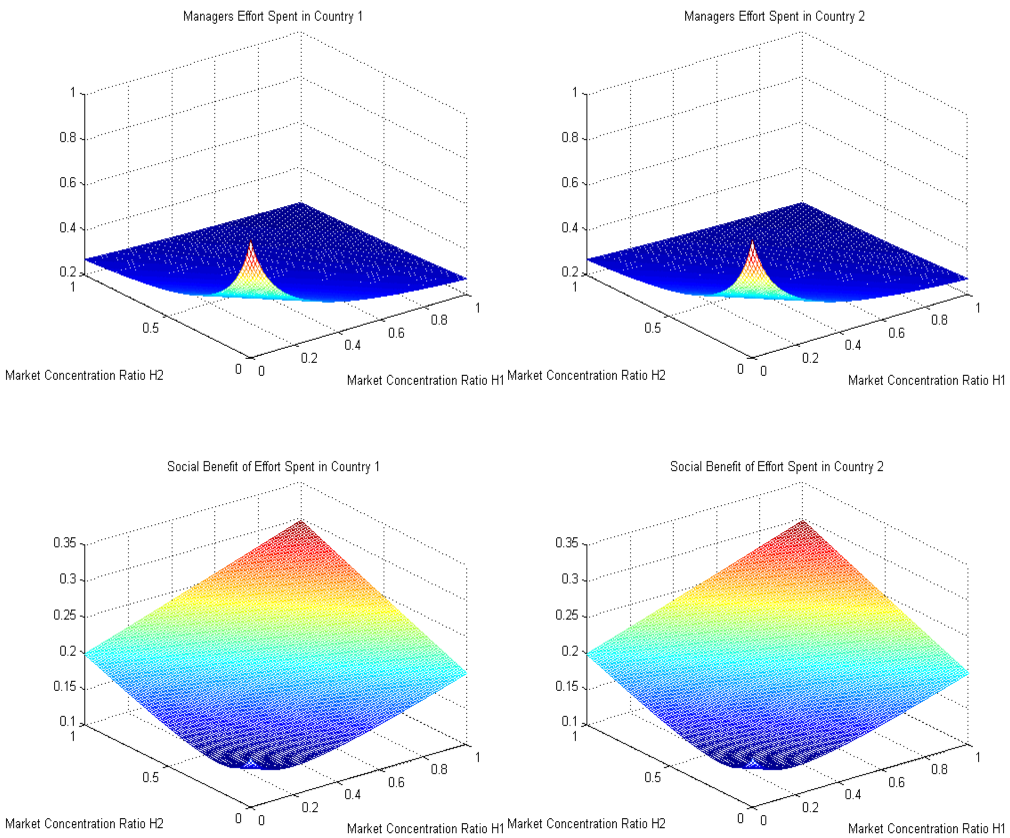
We provide a numerical analysis for our model, and set the following parameter values: $W^1 = 100$, $M^1 = 100$, $\mu_p = 0.05$, $\sigma_p = 0.1$, $\sigma_x^1 = 0.05$, $\widehat{a^1} = 0.15$, $\widehat{b^1} = 0.3$, $\sigma_{a^1} = 0.4$, $\sigma_{b^1} = 0.4$.

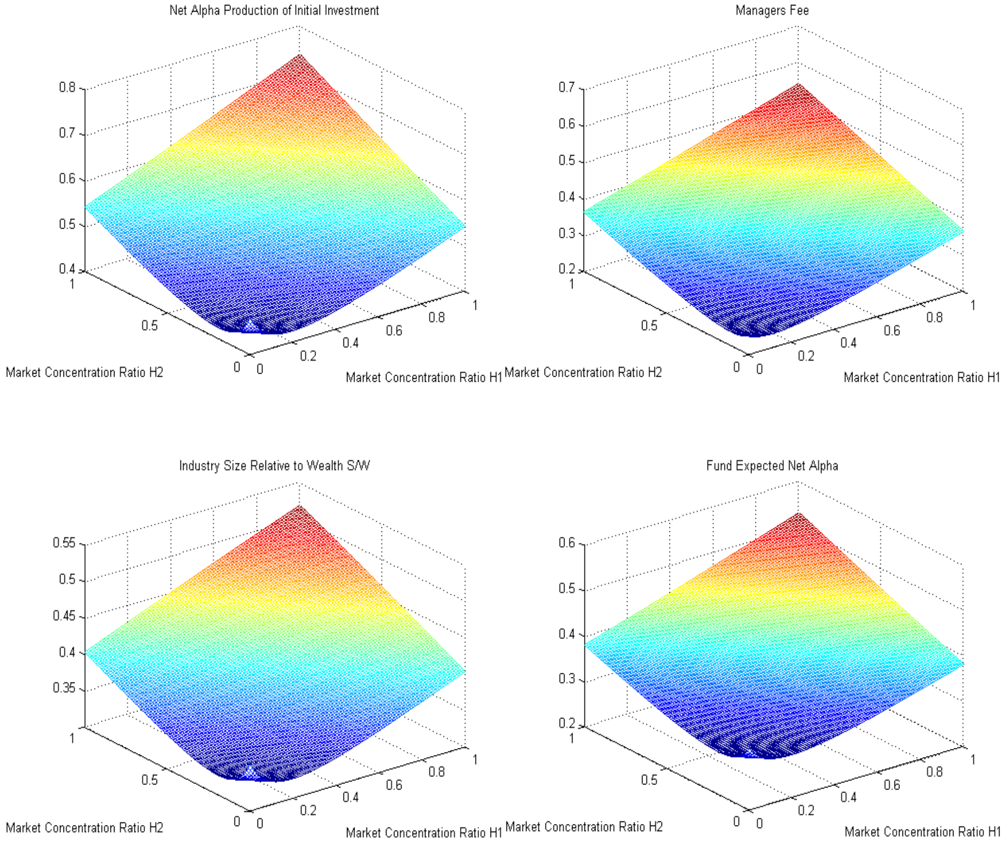
To simplify the case, we assume that fund managers in Country 1 have the same average cost functions, such that funds have the same size in equilibrium. We set the parameters of the average cost functions as $c_0^1 = 0.005, c_1^1 = 0.1$, and the functions $c_2^{11}(e_i^{11};\, H^1, H^2)$ and $c_2^{12}(e_i^{12};\, H^1, H^2)$ as $2(0.1 + H^1 + H^2)e_i^{11}$ and $2(0.1 + H^1 + H^2)e_i^{12}$, respectively. Also, we set the functions $A^{11}(e_i^{11};\, H^1, H^2)$ and $A^{12}(e_i^{12};\, H^1, H^2)$ as $(0.5 + H^1 + H^2)\ln(e_i^{11} + 1)$ and $(0.5 + H^1 + H^2)\ln(e_i^{12} + 1)$, respectively. We choose 100 points evenly spread on $[0, 0.999]$ to be the values of the market concentrations H^1 and H^2.

Figure 4.2 illustrates the numerical results of the relevant variables in equilibrium. As $c_2^{11}(e_i^{11};\, H^1, H^2)$ and $c_2^{12}(e_i^{12};\, H^1, H^2)$ have the same functional forms, and $A^{11}(e_i^{11};\, H^1, H^2)$ and $A^{12}(e_i^{12};\, H^1, H^2)$ have the same functional forms, in equilibrium, $e_i^{11^*} = e_i^{12^*}$ and $B^{11}\left(e_i^{11^*};\, H^1, H^2\right) = B^{12}\left(e_i^{12^*};\, H^1, H^2\right)$ for $\forall\, H^1, H^2$. Due to the parameter assumptions, higher H^1 and H^2 induce smaller marginal impact of $e_i^{11^*}$ ($e_i^{12^*}$) on gross alpha than marginal cost of $e_i^{11^*}$ ($e_i^{12^*}$); thus, $e_i^{11^*}$ ($e_i^{12^*}$) decrease with H^1 and H^2, and they achieve the highest value at $H^1 = H^2 = 0$. On the other hand, as higher H^1 and H^2 induces larger impact of $e_i^{11^*}$ ($e_i^{12^*}$) on gross alpha than cost of $e_i^{11^*}$ ($e_i^{12^*}$) where H^1 and H^2 are above 0.2, $B^{11}\left(e_i^{11^*};\, H^1, H^2\right)$ ($B^{12}\left(e_i^{12^*};\, H^1, H^2\right)$) increases with H^1 and H^2 after this point, and achieve the highest value at $H^1 = H^2 = 0.999$. Moreover, because higher H^1 and H^2 induce a positive (negative) sum of direct benefits of $e_i^{11^*}$ and $e_i^{12^*}$ where H^1 and H^2 are high (low), $X\left(e_i^{11^*}, e_i^{12^*};\, H^1, H^2\right)$, $(S^1/W^1)^*$, and $\mathrm{E}(\alpha_i^1 | D)\big|_{\left\{\mathbf{e^{11^*}}, \mathbf{e^{12^*}}, \mathbf{f^{1^*}}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}}$ increase (decrease) with H^1 and H^2 where H^1 and H^2 are high (low). We can also observe that $(S^1/W^1)^*$ and $\mathrm{E}(\alpha_i^1 | D)\big|_{\left\{\mathbf{e^{11^*}}, \mathbf{e^{12^*}}, \mathbf{f^{1^*}}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}}$ are convex in H^1 and H^2, and the cross partial derivatives of $X\left(e_i^{11^*}, e_i^{12^*};\, H^1, H^2\right)$, $(S^1/W^1)^*$, and $\mathrm{E}(\alpha_i^1 | D)\big|_{\left\{\mathbf{e^{11^*}}, \mathbf{e^{12^*}}, \mathbf{f^{1^*}}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}}$ with respect to H^1 and H^2 are positive. In addition, due to our parameter values, f_i^* increases with H^1 and H^2.

Figure 4.2. Numerical Results

This figure shows the numerical results in equilibrium given the parameter values above. For each of the graphs, the x-axis and y-axis are the values of H^1 and H^2, respectively, and the z-axis shows the equilibrium values of, from the left-top to the right-bottom, $e_i^{11^*}$, $e_i^{12^*}$, $B^{11}\left(e_i^{11^*}; H^1, H^2\right)$, $B^{12}\left(e_i^{12^*}; H^1, H^2\right)$, $X\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)$, f_i^*, $(S^1/W^1)^*$, $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11^*}}, \mathbf{e^{12^*}}, \mathbf{f^{1^*}}, \boldsymbol{\delta}^{\mathbf{1^*}}\}}$, respectively.




Net Alpha Production of Initial Investment
Market Concentration Ratio H2
Market Concentration Ratio H1
Managers Fee
Market Concentration Ratio H2
Market Concentration Ratio H1
Industry Size Relative to Wealth S/W
Market Concentration Ratio H2
Market Concentration Ratio H1
Fund Expected Net Alpha
Market Concentration Ratio H2
Market Concentration Ratio H1

Endogenous Market Concentrations

If we define H^1 and H^2 as the Herfindahl-Hirschman index (HHI) and allow it to be endogenous, then the equilibrium HHI's of Country 1 and Country 2 are, following Proposition I2,

$$H^{1^*} \triangleq \sum_{i=1}^{M^1} \left(\frac{S_i^1}{S^1}\right)^2 = \sum_{i=1}^{M^1} \left[c_{1,i}^1 \sum_{j=1}^{M^1} \left(c_{1,i}^1\right)^{-1}\right]^{-2}. \tag{4.21}$$

$$H^{2^*} \triangleq \sum_{i=1}^{M^2} \left(\frac{S_i^2}{S^2}\right)^2 = \sum_{i=1}^{M^2} \left[c_{1,i}^2 \sum_{j=1}^{M^2} \left(c_{1,i}^2\right)^{-1}\right]^{-2}. \tag{4.22}$$

Depending on the size of $c_{1,i}^1$ relative to $c_{1,j}^1$, $\forall j \neq i$, an increase in $c_{1,i}^1$, holding $c_{1,j}^1$, $\forall j \neq i$ unchanged, increases or decreases H^{1^*}. The corresponding results in Country 2 are similar.

Similar to the endogenous framework in Chapter 2, an increase in $c_{1,i}^1$ affects Country 1's equilibrium fund expected net alphas in two ways: 1) its direct impact leads to lower equilibrium fund expected net alphas (Proposition I7), and 2) depending on fund i's size relative to rivals', it increases or decreases H^{1^*}, which consequently increases (decreases) equilibrium fund expected net alphas if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^1} \geq 0\ (< 0)$ (Proposition I6). Also, an increase in $c_{1,i}^1$ affects Country 1's equilibrium AFMI size in two ways: 1) its direct impact leads to smaller AFMI size because its direct impact leads to lower equilibrium fund expected net alphas which discourages investments, and 2) depending on fund i's size relative to that of rivals, it increases or decreases H^{1^*}, which consequently increases (decreases) equilibrium AFMI size if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^1} \geq 0\ (< 0)$ (Proposition I5). The second-order relationships between Country 1's concentration and its equilibrium fund expected net alphas, and between Country 1's concentration and its AFMI size, are more complicated.

On the other hand, an increase in $c_{1,i}^2$ affects Country 1's equilibrium fund expected net alphas in the following way: depending on fund i's size relative to rivals', it increases or decreases H^{2^*}, which consequently increases (decreases) equilibrium fund expected net alphas if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^2} \geq 0\ (< 0)$ (Proposition I6). Also, an increase in $c_{1,i}^2$ affects Country 1's equilibrium AFMI size in the following way: depending on fund i's size relative to rivals', it increases or decreases H^{2^*}, which consequently increases (decreases) equilibrium AFMI size if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^2} \geq 0 \; (< 0)$ (Proposition I5). Notice that an increase in $c_{1,i}^2$ does not have direct impact as that of $c_{1,i}^1$ on Country 1's fund expected net alphas and AFMI size.

Thus, where the market concentration is endogenous, the relations between local market concentration and equilibrium local fund expected net alphas and AFMI size are more complex. On the other hand, the relations between foreign market concentration and the local market's equilibrium fund expected net alphas and AFMI size are similar to those under the exogenous concentrations framework.

Similarly to previous chapters, we now proceed with an empirical analysis of the benefits and costs of changing market concentration levels of the AFMI using the version of our model with endogenous concentration. In this sense, this version of our model befits available data of empirical market concentration levels, such as the HHI. Popular empirical market concentration measures, such as HHI, are functions of rivals' relative fund sizes. We expect that market characteristics, such as regulation, transaction costs, tax rates, and barriers to entry, affect funds' cost sensitivity to size (i.e., $c_{1,i}^1$'s and $c_{1,i}^2$'s). As a result, they affect relative fund sizes and, thus, the level of empirical market concentration measures. We use empirical techniques to control potential endogeneity of market concentration measures.

Whether local fund net alphas and AFMI size move in the same direction with local (foreign) market concentration and whether both are concave or convex in it become empirical questions. Further, in cases where active fund management creates value, if fund net alphas and AFMI size increase with local (foreign) market concentration, our model predicts positive marginal direct benefits of effort, for plausible parameter values. We note that both signs of the benefits of changing concentration levels are plausible alternatives to a null hypothesis of no benefit of active fund manager efforts.

4.3. Empirical Study

We now provide analysis of the market concentration-net alpha and market concentration-AFMI size relations using international data of active equity mutual funds. We regard the U.S. active equity mutual fund market as a foreign fund market, whose concentration levels might affect another market's fund net alphas and AFMI size. This is because U.S. has the largest active mutual fund market, which influences global fund markets. We analyze how local active equity mutual fund market concentration levels, and more importantly, how the U.S. active equity mutual fund market concentration levels influence a global AFMI market's fund net alphas and AFMI size.

Methodology

We describe how we measure concentration, fund net alpha, and our econometric strategy to estimate the effects of changing local and foreign concentration levels on fund net alpha and AFMI size, controlling for endogeneity and omitted-variable bias-related issues.

Concentration Measure

Following Chapter 3, and many other empirical papers, we use the following three indices to measure market concentration:

1. HHI

$$HHI_{j,t} = \sum_{i}^{M_{j,t}} MS_{i,j,t}^{2} \tag{4.23}$$

2. normalized HHI (NHHI)

$$NHHI_{j,t} = \frac{M_{j,t} \times HHI_{j,t} - 1}{M_{j,t} - 1} \tag{4.24}$$

3. sum of the first five largest funds' market shares (5-Fund-Index)

$$5\_Fund\_Index_{j,t} = \sum_{i=1}^{5} MS_{i,j,t} \tag{4.25}$$

The indices i, j, and t represent the fund, fund market, and time indicators. $MS_{i,j,t}$ is the market share of a fund, measured as the fund's asset under management divided by the total assets under management in its market, and $M_{j,t}$ is the number of funds in the corresponding market. As some markets tend to have a large number of funds whereas others tend to have a small number of funds, we mainly focus on the results of using NHHI as the market concentration measure, because it adjusts the effect of the number of funds on market concentration levels (Cremers, Nair and Peyer (2008)). For a robustness check, we redo the analyses using HHI and 5-Fund-Index.

Style-Matching Model and Net Alpha Estimation

Following Chapter 3, we develop our style-matching model to estimate funds' passive benchmarks and then calculate fund net alphas. We use the following return-generating process:

$$R_{i,j,t} = \alpha_{i,j,t} + b_{i,j,t}^{1} F_{j,t}^{1} + \cdots + b_{i,j,t}^{n_j} F_{j,t}^{n_j} \tag{4.26}$$

where $R_{i,j,t}$ is the return net of management fee of an active fund, $\alpha_{i,j,t}$ is fund net alpha, $F_{j,t}^{1}$ through $F_{j,t}^{n_j}$ are returns net of management fees of local tradable index funds of different asset classes, a U.S. large-cap equity tradable index fund, and a local risk-free asset, $b_{i,j,t}^{1}$ through $b_{i,j,t}^{n_j}$ are the coefficients with respect to these factors, and n_j is the number of these factors in a particular market. In our algorithm, we minimize, in each fund market, the variance of the residual when projecting $R_{i,j,t}$ on $F_{j,t}^{1}$ through $F_{j,t}^{n_j}$, and we constrain the coefficients $\hat{b}_{i,j,t}^{1}$ through $\hat{b}_{i,j,t}^{n_j}$ to be positive and sum up to one (as we do not allow short selling). We use a rolling window, from months $t-60$ to $t-1$, to estimate $\hat{b}_{i,j,t}^{1}$ through $\hat{b}_{i,j,t}^{n_j}$. The predicted value $\hat{b}_{i,j,t}^{1} F_{j,t}^{1} + \cdots + \hat{b}_{i,j,t}^{n_j} F_{j,t}^{n_j}$ is the passive benchmark at time t, and we estimate $\alpha_{i,j,t}$ by subtracting $R_{i,j,t}$ from $\hat{b}_{i,j,t}^{1} F_{j,t}^{1} + \cdots + \hat{b}_{i,j,t}^{n_j} F_{j,t}^{n_j}$.

Notice that this method is similar to the style-matching model developed by Sharpe (1992). Also, as our passive benchmark is tradable, our net alpha estimation is consistent with the argument of Berk and Binsbergen (2015) that to measure the value added by a fund, its performance should be compared to the next-best investment opportunity available to investors. Moreover, our style-matching passive benchmark is similar to the characteristic-based benchmark developed by Daniel, Grinblatt, Titman, and Wermers (1997). Our model is similar to the style-matching model of Chapter 3 except that besides the local tradable index funds, it contains a U.S. large-cap equity tradable index fund. This is because we need to develop an international passive benchmark, and a U.S. large-cap equity tradable index fund can be a potential factor in this benchmark.

Market Concentration-Net Alpha Relation

Pastor, Stambaugh, and Taylor (2015) (PST) develop a recursive demeaning (RD) estimator to control endogeneity bias created in a simple demeaned model, and we adopt their method here to analyze the market concentration-net alpha relation. The model we use is

$$\overline{\alpha_{i,j,t}} = \beta_{1,j}\overline{NHHI_{j,t-1}^{L}} + \beta_{2,j}\overline{{NHHI_{j,t-1}^{L}}^{2}} + \beta_{3,j}\overline{NHHI_{t-1}^{US}} + \beta_{4,j}\overline{{NHHI_{t-1}^{US}}^{2}} + \beta_{5,j}\overline{NHHI_{j,t-1}^{L} * NHHI_{t-1}^{US}} + \overline{\varepsilon_{i,j,t}} \tag{4.27}$$

where the superscription "L" and "US" represent the local and the U.S. market concentration measures, respectively. The bar on top of the variables represents the forward-demeaning operator. The forward-demeaned value of a time-series variable X_t is

$$\overline{X_t} = X_t - \frac{1}{T-t+1}\sum_{s=t}^{T} X_s \tag{4.28}$$

where T is the total number of observation of this time-series.

This model is similar to the model of market concentration-net alpha relation in Chapter 3, except for two points: 1) this model includes the U.S. market concentration measure as explanatory variables, so it fits our international model and studies how U.S. market concentration is associated with the fund net alphas in market j; and 2) this model does not include the fund market share as a control, because in our unreported tests, we find that the fund market share is insignificant. We exclude this insignificant variable to reduce noise in the estimations.

Market Concentration-AFMI Size Relation

We use the vector auto-regression (VAR) method to study the market concentration-AFMI size relation because both market concentration measures and AFMI size are market-level time-series variables, and empirically, local market concentration and local AFMI size can be endogenous. The model is

$$\begin{bmatrix} AFMI\_Size_{j,t} \\ NHHI_{j,t}^{L} \\ {NHHI_{j,t}^{L}}^{2} \end{bmatrix} = \begin{bmatrix} a_{0,j} \\ b_{0,j} \\ c_{0,j} \end{bmatrix} + \begin{bmatrix} a_{1,j} & a_{2,j} & a_{3,j} \\ b_{1,j} & b_{2,j} & 0 \\ 0 & 0 & c_{3,j} \end{bmatrix} \begin{bmatrix} AFMI\_Size_{j,t-1} \\ NHHI_{j,t-1}^{L} \\ {NHHI_{j,t-1}^{L}}^{2} \end{bmatrix} + \begin{bmatrix} a_{4,j}NHHI_{t-1}^{US} + a_{5,j}{NHHI_{t-1}^{US}}^{2} + a_{6,j}NHHI_{t-1}^{US} * NHHI_{j,t-1}^{L} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{a,j,t} \\ \epsilon_{b,j,t} \\ \epsilon_{c,j,t} \end{bmatrix}. \tag{4.29}$$

This model is similar to the model of market concentration-AFMI size relation in Chapter 3, except that in the first equation, it includes the U.S. market concentration measure and its higher order terms in order to study the effect of the U.S. market concentration on in market j's AFMI size. We regard the U.S. market concentration as an exogenous variable in this VAR system, because we believe empirically there is little causality between the local and the U.S. market concentration. In our analysis, we focus on the first equation, the equation studying how market concentration levels affect market j's AFMI size.

Data

We obtain our data from the Global Databases of Morningstar Direct. Our sample contains 30 active equity mutual fund markets. Due to data availability, the sample periods of different markets are different. The sample periods of each market are reported in Table 4.1; most of them have sample periods starting from 1990s to the end of 2015. Our Appendix supplements the data description below.

Table 4.1. Sample Periods of the 30 Global Markets

This table reports the sample period of the 30 global markets in our study.

Global Market	Starting Period	Ending Period
Australia	1992/01	2015/12
Austria	1999/02	2001/02
Belgium	1999/02	2015/12
Brazil	1997/02	2015/12
Canada	1992/01	2015/12
Chile	2006/01	2015/12
China (Mainland)	2003/05	2015/12
Denmark	1992/01	2015/12
Finland	1999/02	2015/12
France	1999/02	2015/12
Germany	1999/02	2015/12
Greece	1999/02	2012/08
Hong Kong	1998/08	2015/12
India	1999/08	2015/12
Israel	2001/03	2015/12
Italy	1999/02	2015/12
Japan	1992/01	2015/12
Korea	2001/10	2015/12
Mexico	1999/11	2015/12
Netherlands	1999/02	2015/12
Norway	1992/01	2015/12
Portugal	1999/02	2015/09
Singapore	1997/01	2015/12
South Africa	1995/02	2015/12
Spain	1999/02	2015/12
Sweden	1992/01	2015/12
Switzerland	1992/01	2015/12
Taiwan	2004/10	2015/12
Thailand	1996/09	2015/12
United Kingdom	1997/11	2015/12

We use keywords in Morningstar to identify active equity mutual funds. We require the mutual funds to be open-ended and non-restricted. In each mutual fund market dataset, we exclude index funds, enhanced index funds, funds of funds, and in-house funds of funds. Also, we require funds to be classified as "Equity" in the Global Broad Category Group, and we further identify equity funds based on their Morningstar Category. Next, we use the Fund ID provided by Morningstar to aggregate fund share class-level information to fund-level information. Since we use a 5-year rolling window to estimate fund net alphas, when analyzing the market concentration-net alpha relation, we require each of our active equity mutual funds to have at least 10 years' return observations.[43] On the other hand, when analyzing the market concentration-net alpha relation, we use the data of all the active equity funds in our sample.

The index funds used in the style-matching model are also from Morningstar. We require index funds to have no missing observations in our sample period so that the style-matching model is consistent and stable. The factors used in the style-matching model include the local equity index funds, a U.S. large cap index fund, and a local risk-free asset. The information of the risk-free rate of each country is provided by the International Financial Statistics on the official website of International Monetary Fund (IMF).

For each market, the AFMI size is calculated as total funds' net assets under management divided by stock market capitalization, which is a relative size measure and which is consistent with Chapter 3 and PST. Each market's fund net assets under management and stock market capitalization are also provided by the Global Databases of Morningstar Direct.

All the fund returns are net of administrative and management fees and other costs taken out of fund assets; thus, the fund alphas we estimate are net alphas (net of fees). For comparison purpose and to be consistent with our international model, we measure the fund returns, risk-free returns, fund net assets under management, and stock market capitalization in U.S. dollar.

Table 4.2 reports the summary statistics of these global active equity mutual fund markets. Panel A shows the summary statistics of market-level variables. It shows that the average AFMI size varies a lot across the global markets, from around 17% in Denmark to 0.015% in Germany. The market concentration level also varies a lot across

[43] We also omit some rare cases where there is a gap with more than 5 years' return observations missing.

the global markets. The average NHHI value ranges from around 0.49 in Austria to around 0.01 in Taiwan. Panel B shows the summary statistics of fund-level variables. First, the average R-squared of the style-matching model is quite high in each market (ranging from 97% in Chile to 83% in Mexico), with a low standard deviation in each market. This result shows that our style-matching benchmarks perform well in tracking the style of the active equity mutual funds, so it is unlikely that our style-matching models omit relevant factors in developing the passive benchmarks. Also, most markets' average fund net alphas are positive with a large standard deviation.

Table 4.2. Summary Statistics

Monthly data is used. Panel A reports the summary statistics for market-level data, and Panel B reports those for fund-level data. We report the number of observations, mean, and standard deviation of each variable. AFMI Size is the sum of funds' net assets under management divided by the stock market capitalization in the same month. The Style-Matching Model R^2, AFMI Share, NHHI, HHI, and 5-Fund-Index are in decimals. Net Return and Net Alpha are in percentages, and both are net of administrative and management fees and other costs taken out of fund assets.

Panel A

	AFMI Size			NHHI			HHI			5-Fund-Index		
Global Market	Obs	Mean	Sd	Obs	Mean	Sd	Obs	Mean	Sd	Obs	Mean	Sd
Australia	228	0.037	0.009	228	0.029	0.016	228	0.035	0.019	228	0.322	0.126
Austria	85	0.018	0.008	85	0.492	0.223	85	0.553	0.211	85	0.936	0.077
Belgium	143	0.002	0.002	143	0.122	0.120	143	0.173	0.138	143	0.689	0.166
Brazil	167	0.001	0.002	167	0.032	0.027	167	0.038	0.031	167	0.314	0.128
Canada	228	0.077	0.021	228	0.015	0.004	228	0.019	0.006	228	0.206	0.039
Chile	51	0.001	0.002	51	0.017	0.001	51	0.038	0.001	51	0.312	0.012
China (Mainland)	92	0.003	0.001	92	0.077	0.024	92	0.114	0.048	92	0.647	0.191
Denmark	192	0.170	0.646	228	0.100	0.070	228	0.136	0.085	228	0.623	0.165
Finland	143	0.008	0.005	143	0.102	0.050	143	0.180	0.090	143	0.718	0.212
France	143	0.011	0.003	143	0.025	0.010	143	0.028	0.010	143	0.290	0.048
Germany	143	0.000	0.000	143	0.062	0.032	143	0.072	0.035	143	0.522	0.099
Greece	103	0.019	0.007	103	0.180	0.270	103	0.230	0.261	103	0.754	0.130
Hong Kong	149	0.001	0.001	130	0.133	0.100	149	0.305	0.293	149	0.823	0.124
India	137	0.004	0.002	137	0.068	0.087	137	0.144	0.193	137	0.503	0.304
Israel	118	0.013	0.005	118	0.017	0.008	118	0.025	0.007	118	0.255	0.061
Italy	143	0.006	0.003	143	0.027	0.012	143	0.039	0.017	143	0.330	0.092
Japan	200	0.014	0.044	228	0.073	0.115	228	0.077	0.114	228	0.357	0.140
Korea	111	0.052	0.022	111	0.014	0.003	111	0.016	0.003	111	0.190	0.032
Mexico	134	0.000	0.000	134	0.125	0.086	134	0.156	0.100	134	0.640	0.126
Netherlands	143	0.002	0.004	143	0.084	0.076	143	0.145	0.097	143	0.706	0.150
Norway	192	0.031	0.019	228	0.082	0.043	228	0.129	0.080	228	0.644	0.215
Portugal	140	0.002	0.002	114	0.093	0.036	114	0.147	0.031	114	0.749	0.059
Singapore	168	0.001	0.001	145	0.180	0.108	168	0.396	0.291	168	0.859	0.113
South Africa	191	0.016	0.008	175	0.088	0.146	191	0.190	0.298	191	0.542	0.231
Spain	143	0.003	0.002	143	0.024	0.009	143	0.032	0.008	143	0.298	0.040
Sweden	192	0.015	0.012	228	0.123	0.164	228	0.230	0.255	228	0.622	0.342
Switzerland	205	0.008	0.008	228	0.115	0.134	228	0.140	0.156	228	0.500	0.295
Taiwan	75	0.010	0.003	75	0.010	0.001	75	0.016	0.001	75	0.186	0.009
Thailand	172	0.043	0.148	172	0.021	0.006	172	0.027	0.007	172	0.279	0.041
United Kingdom	158	0.007	0.006	158	0.071	0.113	158	0.085	0.135	158	0.407	0.279

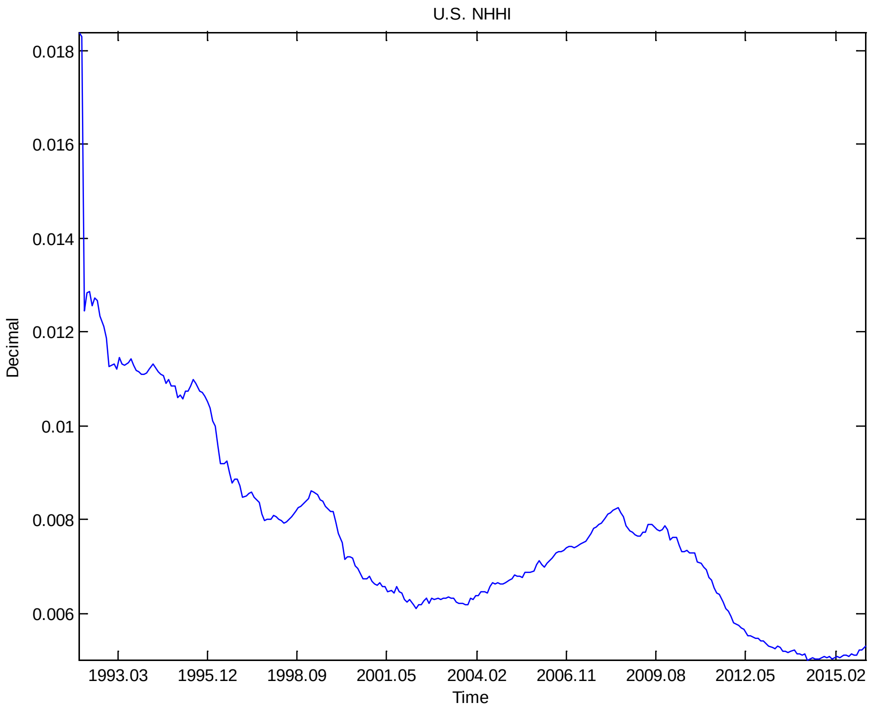
Panel B

Global Market	Net Return (%)			Net Alpha (%)			Stlye-Matching Model R^2		
	Obs	Mean	Sd	Obs	Mean	Sd	Obs	Mean	Sd
Australia	45,457	0.984	6.773	32,865	0.213	1.995	32,884	0.919	0.084
Austria	571	1.137	8.566	571	0.182	2.218	571	0.902	0.042
Belgium	2,774	0.770	6.118	2,690	0.227	1.896	2,690	0.896	0.080
Brazil	15,543	1.071	10.698	11,995	0.143	5.123	11,995	0.891	0.100
Canada	56,414	0.717	5.953	44,250	0.033	2.189	44,250	0.865	0.126
Chile	1,500	-1.236	6.092	1,500	-0.144	1.096	1,500	0.972	0.027
China (Mainland)	368	0.537	8.480	325	0.156	2.476	325	0.948	0.052
Denmark	4,526	1.089	6.048	3,670	0.086	1.465	3,670	0.907	0.123
Finland	3,716	0.867	6.939	3,356	0.049	1.703	3,356	0.929	0.075
France	24,210	0.567	6.158	22,324	0.127	1.982	22,324	0.875	0.121
Germany	11,305	0.794	6.734	10,630	0.107	2.059	10,633	0.907	0.102
Greece	1,709	-0.404	9.161	1,656	-0.033	2.028	1,656	0.941	0.046
Hong Kong	2,526	1.033	6.267	2,177	0.185	2.143	2,177	0.862	0.171
India	14,334	1.496	8.885	11,090	0.385	2.707	11,090	0.888	0.093
Israel	9,216	0.675	6.972	8,466	-0.162	2.984	8,466	0.829	0.135
Italy	5,894	0.322	6.831	5,601	0.124	1.296	5,601	0.955	0.057
Japan	56,454	0.379	5.738	39,763	0.095	2.437	39,763	0.849	0.148
Korea	22,019	0.620	8.274	16,819	0.013	1.885	16,823	0.949	0.041
Mexico	3,709	0.689	6.688	3,320	0.049	2.691	3,356	0.828	0.173
Netherlands	1,896	0.668	6.759	1,876	0.152	2.488	1,878	0.873	0.152
Norway	9,155	0.984	8.109	7,110	0.088	2.291	7,188	0.910	0.101
Portugal	2,220	0.240	7.157	2,206	0.075	1.968	2,206	0.930	0.024
Singapore	2,191	0.964	6.387	1,914	0.068	1.441	1,914	0.931	0.057
South Africa	13,193	1.015	7.267	9,973	0.145	2.269	9,973	0.910	0.074
Spain	10,605	0.550	6.959	9,962	0.002	1.296	9,962	0.956	0.076
Sweden	18,262	0.948	7.259	14,282	0.058	1.785	14,282	0.933	0.087
Switzerland	21,039	0.770	5.344	15,429	0.049	2.082	15,429	0.877	0.144
Taiwan	7,338	0.420	5.543	7,308	-0.130	2.483	7,308	0.836	0.067
Thailand	20,298	1.340	7.096	15,110	0.212	1.757	15,110	0.945	0.043
United Kingdom	60,169	0.837	5.364	52,583	0.013	1.545	52,589	0.916	0.083

Figure 4.3 illustrates the monthly NHHI of the U.S. active equity mutual fund market from January 1992 to December 2015. It shows that the concentration level of this U.S. market decreased substantially from January 1992 to the end of 2003. It started to increase gradually, decreased again, and has reached the lowest point at the current time.

Figure 4.3. NHHI of the U.S. Active Equity Mutual Fund Market

The NHHI value is in decimals. The sample period is from January 1992 to December 2015.



Empirical Results

Table 4.3 reports the empirical results of the market concentration-net alpha relation for the 30 active equity mutual fund markets in our sample. Here we focus on how the fund net alphas are associated with the concentration of the U.S. active equity mutual fund market. Panel A reports the results of the model which contains only the first-order terms of the local market NHHI and the U.S. market NHHI. We can see that most markets' fund net alphas, on average, are significantly associated with the U.S. NHHI after controlling for the local market NHHI. In particular, 17 (5) markets' fund net alphas, on average, are significantly negatively (positively) associated with the U.S. NHHI. Also, most markets' fund net alphas, on average, are significantly associated with the local NHHI; in particular, 9 (13) markets' fund net alphas, on average, are significantly negatively (positively) associated with the local NHHI. Interestingly, in 19 markets, the sign of the coefficient of the U.S. NHHI is different from that of the local NHHI, and future research can explore the reason behind this phenomenon.

Panel B shows the results of the model and contains both the first-order terms and second-order terms of the local market NHHI and the U.S. market NHHI. We can see that 14 (6) markets' fund net alphas, on average, are concave (convex) in the U.S. NHHI, as the second-order term of the U.S. market NHHI is significantly negative (positive). On the other hand, 13 (7) markets' fund net alphas, on average, are concave (convex) in the local NHHI.

We pool the datasets of all markets together and run the model again. We present the results in Panel C. We find that on average, fund net alphas are significantly negatively associated with the U.S. NHHI, and concave in the U.S. NHHI. On the other hand, although fund net alphas are significantly negatively associated with the local NHHI in model specification 1, the coefficient of the local NHHI loses significance after we include the U.S. NHHI into the model (i.e., model specifications 2 to 4).

Table 4.3. Market Concentrations and Fund Net Alpha

This table reports the results of our RD panel regression model for each market. The dependent variable is fund net alpha. Panel A reports the results of the model including only the first-order terms of the lagged value of the forward demeaning local NHHI and U.S. NHHI, whereas Panel B reports the results of the model including the first-order and second-order terms of lagged value of the forward demeaning local NHHI and U.S. NHHI. Panel C reports the results when pooling the datasets of all markets together. NHHI is the Normalized-Herfindahl-Hirschman index measured in decimal. The unit of coefficients is percentage. Standard errors are clustered by fund and presented in parentheses. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level in a two-tail t-test, respectively.

Panel A Result of Each Market, First-Order

	Lagged Local NHHI		Lagged U.S. NHHI				
Global Market	Coefficient	S.E.	Coefficient	S.E.	Obs	R^2	Adj-R^2
Australia	1.2906	(1.1457)	-75.5710***	(13.5934)	32,608	0.002	0.001
Austria	0.0897	(0.1684)	-458.9187**	(125.5158)	564	0.012	0.008
Belgium	0.4893**	(0.2217)	-25.8232	(22.2164)	2,668	0.001	0.000
Brazil	-11.2486***	(2.4068)	322.5425***	(53.5838)	11,893	0.006	0.005
Canada	0.4756	(4.5740)	-61.8306***	(12.1148)	43,949	0.001	0.001
Chile	-11.8376	(10.5419)	-215.6667***	(75.0880)	1,475	0.016	0.015
China (Mainland)	-15.3670	(9.2375)	241.2103	(144.8966)	321	0.018	0.011
Denmark	-0.0754	(0.7729)	-75.4036**	(28.7389)	3,646	0.003	0.002
Finland	3.4390***	(0.7475)	-12.1600	(24.5917)	3,329	0.009	0.008
France	6.6353***	(1.2285)	-50.3707***	(11.5293)	22,144	0.001	0.001
Germany	1.0358**	(0.4296)	-52.9686***	(12.4963)	10,541	0.001	0.001
Greece	-0.7813***	(0.1655)	-3.7675	(58.4727)	1,639	0.009	0.008
Hong Kong	0.7358***	(0.1301)	147.1041***	(33.2653)	2,159	0.011	0.011
India	2.5388***	(0.3792)	-126.1794***	(25.7825)	10,981	0.006	0.005
Israel	-20.3981***	(3.2614)	-134.7909***	(28.6698)	8,384	0.002	0.002
Italy	-0.4211	(1.1088)	-21.7150	(18.0456)	5,554	0.000	0.000
Japan	-0.7009***	(0.1262)	-59.9246***	(14.0838)	39,445	0.003	0.003
Korea	83.9876***	(6.4067)	-149.2417***	(17.1097)	16,609	0.009	0.009
Mexico	2.9666***	(0.9298)	-235.7848***	(47.9464)	3,291	0.006	0.006
Netherlands	-0.9892*	(0.4906)	94.1368**	(39.7610)	1,861	0.002	0.001
Norway	-4.9509***	(0.7102)	38.1465*	(20.9406)	7,065	0.005	0.005
Portugal	-10.7860***	(2.0232)	-594.4104***	(59.1839)	1,761	0.032	0.031
Singapore	0.4053	(0.2457)	-44.1550	(30.5739)	1,900	0.008	0.007
South Africa	-0.9647***	(0.2430)	24.1508	(25.5526)	9,895	0.005	0.005
Spain	-17.9681***	(1.8727)	-110.4906***	(14.7578)	9,877	0.014	0.013
Sweden	0.4302***	(0.1560)	21.8705	(16.4268)	14,186	0.001	0.001
Switzerland	0.4577**	(0.2180)	56.2353***	(13.3695)	15,311	0.001	0.001
Taiwan	164.5697***	(40.6229)	-96.5752***	(27.0815)	7,210	0.005	0.004
Thailand	22.3072***	(3.1148)	-31.2096**	(12.4564)	14,978	0.005	0.004
United Kingdom	0.2960***	(0.0762)	-78.7195***	(7.7468)	52,156	0.003	0.003

Panel B Result of Each Market, First-Order and Second-Order

	Lagged Local NHHI		(Lagged Local NHHI)^2		Lagged U.S. NHHI		(Lagged U.S. NHHI)^2		(Lagged Local NHHI)*(Lagged U.S. NHHI)				
Global Market	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Obs	R^2	Adj-R^2
Australia	12.1344	(15.0065)	-620.8288***	(211.7154)	1,671.5690***	(220.4302)	-140018.0767***	(17,983.1620)	3,848.2723**	(1,910.1458)	32,608	0.007	0.006
Austria	-13.4248	(10.7595)	1.2624	(2.5138)	15,469.9485***	(3,308.6370)	-1.1498e+06***	(251,478.2428)	1,717.9306	(1,221.7965)	564	0.033	0.025
Belgium	14.9237***	(3.8047)	-18.0470***	(2.7051)	-919.5150	(568.4424)	70,365.0348	(45,487.0366)	-881.2612*	(474.4427)	2,668	0.007	0.005
Brazil	-223.3929***	(73.4331)	-172.3538	(137.7407)	8,235.7286***	(773.0714)	-660744.6012***	(63,520.0179)	34,326.9545***	(10,002.9437)	11,893	0.014	0.014
Canada	95.7283	(58.2449)	24,143.0246***	(2,975.0089)	3,194.2467***	(225.4686)	-130089.5009***	(15,966.0459)	-118724.2097***	(16,624.3884)	43,949	0.012	0.012
Chile	1,231.5547***	(230.7132)	58,590.2686**	(25,494.8691)	-3,224.8274	(2,126.0537)	1266803.3425***	(322,320.2068)	-602982.3203***	(179,010.7106)	1,475	0.037	0.034
China (Mainland)	-53.5502	(70.2955)	434.4585	(322.7890)	299.2410	(1,784.1102)	-10,881.0737	(209,526.1991)	-1,111.8371	(17,053.1777)	321	0.025	0.010
Denmark	-13.8217**	(6.4208)	-8.2598	(18.9726)	642.1270	(561.5610)	-65,256.6056	(45,583.5397)	2,189.5172**	(960.4949)	3,646	0.006	0.004
Finland	-10.8887	(9.5915)	-39.2853**	(17.9864)	2,546.8754***	(546.7124)	-224822.4020***	(50,069.7658)	3,499.5652*	(1,920.1950)	3,329	0.015	0.013
France	-440.0335***	(36.4607)	-1,563.3112***	(148.4873)	632.5885***	(196.1863)	-200861.7068***	(20,283.8866)	85,167.4164***	(6,495.1462)	22,144	0.013	0.013
Germany	-294.6229***	(38.3071)	-51.7654***	(9.2825)	-1,612.6980***	(378.9749)	-69,988.6949***	(20,751.7994)	46,490.7986***	(6,150.1818)	10,541	0.009	0.008
Greece	2.6161	(7.4420)	2.5538*	(1.2781)	1,685.6667	(1,142.8328)	-118750.0354	(82,244.9924)	-934.2699	(1,042.7285)	1,639	0.011	0.008
Hong Kong	4.2212	(5.2975)	-1.7773*	(0.8884)	2,854.8654***	(589.8847)	-206130.0127***	(42,669.0882)	-297.9554	(700.5502)	2,159	0.018	0.016
India	124.6320***	(9.1950)	-18.1484***	(3.6192)	-4,525.2875***	(419.1960)	377,977.4446***	(32,533.0687)	-16,823.5393***	(1,257.2852)	10,981	0.025	0.024
Israel	374.1449***	(56.5913)	-8,319.4400***	(726.1704)	-824.5578	(652.2047)	61,537.1866	(47,116.3854)	-9,877.1457	(6,798.3432)	8,384	0.011	0.010
Italy	-202.5231***	(74.5756)	254.9126	(165.9769)	-409.9998	(466.6043)	-18,096.3287	(27,326.8079)	27,405.4333***	(9,778.0061)	5,554	0.003	0.002
Japan	-2.8400	(2.7628)	57.8573***	(4.8129)	938.3056**	(445.0549)	-70,039.0899**	(35,105.6898)	-2,648.6909***	(284.3859)	39,445	0.013	0.013
Korea	80.8340	(69.0326)	36,458.1654***	(3,376.2972)	1,733.9000***	(513.7604)	6,385.4397	(45,137.0800)	-147980.9918***	(12,171.7114)	16,609	0.022	0.021
Mexico	44.7992***	(14.3663)	-54.8955***	(6.3667)	1,717.8313	(1,092.1995)	-148639.0269	(92,200.6805)	-3,268.9933	(1,948.5128)	3,291	0.015	0.014
Netherlands	125.1996**	(47.5393)	-3.8764	(2.2596)	-1,177.0623**	(489.5913)	173,513.2144**	(60,671.7974)	-17,185.1809**	(6,525.6993)	1,861	0.010	0.008
Norway	-60.0770***	(22.2524)	6.1427	(44.2803)	2,499.4191***	(835.4820)	-220908.8625***	(71,762.3241)	7,900.8989***	(2,570.0113)	7,065	0.011	0.010
Portugal	344.9390***	(79.2038)	-625.9653***	(144.4188)	2,071.2786	(2,355.7691)	36,128.5548	(127,655.2587)	-40,339.0456***	(8,351.0361)	1,761	0.069	0.067
Singapore	-28.1398***	(9.2428)	3.9966***	(0.8335)	-1,512.4202	(1,122.8143)	59,325.6407	(76,499.2671)	3,802.2678**	(1,355.7160)	1,900	0.019	0.016
South Africa	16.7911***	(5.0119)	-1.6189	(1.1140)	3,824.4959***	(504.0262)	-285469.7106***	(38,524.3906)	-2,443.4683***	(768.7778)	9,895	0.022	0.022
Spain	174.5518***	(33.7259)	-1,199.7413***	(236.3992)	1,137.0467***	(266.6711)	-60,279.1440***	(18,663.7125)	-20,038.6924***	(3,929.2799)	9,877	0.020	0.019
Sweden	1.0919	(1.5475)	-1.8420*	(0.9988)	-547.1235**	(252.4415)	43,311.0509**	(19,748.9973)	28.1569	(240.2488)	14,186	0.002	0.001
Switzerland	-9.1391***	(1.8490)	-10.7796***	(2.8832)	565.5256***	(207.9305)	-43,742.3355***	(16,270.5384)	1,806.5763***	(371.0099)	15,311	0.004	0.003
Taiwan	4,456.9070***	(1,239.8500)	-45,785.1589	(47,549.6937)	-1,848.7771**	(811.3109)	597,892.0507***	(37,756.4960)	-581486.0656***	(78,407.1888)	7,210	0.019	0.018
Thailand	322.4031***	(35.4762)	1,990.5080***	(523.3670)	-363.7719	(304.6007)	111,454.3335***	(24,155.8015)	-57,209.8698***	(4,421.7657)	14,978	0.027	0.026
United Kingdom	-2.6171	(2.2673)	0.0429	(0.3408)	383.2011***	(110.0330)	-36,252.9033***	(8,528.5200)	432.5343	(351.8808)	52,156	0.004	0.003

Panel C Result of Pooled Panel Data

	(1)	(2)	(3)	(4)
Lagged Local NHHI	-0.1420**		-0.1002	0.6683
	(0.0682)		(0.0680)	(0.6798)
(Lagged Local NHHI)^2				-0.1613
				(0.2421)
Lagged U.S. NHHI		-40.8668***	-40.2406***	946.6345***
		(4.2360)	(4.2267)	(83.7309)
(Lagged U.S. NHHI)^2				-75,422.6531***
				(6,508.1317)
(Lagged Local NHHI)*(Lagged U.S. NHHI)				-118.5105
				(93.2153)
Obs	357,400	357,400	357,400	357,400
R^2	0.0000	0.0004	0.0004	0.0013
Adj-R^2	0.0000	0.0004	0.0004	0.0013

Table 4.4 reports the empirical result of the market concentration-AFMI size relation for the 30 active equity mutual fund markets. Panel A reports the results of the model which contains only the first-order terms of the concentration measures. We can see that most markets' AFMI sizes, on average, are not significantly associated with the U.S. NHHI, and instead, they are mainly explained by its lag values. In particular, we find that only 9 (2) markets' AFMI sizes, on average, are significantly negatively (positively) associated with the U.S. NHHI. On the other hand, we find that only 7 (7) markets' AFMI sizes, on average, are significantly negatively (positively) associated with the local NHHI. The insignificance of the coefficients of the U.S. NHHI and the local NHHI probably is due to the small sample periods of most of the global fund markets.

Panel B reports the results of the model and contains both the first-order and second-order terms of the concentration measures. We find that only 3 (4) markets' AFMI sizes, on average, are concave (convex) in the U.S. NHHI, as the second-order term of the U.S. market NHHI is significantly negative (positive). On the other hand, we find that only 3 (6) markets' AFMI sizes, on average, are concave (convex) in the local NHHI.

We pool the datasets of all markets together, run a panel VAR model and present the results in Panel C. Although panel VAR techniques become more and more popular, the algorithm of estimating a panel VAR is still under developed. Here, we use the most updated algorithm that we can find in STATA[44], and it requires all variables in the panel VAR to be endogenous variables. Here, we include only AFMI size, the local NHHI, and the U.S. NHHI in the panel VAR. We find that on average, AFMI are significantly negatively associated with the U.S. NHHI, but not significantly associated with the local NHHI. As an aside, from the last two columns, we also note that on average, the U.S. NHHI and the local NHHI are not highly correlated with each other: the lagged U.S. NHHI is not statistically significantly associated with the local NHHI, and the lagged local NHHI is not economically significantly associated with the U.S. NHHI.

[44] We use the "xtvar" function in STATA.

Table 4.4. Market Concentrations and AFMI Size

This table reports the results of our VAR model. Panel A reports the results including only the first-order terms of local NHHI and U.S. NHHI in the model, whereas Panel B reports the results including the first-order and second-order terms of these variables. Only the results of the first equation of our VAR model, i.e., the equation with AFMI size as the dependent variable, are reported in these two Panels. Panel C reports the result of a panel VAR model pooling all markets' data together. AFMI size is the sum of funds' net assets under management divided by the stock market capitalization in the same month, and its value is in decimal. NHHI is the Normalized-Herfindahl-Hirschman index measured in decimal. Lagged AFMI size, Time Trend and a constant term are also included in the model. Time Trend is included as an exogenous variable in the VAR, and is set to be one for the first month of the sample, and to increase by one each month. Small-sample adjusted statistics are used and presented. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level in a two-tail t-test, respectively.

Panel A Result of Each Market, First-Order

	Lagged Local NHHI		Lagged U.S. NHHI			
Global Market	Coefficient	S.E.	Coefficient	S.E.	Obs	R^2
Australia	-0.0711	(0.0518)	-0.7225**	(0.3367)	227	0.885
Austria	0.0065*	(0.0038)	-2.5065**	(0.9665)	84	0.821
Belgium	0.0032**	(0.0016)	0.0298	(0.0559)	142	0.884
Brazil	0.0068	(0.0050)	0.2356***	(0.0905)	166	0.894
Canada	0.3638*	(0.1916)	-0.1800	(0.2571)	227	0.979
Chile	0.0595	(0.0362)	-0.1704	(0.2614)	59	0.982
China (Mainland)	-0.0135***	(0.0046)	-0.0300	(0.1638)	91	0.617
Denmark	0.7974	(0.6268)	14.5061	(18.7044)	191	0.911
Finland	-0.0008	(0.0018)	0.1453	(0.1003)	142	0.981
France	-0.0080	(0.0143)	0.0737	(0.1192)	142	0.927
Germany	-0.0013***	(0.0004)	-0.0048*	(0.0025)	142	0.201
Greece	-0.0059***	(0.0020)	-1.4945***	(0.4690)	102	0.859
Hong Kong	-0.0002**	(0.0001)	-0.0233**	(0.0106)	129	0.988
India	-0.0028**	(0.0014)	0.0966	(0.1330)	136	0.914
Israel	0.0876*	(0.0465)	-0.4073	(0.5088)	117	0.745
Italy	0.0055	(0.0083)	-0.0853	(0.1274)	142	0.942
Japan	-0.0082	(0.0126)	1.8370	(1.8165)	199	0.801
Korea	-0.1918	(0.3173)	2.7039*	(1.4235)	110	0.903
Mexico	-0.0002***	(0.0001)	-0.0052*	(0.0031)	133	0.975
Netherlands	0.0121	(0.0197)	0.0823	(0.0598)	142	0.005
Norway	0.0195	(0.0201)	0.0349	(0.5369)	191	0.903
Portugal	0.0260***	(0.0056)	-0.2390***	(0.0829)	113	0.907
Singapore	-0.0003	(0.0002)	-0.0398	(0.0461)	144	0.960
South Africa	-0.0020*	(0.0010)	0.0933	(0.1306)	174	0.961
Spain	0.0047*	(0.0027)	-0.0746**	(0.0309)	142	0.993
Sweden	-0.0035	(0.0031)	-0.0650	(0.1761)	191	0.976
Switzerland	0.0025	(0.0041)	0.3326	(0.2372)	204	0.924
Taiwan	0.1689*	(0.0928)	-0.0535	(0.1151)	74	0.979
Thailand	0.2471	(0.9116)	-4.3840	(5.9990)	171	0.718
United Kingdom	-0.0029	(0.0020)	-0.7151***	(0.2317)	157	0.928
Lagged AFMI Size			Yes			
Time Trend			Yes			
Constant			Yes			

Panel B Result of Each Market, First-Order and Second-Order

	Lagged Local NHHI		(Lagged Local NHHI)^2		Lagged U.S. NHHI		(Lagged U.S. NHHI)^2		(Lagged Local NHHI)*(Lagged U.S. NHHI)			
Global Market	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Obs	R^2
Australia	-0.2192	(0.2840)	4.9728	(3.6708)	-1.4935	(5.0678)	92.0455	(393.0443)	-35.7700	(24.9172)	227	0.887
Austria	0.0696*	(0.0385)	-0.0064	(0.0122)	-11.5497	(15.8488)	777.1114	(1,083.7996)	-7.5277*	(3.9499)	84	0.818
Belgium	-0.0133	(0.0115)	0.0031	(0.0064)	-1.1532	(0.7169)	77.5717	(55.9782)	1.7651	(1.3490)	142	0.878
Brazil	0.1340	(0.0838)	-0.0890	(0.1407)	0.7521	(1.3696)	-12.0716	(102.1999)	-18.4832	(11.5982)	166	0.896
Canada	-1.3296*	(0.7905)	233.3294***	(40.0376)	-6.1615	(5.1675)	1,246.2961***	(453.4508)	-839.6467***	(187.1214)	227	0.981
Chile	-0.2491	(0.2917)	2.5488	(25.3728)	9.3029**	(3.8543)	-907.2576**	(439.7056)	52.4162	(178.0283)	59	0.984
China (Mainland)	-0.0263	(0.0215)	0.4428***	(0.1526)	-0.3591	(0.9395)	58.0120	(71.3939)	-7.3075	(4.7038)	91	0.648
Denmark	-15.8950***	(5.9205)	15.5750	(11.4372)	-35.5479	(359.1966)	-4,480.4681	(27,826.2122)	1,876.3508*	(1,015.3928)	191	0.916
Finland	0.0374	(0.0261)	0.0491	(0.0406)	0.8491	(1.6501)	-0.6120	(143.3447)	-7.0856	(4.8149)	142	0.982
France	0.1594	(0.2816)	2.8767***	(0.8947)	0.3133	(1.6465)	101.9600	(161.4764)	-60.7448	(41.3272)	142	0.935
Germany	-0.0024	(0.0027)	-0.0014	(0.0011)	0.0063	(0.0387)	-2.4138	(2.2264)	0.3915	(0.4211)	142	-0.160
Greece	0.0078	(0.0466)	-0.0023	(0.0119)	5.0494	(6.7230)	-442.6780	(493.9509)	-1.4952	(6.4397)	102	0.860
Hong Kong	0.0015	(0.0012)	-0.0005	(0.0004)	0.0598	(0.1265)	-5.2909	(8.6571)	-0.2030	(0.1565)	129	0.989
India	-0.0254	(0.0202)	0.0274***	(0.0098)	-1.4189	(1.0177)	97.3713	(75.1043)	1.5355	(2.5322)	136	0.922
Israel	-3.3545***	(1.0891)	13.6453	(9.4577)	4.4281	(7.4120)	-690.1643	(501.0213)	554.3161***	(141.8888)	117	0.764
Italy	-0.0069	(0.1804)	-0.7317	(0.5187)	-1.3109	(1.4909)	71.3317	(89.7263)	10.5638	(23.6822)	142	0.946
Japan	0.2851	(0.2229)	-0.2560	(0.3593)	-127.6519***	(31.0744)	9,783.1203***	(2,345.5046)	-26.3394	(27.0975)	199	0.817
Korea	9.3334*	(4.8315)	-506.5681***	(182.5274)	-7.6466	(23.3194)	-209.7097	(2,287.1588)	734.0478	(720.2526)	110	0.911
Mexico	-0.0005	(0.0009)	0.0004	(0.0005)	0.0115	(0.0496)	-1.3550	(4.1335)	0.0276	(0.1079)	133	0.975
Netherlands	0.0004	(0.0091)	-0.0528***	(0.0074)	0.4010*	(0.2131)	-31.4706*	(18.3690)	0.6027	(1.1448)	142	-0.050
Norway	-1.1470***	(0.3237)	1.8692***	(0.6539)	8.6998	(10.3119)	-1,054.8075	(843.4399)	123.0075***	(38.2893)	191	0.909
Portugal	0.0824	(0.0865)	-0.0525	(0.1543)	2.5054	(2.6280)	-157.5255	(142.0225)	-7.2516	(9.0960)	113	0.910
Singapore	-0.0093*	(0.0050)	-0.0006	(0.0013)	-1.8129***	(0.6777)	120.4225***	(45.7049)	1.4151**	(0.7050)	144	0.962
South Africa	-0.0203	(0.0384)	-0.0002	(0.0041)	3.8373*	(2.2840)	-291.7238*	(167.3644)	2.8387	(6.3129)	174	0.962
Spain	0.0708*	(0.0395)	-0.5463**	(0.2376)	0.6545	(0.5726)	-48.0717	(45.9861)	-4.7948	(5.4162)	142	0.993
Sweden	-0.0275	(0.0649)	0.0265	(0.0671)	2.9276	(2.9221)	-237.5882	(220.9121)	2.6127	(8.4581)	191	0.976
Switzerland	-0.1467***	(0.0344)	0.0867	(0.0627)	2.9796	(3.4156)	-251.8242	(254.1418)	18.9163***	(4.3578)	204	0.932
Taiwan	-3.3007	(3.0760)	104.4763	(108.9223)	-1.2560	(1.7921)	-84.9171	(107.8926)	243.5878	(199.6846)	74	0.980
Thailand	0.3421	(11.9248)	-52.0553	(114.9224)	-74.1993	(126.7540)	4,766.8786	(9,451.4251)	365.4583	(1,382.0903)	171	0.719
United Kingdom	-0.0488	(0.0494)	0.0155**	(0.0078)	-9.2242***	(3.1868)	589.7990***	(227.6833)	5.4561	(7.6308)	157	0.932
Lagged AFMI Size			Yes									
Time Trend			Yes									
Constant			Yes									

Panel C Result of Panel VAR

	AFMI Size	Local NHHI	U.S. NHHI
Lagged AFMI Size	0.8777***	-0.0382	0.0001
	(0.0066)	(0.0470)	(0.0003)
Lagged Local NHHI	-0.0004	0.9590***	0.0002***
	(0.0007)	(0.0046)	(0.0000)
Lagged U.S. NHHI	-0.0744***	-0.0783	0.9987***
	(0.0262)	(0.1852)	(0.0011)
Constant	Yes	Yes	Yes
Obs	3,210	3,210	3,210
R^2	0.9877	0.9713	0.9964

Table 4.5 summaries the results in Table 4.3 and Table 4.4. It shows that global fund markets' fund net alphas and AFMI size tend to move in the same direction with the U.S. AFMI concentration: 15 (5) markets' fund net alphas and AFMI size are both on average, negatively (positively) associated with the U.S. NHHI; among them, 7 (1) markets' fund net alphas and AFMI size are both significantly negatively (positively) associated with the U.S. NHHI. On the other hand, global fund markets' fund net alphas and AFMI size do not tend to move in the same direction with the local AFMI concentration: only 4 (6) markets' fund net alphas and AFMI size are both on average, negatively (positively) associated with the local NHHI; among them, 2 (2) markets' fund net alphas and AFMI size are both significantly negatively (positively) associated with the local NHHI. Also, from the Panel Cs of Table 4.3 and Table 4.4, we can see that when pooling all the markets' data together, on average, fund net alphas and AFMI size, are both significantly negatively associated with the U.S. NHHI but are insignificantly associated with the local NHHI.

The fact that global fund markets' fund net alphas and AFMI size tend to move in the same direction with the U.S. AFMI concentration is consistent with the prediction of our theoretical model under both the exogenous concentration framework and the endogenous concentration framework. Our global fund market samples contain the funds that invest only in the local stock market. Based on our theoretical model, a higher U.S. AFMI concentration would affect these funds in two aspects: 1) it implies more unexplored opportunities in U.S. stock markets, which attracts local AFMI managerial efforts, leaves more unexplored opportunities in the local stock market in equilibrium (i.e., the substitution effect) and makes managerial effort spent in local stock market more efficient in producing gross alphas; 2) it implies that it is easier for U.S. managers to ask for higher salary, which consequently increases the reservation price of effort of local managers, thus increasing the costs of local managerial efforts. Therefore, in some markets, if both fund net alphas and AFMI size are both positively (negatively) associated with the U.S. AFMI concentration, then it implies that a higher U.S. AFMI concentration increases local AFMI managerial effort impacts on gross alpha production more (less) than it increases effort costs.

The current low and probably decreasing concentration in the U.S. AFMI, given the trade-off of higher U.S. AFMI concentration is not changed, would benefit (harm) the global AFMIs whose fund net alphas and AFMI size are on average, negatively (positively) associated with the U.S. NHHI. Our results show that half the global AFMIs in our sample are likely to benefit from that.

Robustness Check

We conduct the robustness checks on our empirical results. We analyze the sensitivity of our results to various measures of market concentration, by replacing the local (U.S.) NHHI by the local (U.S.) HHI or the local (U.S.) 5-Fund-Index and then redoing the analyses. We find consistent results. Also, we analyze the sensitivity of the results in Table 4.3 by using fund fixed-effect regressions instead of the RD method. We find consistent results.

Table 4.5. Summary of Results

This table summarizes the results of the previous two tables and shows the number of markets that present the corresponding results. The numbers of positive or negative first-order relationships are counted from Panel A of the previous two tables, whereas the numbers of positive or negative second-order relationships are counted from Panel B of the previous two tables.

U.S. NHHI -- Net Alpha			
Positive 1st-order	8	Negative 1st-order	22
Significantly Positive 1st-order	5	Significantly Negative 1st-order	17
Positive 2nd-order	11	Negative 2nd-order	19
Significantly Positive 2nd-order	6	Significantly Negative 2nd-order	14

U.S. NHHI -- AFMI Size			
Positive 1st-order	12	Negative 1st-order	18
Significantly Positive 1st-order	2	Significantly Negative 1st-order	9
Positive 2nd-order	12	Negative 2nd-order	18
Significantly Positive 2nd-order	4	Significantly Negative 2nd-order	3

U.S. NHHI -- Net Alpha & AFMI Size			
Both Positive 1st-order	5	Both Negative 1st-order	15
Both Significantly Positive 1st-order	1	Both Significantly Negative 1st-order	7
Both Positive 2nd-order	4	Both Negative 2nd-order	11
Both Significantly Positive 2nd-order	0	Both Significantly Negative 2nd-order	1

Local NHHI -- Net Alpha			
Positive 1st-order	17	Negative 1st-order	13
Significantly Positive 1st-order	13	Significantly Negative 1st-order	9
Positive 2nd-order	12	Negative 2nd-order	18
Significantly Positive 2nd-order	7	Significantly Negative 2nd-order	13

Local NHHI -- AFMI Size			
Positive 1st-order	15	Negative 1st-order	15
Significantly Positive 1st-order	7	Significantly Negative 1st-order	7
Positive 2nd-order	16	Negative 2nd-order	14
Significantly Positive 2nd-order	6	Significantly Negative 2nd-order	3

Local NHHI -- Net Alpha & AFMI Size			
Both Positive 1st-order	6	Both Negative 1st-order	4
Both Significantly Positive 1st-order	2	Both Significantly Negative 1st-order	2
Both Positive 2nd-order	5	Both Negative 2nd-order	7
Both Significantly Positive 2nd-order	1	Both Significantly Negative 2nd-order	1

4.4. Conclusion

We introduce a theoretical model of an international AFMIs equilibrium, where we investigate how local and foreign AFMI market concentrations affect local AFMI performance, size, and managerial efforts. We extend Chapter 2’s AFMI model to a two-country framework, where in each country, investors can only invest in local AFMI funds due to transaction costs, while AFMI funds can invest in both local and foreign stock markets. Higher local market concentration levels imply more unexplored investment opportunities in the local stock market, making managers’ efforts spent in the local stock market more productive. In equilibrium, it also attracts managers in the foreign country to invest in the local stock market, leaving more unexplored investment opportunities in the foreign market and, consequently, making managers’ efforts spent in the foreign stock market more productive. At the same time, however, higher local market concentration levels allow local managers to require higher compensation for their efforts, making managerial efforts spent in both local and foreign stock markets more costly. On the other hand, higher foreign market concentration levels imply more unexplored investment opportunities in the foreign stock market, making managers’ efforts spent in the foreign stock market more productive. In equilibrium, it also attracts managers in the local market to invest in the foreign stock market, leaving more unexplored investment opportunities in the local market, and consequently making managers’ efforts spent in the local stock market more productive. At the same time, however, higher foreign market concentration levels allow foreign managers to require higher compensation for their efforts, increasing local managers’ reservation price of efforts and, consequently, making local managerial efforts more costly.

Our model’s comparative statistics offer the market concentration-net alpha relations and market concentration-AFMI size relations. In particular, in equilibrium, if higher local (foreign) market concentration increases effort impacts on alpha production more than it increases effort costs, i.e., higher fund local (foreign) market concentration induces a larger sum of direct benefits of managerial efforts, then fund expected net alphas and AFMI size will both increase, and the opposite is also true. In addition, the concavity of fund expected net alphas in local (foreign) market concentration indicates that the sum of direct benefits of managerial efforts is concave in local (foreign) market concentration. This further induces concavity of AFMI size in local (foreign) market concentration. On the other hand, the convexity of AFMI size in local (foreign) market concentration implies that the sum of direct benefits of managerial efforts is convex in local (foreign) market concentration, and this induces convexity of AFMI size in local (foreign) market concentration.

We specialize our model to allow for endogenous market concentration levels, which befits empirical market concentration measures and facilitates empirical studies. Although the local market concentration-net alpha relation and the local market concentration-AFMI size relation become more complex in this framework, for the foreign counterparts, we are still able to conclude that fund expected net alphas and AFMI size, in equilibrium, move in the same direction with foreign market concentration.

We use the data of 30 active equity mutual fund markets in Morningstar Direct to test our theoretical findings. We find that, 17 (5) markets' fund net alphas, on average, are significantly negatively (positively) associated with the U.S. NHHI, while 9 (13) markets' fund net alphas, on average, are significantly negatively (positively) associated with the local NHHI. Also, we find that only 9 (2) markets' AFMI size, on average, are significantly negatively (positively) associated with the U.S. NHHI while only 7 (7) markets' AFMI size, on average, is significantly negatively (positively) associated with the local NHHI. More importantly, we find that 15 (5) markets' fund net alphas and AFMI size are both on average, negatively (positively) associated with the U.S. NHHI; among them, 7 (1) markets' fund net alphas and AFMI size are both significantly negatively (positively) associated with the U.S. NHHI. If we pool all the markets' data together and redo our analyses, we find that on average, fund net alphas and AFMI size are both significantly negatively associated with the U.S. NHHI but are insignificantly associated with the local NHHI. The fact that global fund markets' fund net alphas and AFMI size tend to move in the same direction with the U.S. AFMI concentration is consistent with our theoretical implications.

Our findings provide relevant implications for fund managers, investors, and regulators. If market parameters leading to the current equilibrium persist, the current low, and probably decreasing, concentration in the U.S. AFMI, would benefit (harm) the global AFMIs whose fund net alphas and AFMI size are on average, negatively (positively) associated with the U.S. NHHI. Our empirical results suggest that a large proportion of the global AFMIs would benefit from the declining U.S. AFMI concentration.

Chapter 5: Conclusion

This thesis studies how AFMI market concentration levels affect AFMI performance, size, managerial efforts, and fees. In Chapter 2, we develop a theoretical model to analyze an AFMI equilibrium where we investigate performance, size, managers' costly (optimal) efforts and fees, under a continuum of exogenous market concentration levels. We provide comparative statistics that characterize the market concentration-net alpha relation and the market concentration-AFMI size relation. In particular, funds' expected net alphas and AFMI size increase with market concentration if and only if higher concentration induces a larger impact on gross alpha production than on the costs of effort (i.e., higher concentration induces higher AFMI direct benefits of net alpha production). Then we specialize our model to allow for endogenous market concentration levels, which befits empirical market concentration measures and enables us to study the model empirically.

In Chapter 3, we use Morningstar's U.S. active equity mutual fund data to test the implications of our model developed in Chapter 2. We find that in the U.S. AFMI, on average, both fund net alphas and AFMI size are increasing concave with market concentration.

In Chapter 4, we extend our study and introduce a theoretical model of an international AFMI equilibrium, where we investigate how local and foreign AFMI market concentrations affect local AFMI performance, size, managerial efforts and fees. We show that in equilibrium, if higher local (foreign) market concentration increases effort impacts on alpha production more than it increases effort costs, i.e., if higher fund local (foreign) market concentration induces a larger sum of direct benefits of managerial efforts, then fund expected net alphas and AFMI size will both increase, and the opposite is also true. We specialize our model to allow for endogenous market concentration levels to facilitate empirical studies. Although the local market concentration-net alpha relation and the local market concentration-AFMI size relation become more complex in this framework, we are still able to conclude that fund expected net alphas and AFMI size, in equilibrium, move in the same direction as foreign market concentration.

We use data of 30 active equity mutual fund markets in Morningstar Direct to test our theoretical findings. We find that 17 (5) markets' fund net alphas, on average, are significantly negatively (positively) associated with the U.S. AFMI market concentration, while 9 (13) markets' fund net alphas, on average, are significantly negatively (positively) associated with the local AFMI market concentration. Also, we find that only 9 (2) markets' AFMI size, on average, are significantly negatively (positively) associated with the U.S. AFMI market concentration while only 7 (7) markets' AFMI size, on average, are significantly negatively (positively) associated with the local AFMI market concentration. More importantly, we find that 15 (5) markets' fund net alphas and AFMI size are both, on average, negatively (positively) associated with the U.S. AFMI market concentration, and among them, 7 (1) markets' fund net alphas and AFMI size are both significantly negatively (positively) associated with the U.S. AFMI market concentration. If we pool all the markets' data together, we find that on average, fund net alphas and AFMI size, are both significantly negatively associated with the U.S. AFMI market concentration, but they are insignificantly associated with the local AFMI market concentration.

This thesis provides relevant implications for fund managers, investors, and regulators. Under the current, empirically identified, tradeoff between changes in managerial productivity and in effort costs due to changes in the U.S. AFMI market concentration level, the current low, and probably decreasing, U.S. AFMI market concentration is likely to decrease fund net alphas, AFMI size, and AFMI direct benefits of net alpha production in the U.S. AFMI. On the other hand, this low and probably decreasing U.S. AFMI market concentration is likely to benefit a large proportion of the global AFMI markets in terms of higher fund net alphas and larger AFMI size.

The findings in this thesis motivate future studies. First, our empirical analysis can be extended to the pension fund industry and hedge fund industry, and it is interesting to check whether the difference in these fund industries' regulations affect the AFMI market concentration-net alpha and the AFMI market concentration-AFMI size relations. Second, in Chapter 4, when we study how local and U.S. AFMI market concentrations affect fund net alphas, we find that in most of the markets, the sign of the coefficient of the U.S. AFMI market concentration is opposite to that of the local AFMI market concentration. Future research can explore the reason behind this phenomenon. In addition, as AFMI concentration is a relevant factor influencing AFMI net alphas and size, future research can study how it affects the flow-performance relation in fund markets.

References

Aguerrevere, F.L., 2009, Real Options, Product Market Competition, and Asset Returns, *The Journal of Finance* 64, 957–983.

Ambrose, B., Diop, M. and Yoshida, J., 2016, Product Market Competition and Corporate Real Estate Investment under Demand Uncertainty, *Real Estate Economics*, 1-70.

Berger, A.N., 1995, The Profit-Structure Relationship in Banking—Tests of Market-Power and Efficient-Structure Hypotheses, *Journal of Money, Credit and Banking* 27, 404–431.

Berger, A.N., Bonime, S.D., Covitz, D.M. and Hancock, D., 1999, Why Are Bank Profits So Persistent? The Roles of Product Market Competition, Informational Opacity, and Regional/Macroeconomic Shocks, *Journal of Banking and Finance* 24, 1203–1235.

Berger, A.N. and Hannan, T.H., 1989, The Price-Concentration Relationship in Banking, *The Review of Economics and Statistics* 71, 291–299.

Berk, J.B. and Binsbergen, J.H., 2015, Measuring Skill in the Mutual Fund Industry, *Journal of Financial Economics* 118, 1–20.

Berk, J.B. and Green, R.C., 2004, Mutual Fund Flows and Performance in Rational Markets, *Journal of Political Economy* 112, 1269–1295.

Chan, K., Covrig, V. and Ng, L., 2005, What Determines the Domestic Bias and Foreign Bias? Evidence from Mutual Fund Equity Allocations Worldwide, *The Journal of Finance* 3, 1495-1534.

Claessens, S. and Laeven, L., 2003, What Drives Bank Competition? Some International Evidence, *Journal of Money, Credit and Banking* 36, 563–583.

Connor, G. and Korajczyk, R.A., 1988, Risk and Return in an Equilibrium APT Application of a New Test Methodology, *Journal of Financial Economics* 21, 255–289.

Cremers, K.J.M., Nair, V.B. and Peyer, U., 2008, Takeover Defenses and Competition: The Role of Stakeholders, *Journal of Empirical Legal Studies* 5, 791–818.

Daniel, K., Grinblatt, M., Titman, S. and Wermers, R., 1997, Measuring Mutual Fund Performance with Characteristic-Based Benchmarks, *Journal of Finance* 52, 1035–1058.

DeFond, M., Hu, X., Hung, M. and Li, S., 2011, The Impact of Mandatory IFRS Adoption on Foreign Mutual Fund Ownership: The Role of Comparability, *Journal of Accounting and Economics* 51, 240-258.

Ferreira, M.A., Keswani, A., Miguel, A.F. and Ramos, S.B., 2012, The Determinants of Mutual Fund Performance: A Cross-Country Study, *Review of Finance* 17, 483-525.

Ferreira, M.A., Keswani, A., Miguel, A.F. and Ramos, S.B., 2012, The Flow-Performance Relationship Around the World, *Journal of Banking and Finance* 36, 1759-1780.

Geroski, P.A., 1990, Innovation, Technological Opportunity, and Market Structure, *Oxford Economic Papers* 42, 586–602.

Giroud, X. and Mueller, H.M., 2011, Corporate Governance, Product Market Competition, and Equity Prices, *The Journal of Finance* 2, 563–600.

Goldberg, L.G. and Rai, A., 1996, The Structure-Performance Relationship for European Banking, *Journal of Banking and Finance* 20, 745–771.

Goldstein, I. and Pauzner, A., 2004, Contagion of Self-Fulfilling Financial Crises Due to Diversification of Investment Portfolios, *Journal of Economic Theory* 119, 151-183.

Gruber, M.J., 1996, Another Puzzle: The Growth in Actively Managed Mutual Funds, *The Journal of Finance* 51, 783–810.

Hoberg, G., Kumar, N. and Prabhala, N.R., 2016, Mutual Fund Competition, Managerial Skill, and Alpha Persistence, Working Paper.

Jensen, M.C., 1968, The Performance of Mutual Funds in the Period 1945–1964, *The Journal of Finance* 23, 389–416.

Jotikasthira, C., Lundblad, C. and Ramadorai, T., 2012, Asset Fire Sales and Purchases and the International Transmission of Funding Shocks, *The Journal of Finance* 6, 2015-2050.

Khorana, A., Servaes, H. and Tufano, P., 2005, Explaining the Size of the Mutual Fund Industry Around the World, *Journal of Financial Economics* 78, 145-185.

Khorana, A., Servaes, H. and Tufano, P., 2008, Mutual Fund Fees Around the World, *The Review of Financial Studies* 22, 1279-1310.

Massa, M., 2003, How do Family Strategies Affect Fund Performance? When Performance-Maximization is Not the Only Game in Town, *Journal of Financial Economics* 67, 249–304.

Nickell, S.J., 1996, Competition and Corporate Performance, *The Journal of Political Economy* 104, 724–746.

Pastor, L. and Stambaugh, R.F., 2012, On the Size of the Active Management Industry, *Journal of Political Economy* 120, 740–781.

Pastor, L., Stambaugh, R.F. and Taylor, L.A., 2015, Scale and Skill in Active Management, *Journal of Financial Economics* 116, 23–45.

Philippon, T. and Reshef, A., 2012, Wages and Human Capital in the U.S. Financial Industry: 1909–2006, *The Quarterly Journal of Economics* 127, 1551–1609.

Sharpe, W.F., 1992, Asset Allocation: Management Style and Performance Measurement, *Journal of Portfolio Management* Winter, 7–19.

Spiegel, M. and Zhang, H., 2013, Mutual Fund Risk and Market Share-Adjusted Fund Flows, *Journal of Financial Economics* 108, 506–528.

Wermers, R., 2000, Mutual Fund Performance: An Empirical Decomposition into Stock-Picking Talent, Style, Transactions Costs, and Expenses, *The Journal of Finance* 55, 1655–1695.

Yu, G. and Wahid, A.S., 2014, Accounting Standards and International Portfolio Holdings, *The Accounting Review* 89, 1895-1930.

Appendices

A. Appendix of Chapter 2

Proof of Equivalence of Managers' Problems (2.12) and (2.13)

Due to competition, if funds offer higher (lower) expected net alphas, investments shift into (out of) it. Thus, in equilibrium all funds offer the same expected net alphas. We show in Proposition 2 and 3 that the equilibrium expected net alpha is the highest one that each manager can achieve at zero profit, and that equilibrium fund sizes are determined by manages costs (which can be viewed as a reflection of their skills).

Suppose that the funds market expected net alpha is $\bar{\alpha}$, where $\bar{\alpha}$ is below the highest level of fund expected net alpha that mangers can produce (implying positive profits). While producing expected net alpha of $\bar{\alpha}$, manager i maximizes profits by choosing optimal effort e_i^* that maximizes fund expected net alpha (i.e., the conditions in Proposition 2 hold), and then charges a fee f_i such that her fund expected net alpha becomes $\bar{\alpha}$. Then, the managerial fee becomes

$$f_i = \hat{a} - \hat{b}\frac{S}{W} + A(e_i^*;H) - \bar{\alpha}\,. \tag{A1}$$

Define the *profit rate* of manager $i, pro_i,$ as $pro_i \triangleq f_i - C^i(e_i^*, s_i; H)$. Then, from the last definition and Equation (A1), we have

$$\bar{\alpha} = \hat{a} - \hat{b}\frac{S}{W} + A(e_i^*;H) - pro_i - c_{0,i} - c_{1,i}s_i - c_{2,i}(e_i^*;H)\,. \tag{A2}$$

As all managers produce the same level of expected net alphas, Equation (A2) implies an equilibrium condition,

$$pro_i + c_{1,i}s_i = pro_j + c_{1,j}s_j, \quad \forall i, j\,. \tag{A3}$$

Next, we consider manager i's profit function

$$s_i[f_i - c_{0,i} - c_{1,i}s_i - c_{2,i}(e_i^*;H)], \tag{A4}$$

and by the first-order condition, the optimal fund size given manager i's profit level is

$$s_i^{opt} = \frac{f_i - c_{0,i} - c_{2,i}(e_i^*;H)}{2c_{1,i}} = \frac{pro_i}{2c_{1,i}} + \frac{s_i}{2}\,. \tag{A5}$$

The latter equality is useful in presenting the optimal size relative to current size. Note that if manager i maximizes her fund's expected net alpha, her profit rate $pro_i = 0$, and the condition in Equation (A5), for ${s_i}^{opt}$, does not exist. For a particular manager j, $j \neq i$, it is possible that pro_j so high that $s_j < {s_j}^{opt}$. In other words, although manager i does not observe other managers' cost functions and profit rates, she knows that it is possible that some other manager(s) might have incentive to lower down the profit rate to attract investments, increase their fund size and increase their fund's profits.

Following this argument, we analyze a simple game between manager i and other managers, grouped as an entity "$-i$". If manager i improves her fund expected net alpha infinitesimally, then, other managers receive no investments and get zero profit, but her profits will be changed by an infinitesimal amount ε_i. If, on the other hand, manager $-i$ (any of the other managers) increases her fund's expected net alpha infinitesimally, then manager i receives no investment and earns zero profit, but manager $-i$'s profit changes by ε_{-i}. If all managers produce the same level of fund expected net alphas, then they can make profit. Note that ε_i (ε_{-i}) can be positive or negative, depending on whether manager i's ($-i$'s) fund size is below optimal level or above optimal level. Assume manager $-i$'s strategy is to improve the fund expected net alpha with probability p and maintain $\bar{\alpha}$ with probability $1-p$. This does not mean that manager $-i$ randomly chooses her action. Instead, it means that manager i knows that it is possible that some other manager(s) improve fund expected net alpha to attract investments in order to improve fund profits, and this probability, p, is nontrivial. Suppose that manager i's strategy is to improve her fund expected net alpha with probability θ and maintain $\bar{\alpha}$ with probability $1-\theta$. The payoffs of the game are illustrated in the following table, with the row (column) representing manager i's ($-i$'s) action, and with manager i's ($-i$'s) payoffs are in the first (second) figures in the brackets.

		Maintain $\bar{\alpha}$	Improve Infinitesimally
		$1-p$	p
Maintain $\bar{\alpha}$	$1-\theta$	$(pro_i s_i, pro_{-i} s_{-i})$	$(0, pro_{-i} s_{-i} + \varepsilon_{-i})$
Improve Infinitesimally	θ	$(pro_i s_i + \varepsilon_i, 0)$	$(pro_i s_i + \varepsilon_i, pro_{-i} s_{-i} + \varepsilon_{-i})$

The expected payoff of manager i is

$$\pi_i = (1-p)[(1-\theta)pro_i s_i + \theta(pro_i s_i + \varepsilon_i)] + p\theta(pro_i s_i + \varepsilon_i) \quad \text{(A6)}$$

The first-order condition is

$$\frac{d\pi_i}{d\theta} = \varepsilon_i + p \times pro_i s_i \quad \text{(A7)}$$

With $\varepsilon_i \to 0$, $d\pi_i/d\theta > 0$. Thus, manager i chooses $\theta = 1$ to maximize π_i. Note that if $\bar{\alpha}$ is the feasibly maximum fund expected net alpha, managers' profit rates are zero, and ε_i and ε_{-i} are negative. Then, in this case, the unique Nash equilibrium is (Maintain $\bar{\alpha}$, Maintain $\bar{\alpha}$).

Therefore, each manager will improve his or her fund expected net alpha as long as it is below the maximum fund expected net alpha, thus, managers' problems of maximizing profits is equivalent to their maximizing funds' expected net alphas.

Theories for the Risk-Neutral Case

Where there are infinitely many small risk-neutral investors (i.e., $N \to \infty$), each investor j's choice of $\boldsymbol{\delta}_\mathbf{j}$ has no effect on funds' sizes, thus no effect on S/W; but each of them imposes a negative externality on other investors: as investors increase their investments in funds when they observe positive expected net alphas, the net alpha received by each investor diminishes because of the decreasing returns to scale at both the industry level and fund level. In other words, investors dilute each other's returns, driving the expected net alphas on all active funds toward zero. A small investor j's problem is

$$\max_{\boldsymbol{\delta}_\mathbf{j}} \mathrm{E}(r_j \mid D) = \max_{\boldsymbol{\delta}_\mathbf{j}} \left\{ \mu_p + \boldsymbol{\delta}_\mathbf{j}^\mathbf{T} \left[\hat{a} - \hat{b}\frac{S}{W} + A(e_i^*; H) - f_i^* \right] \boldsymbol{\iota}_\mathbf{M} \right\} \quad \text{(A8)}$$

subject to

$$\boldsymbol{\delta}_\mathbf{j}^\mathbf{T} \boldsymbol{\iota}_\mathbf{M} \le 1, \quad \text{(A9)}$$

$$\delta_{j,i} \ge 0,\ \forall i, \quad \text{(A10)}$$

$$f_i^* - C^i(e_i^*, s_i; H) = 0,\ \forall i, \quad \text{(A11)}$$

$$A_{e_i}(e_i^*; H) - c_{2e_i}(e_i^*; H) = B_{e_i}(e_i^*; H) = 0,\ \forall i. \quad \text{(A12)}$$

These constraints are the same for their counterparts, investors who are mean-variance risk-averse. We assume a symmetric equilibrium such that each investor makes the same equilibrium optimal investment allocation (i.e., $\boldsymbol{\delta}_\mathbf{j}^*$ is the same for all j), so we have

$$\boldsymbol{\delta}_{\mathbf{j}}^{*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} = S / W . \tag{A13}$$

Where $N = 1$, the single large investor internalizes the whole market and can determine s_i, $i = 1, ..., M$ and S / W to maximize his or her expected portfolio return. In this case, the investor faces a tradeoff between allocating additional dollars to funds to take advantage of fund net alphas and diluting returns on wealth already in funds. Notice that this situation includes the case where there are N investors and they collude to act as a single entity. The single investor's problem is

$$\max_{\boldsymbol{\delta}_1} \mathrm{E}(r_1 \mid D) = \max_{\boldsymbol{\delta}_1} \left\{ \mu_p + \boldsymbol{\delta}_1^{\mathbf{T}} \left[\hat{a} - \hat{b} \boldsymbol{\delta}_1^{\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} + A(e_i^*; H) - f_i^* \right] \boldsymbol{\iota}_{\mathbf{M}} \right\}, \tag{A14}$$

subject to the conditions from (A9) to (A12). Similarly, we have

$$\boldsymbol{\delta}_1^{*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M}} = S / W . \tag{A15}$$

The next proposition defines the equilibrium in the risk-neutral case.

PROPOSITION RN1, Unique Nash Equilibrium.

Whether $N \to \infty$ or $N = 1$,

1) There exists a unique Nash equilibrium, $\left\{ \mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^* \right\}$, where

   $\mathbf{e}^*$ is a $M \times 1$ vector that aggregates individual managers' optimal effort allocations, e_i^*;

   $\mathbf{f}^*$ is a $M \times 1$ vector that aggregates individual managers' optimal fee allocations, f_i^*; and

   $\boldsymbol{\delta}^*$ is a $M \times N$ matrix that aggregates the vector of individual investors' optimal wealth allocations to funds, $\boldsymbol{\delta}_{\mathbf{j}}^*$.

2) In this equilibrium, managers produce the same expected net alpha that drives their economic profits to zero, by charging only break-even fees, and investors allocate the same wealth proportions to each of the funds.

Proof of Proposition RN1. See the following sections in Appendix.

The following proposition offers some equilibrium results.

PROPOSITION RN2, Equilibrium by Optimal Allocations.

1) $N \to \infty$,

$$\boldsymbol{\delta}_{\mathbf{j}}^{*\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}} = S/W = \min\left\{\frac{X(e_i^*, H)}{\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W}, 1\right\}, \ \forall j.$$

Where $S/W < 1$, we have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} = 0$;

where $S/W = 1$, we have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} = X(e_i^*, H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W\right] \geq 0$.

2) $N = 1$,

$$\boldsymbol{\delta}_1^{*\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}} = S/W = \min\left\{\frac{X(e_i^*, H)/2}{\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W}, 1\right\}.$$

Where $S/W < 1$, we have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} = X(e_i^*, H)/2 > 0$;

Where $S/W = 1$, we have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} = X(e_i^*, H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W\right] > 0$.

Where $N \to \infty$, small investors invest in the AFMI as long as they infer positive fund expected net alphas. If they have additional wealth to allocate (i.e., $S/W < 1$), they drive equilibrium fund expected net alphas to zero. If they have no additional wealth to allocate (i.e., $S/W = 1$), the equilibrium fund expected net alphas are not driven to 0. At this time, the equilibrium fund expected net alphas are higher if $X(e_i^*, H)$ and the equilibrium optimal expected net alpha of an initial marginal investment in the AFMI is higher, whereas the equilibrium fund expected net alphas are lower if the decreasing returns to scale effect in the AFMI (represented by the factor $\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W$), is stronger. Where $N = 1$, the single investor internalizes the whole market; and to maximize the expected portfolio returns, he or she never allocates investments to drive the equilibrium fund expected net alphas to zero. When the investor has additional wealth to allocate, he or she allocates wealth to funds such that the effect of decreasing returns to scale in the market is eliminated. If the investor has no additional wealth to allocate, the equilibrium fund expected net alphas are similar to those where there are infinitely many risk-neutral investors.

Also, we can see that if $N = 1$, the industry size is half as large as the counterpart in the setting with infinitely many small risk-neutral investors. The reason is that the single large investor can internalize the market, in the sense that his or her own investment determines S/W. When observing positive fund expected net alphas, the investor chooses the optimal level of investment based on the tradeoff of increasing investment in funds (i.e., a larger amount investment captures higher expected returns from the AFMI, but the expected net alpha of each unit of investment reduces due to decreasing returns to scale).

COROLLARY 1 to PROPOSITION RN2. Where $N=1$, the equilibrium fund expected net alphas $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ where $S/W=1$ are higher than those where $S/W<1$. That is, the equilibrium fund expected net alphas are higher when the single investor has no additional investments to allocate than those when he or she has additional wealth.

The intuition is as follows. Suppose there is a threshold $\overline{W}$ such that the equilibrium $S/W\big|_{W=\overline{W}}=1$ and $S/W\big|_{W=\overline{W}}$ also achieves the internal solution. Any additional wealth above $\overline{W}$ is optimally invested in the passive benchmark, making $S/W<1$ and not affecting the fund expected net alphas. All wealth is optimally invested in funds if the wealth level is below $\overline{W}$, and at that time $S/W=1$. Due to the decreasing returns to scale feature in our model, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ decreases with wealth invested in funds until $W=\overline{W}$ and become unaffected by wealth after $W>\overline{W}$. Thus, the equilibrium fund expected net alphas $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ where $S/W=1$ are higher than those where $S/W<1$.

COROLLARY 2 to PROPOSITION RN2. For (large enough) W such that $S/W<1$, whether $N\to\infty$ or $N=1$, we have $d\left(S/W\right)/dX(e_i^*;H)>0$ and $d\left(S/W\right)/d\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]<0$. That is, higher initial marginal fund expected net alphas, induce a larger equilibrium AFMI size relative to total wealth, whereas stronger decreasing returns to scale effect in the AFMI induces a smaller equilibrium AFMI size relative to total wealth.

PROPOSITION RN3, AFMI Size Sensitivity to Concentration.

Whether $N\to\infty$ or $N=1$,

1) where $S/W<1$, we have

a. $\dfrac{d\left(S/W\right)}{dH}=\dfrac{d\left(S/W\right)}{dX(e_i^*;H)}\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]$.

$d\left(S/W\right)/dH\geq 0(<0)$ iff $A_H(e_i^*;H)-c_{2H}(e_i^*;H)\geq 0(<0)$ (where concentration is higher, equilibrium industry size is larger (smaller) if and only if higher concentration induces a larger (smaller) impact on gross alphas than on costs.).

b. $\dfrac{d^2\left(S/W\right)}{dH^2}=\dfrac{d\left(S/W\right)}{dX(e_i^*;H)}\dfrac{d^2B(e_i^*;H)}{dH^2}$.

$d^2(S/W)/dH^2\geq 0(<0)$ iff $d^2B(e_i^*;H)/dH^2\geq 0(<0)$ (S/W is concave (convex) in H if and only if $B(e_i^*;H)$ is concave (convex) in H).

2) Where $S/W=1$, S/W is unrelated to market concentration.

The intuition is as follows. Whether $N\to\infty$ or $N=1$, if an increase in H induces a higher (lower) impact on net alphas than on costs, the initial marginal fund expected net alpha, $X(e_i^*,H)$, is higher (lower). Consequently, this higher (lower) initial marginal fund expected net alpha attracts (discourages) investments if investors have additional wealth to allocate to funds (i.e., $S/W<1$). In this case, the change of the rate at which H drives up S/W is positively proportional to the change of the rate at which H drives up $B(e_i^*;H)$. If investors have no additional wealth to allocate to funds (i.e., $S/W=1$), the market is at a corner solution and H has no effect on S/W.

PROPOSITION RN4, Net alpha Sensitivity to Concentration.

1) Where $N\to\infty$ and $S/W<1$, we have

a. $d\mathrm{E}(\alpha_i\mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=0$,

b. $d^2\mathrm{E}(\alpha_i\mid D)/dH^2\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=0$.

(Equilibrium fund expected net alphas are unrelated to H.)

2) Where $N\to\infty$ and $S/W=1$, we have

a. $d\mathrm{E}(\alpha_i\mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=A_H(e_i^*;H)-c_{2H}(e_i^*;H)$.

$d\mathrm{E}(\alpha_i\mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}\geq 0(<0)$ iff $A_H(e_i^*;H)-c_{2H}(e_i^*;H)\geq 0(<0)$ (where concentration is higher, equilibrium optimal expected net alphas are larger (smaller) if and only if higher concentration induces a larger (smaller) impact on gross alphas than on costs).

b. $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = d^2B(e_i^*;H)/dH^2$

$d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $d^2B(e_i^*;H)/dH^2 \geq (<0)$

($\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is convex (concave) in H if and only if $B(e_i^*;H)$ is convex (concave) in H).

3) Where $N=1$ and $S/W<1$, we have

a. $d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0.5\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]$

$d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $A_H(e_i^*;H)-c_{2H}(e_i^*;H) \geq 0(<0)$ (where concentration is higher, equilibrium optimal expected net alphas are larger (smaller) if and only if higher concentration induces a larger (smaller) impact on gross alphas than on costs).

b. $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0.5d^2B(e_i^*;H)/dH^2$

$d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $d^2B(e_i^*;H)/dH^2 \geq (<0)$

($\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is convex (concave) in H if and only if $B(e_i^*;H)$ is convex (concave) in H).

4) Where $N=1$ and $S/W=1$, we have

a. $d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = A_H(e_i^*;H)-c_{2H}(e_i^*;H)$

$d\mathrm{E}(\alpha_i \mid D)/dH \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $A_H(e_i^*;H)-c_{2H}(e_i^*;H) \geq 0(<0)$ (where concentration is higher, equilibrium optimal expected net alphas are larger (smaller) if and only if higher concentration induces a larger (smaller) impact on gross alphas than on costs).

b. $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = d^2B(e_i^*;H)/dH^2$

$d^2\mathrm{E}(\alpha_i \mid D)/dH^2 \mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0)$ iff $d^2B(e_i^*;H)/dH^2 \geq (<0)$

($\mathrm{E}(\alpha_i \mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is convex (concave) in H if and only if $B(e_i^*;H)$ is convex (concave) in H).

The intuition of Proposition RN4 is as follows. An increase in H affects $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ at two stages. At the first stage, an increase in H changes managers' ability to produce net alphas. At the second stage, investors react to the changes in fund expected net alphas by changing the investment level, consequently affecting $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ under a decreasing returns to scale framework. Where $N \to \infty$ and $S/W < 1$, investors adjust their investments to drive $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ to zero, so $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is unrelated to H. Where $N \to \infty$ and $S/W = 1$, investors have no additional wealth to allocate to funds even though funds' expected net alphas are changed by a higher H. Thus, whether $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ increases depends on whether managers are able to produce higher net alphas under this higher concentration level (i.e., whether $A_H(e_i^*;H) - c_{2H}(e_i^*;H) > 0$). Also, in this case, if H is higher, the change of marginal $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ depends on the change of marginal $B(e_i^*;H)$. Where $N = 1$, if the single investor observes an increase in fund expected net alphas due to a higher H, he or she would not allocate wealth to completely offset the increase in fund expected net alphas due to a tradeoff, if the single investor has additional wealth to allocate (i.e., $S/W < 1$). If the investor has no additional wealth to allocate (i.e., $S/W = 1$), his or her choice does not affect the marginal $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ due to a higher H. In either situation, with a higher H, whether $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ increases depends on whether managers can produce higher net alphas, and the change of marginal $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ depends on the change of marginal $B(e_i^*;H)$.

PROPOSITION RN5, Relation between Net Alpha and Market Share.
Whether $N \to \infty$ or $N = 1$, an increase (decrease) in $c_{1,i}$, while $c_{1,j}$, $\forall j \neq i$ are unchanged, induces a decrease (increase) in s_i / S and an increase (decrease) in s_j / S, $\forall j \neq i$. Also,

1) Where $S/W < 1$, it induces no change in $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and $\mathrm{E}(\alpha_j \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$, $\forall j \neq i$. Thus, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is unrelated to s_i / S, $\forall i$.

2) Where $S/W=1$, it induces a decrease (increase) in $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$, thus $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i/S are positively related—an internality effect; it induces a decrease (increase) in $\mathrm{E}(\alpha_j \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$, $\forall j \neq i$, thus $\mathrm{E}(\alpha_j \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_j/S are negatively related $\forall j \neq i$—an externality effect.

The intuition of Proposition RN5 is as follows. First, based on Proposition 3, we can see that any change in $c_{1,i}$, keeping $c_{1,j}$, $\forall j \neq i$ unchanged, results in a change in s_i/S in the opposite direction and a change in s_j/S, $\forall j \neq i$ in the same direction. Also, an increase in $c_{1,i}$, affects $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ at two stages. At the first stage, it increases manager i's average cost sensitivity to size and induces a decrease in fund expected net alphas. As manager i offers a lower fund expected net alpha, investments shift from fund i to other funds, making all other funds' fund expected net alphas lower due to decreasing returns to scale at the fund level. At the second stage, a decrease in fund expected net alphas discourages investments in the AFMI, which in turn drives up fund expected net alphas due to the effect of decreasing returns to scale. Where $S/W<1$, infinitely many small risk-neutral investors drive fund expected net alphas to zero, whereas a single large risk-neutral investor strategically allocates wealth to funds. In either case, the two stages' effects of changes in $c_{1,i}$ on $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$'s are completely cancelled out; thus, for each fund i, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is unrelated to $c_{1,i}$ and, consequently, unrelated to s_i/S. Where $S/W=1$, investors have no additional wealth to allocate, so the second stage's effect does not exist, and we find $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$'s are driven down by an increase in $c_{1,i}$, keeping $c_{1,j}$, $\forall j \neq i$ unchanged. Consequently, we have a positive relation between $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i/S, and a negative relation between $\mathrm{E}(\alpha_j \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_j/S, $\forall j \neq i$. In other words, in this case, any change in manager i's average cost sensitivity to size induces an internality effect, a positive relation between its market share and equilibrium fund expected net alphas, and induces an externality effect, a negative relation between other funds' market shares and their equilibrium fund expected net alphas.

Proof of Proposition RN2, RN3, RN4, RN5 and the corresponding corollaries. See the following sections in Appendix.

Figure A.1 and Figure A.2 illustrate the numerical results for the case of risk-neutral investors using the same numerical parameters as the risk-averse case. The results of e_i^*, f_i^*, and $B(e_i^*;H)$ are the same as those in the risk-averse case, whether there are infinitely many small risk-neutral investors or there is a single large investor. Where there are infinitely many small risk-neutral investors, when H is small, we have $S/W=1$, and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is positive and is decreasing with H. When H becomes larger, S/W starts to decrease and slightly increase after it achieves the minimum point. In this case, S/W increases with $B(e_i^*;H)$, and we have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ equal to zero. This is because in our calibration, the difference of H's impact on gross alphas and on costs decreases with H when H is small and then slightly increases with H when H is large. In addition, the curvature of S/W and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ in H are convex.

Where there is a single large investor in the market, the results are similar except that the levels of S/W are much smaller than those where there are infinitely many risk-neutral investors, and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$'s are positive across the levels of H. This is because the single investor can internalize the AFMI, limiting the investments in the funds and maximizing portfolio expected returns.

Figure A.1. Infinitely Many Small Risk-Neutral Investors—Comparative Statistics with Respect to Market Concentration

This figure presents the numerical results for the case of infinitely many small risk-neutral investors. The two subplots on the top illustrate the equilibrium optimal management effort and management fees at each market concentration level. The two subplots in the middle reports $B(e_i^*, H)$ at each market concentration level and the equilibrium *S/W* ratio at each $X(e_i^*, H)$ level. The two subplots at the bottom show the equilibrium *S/W* ratio and the equilibrium fund expected net alphas at each market concentration level.

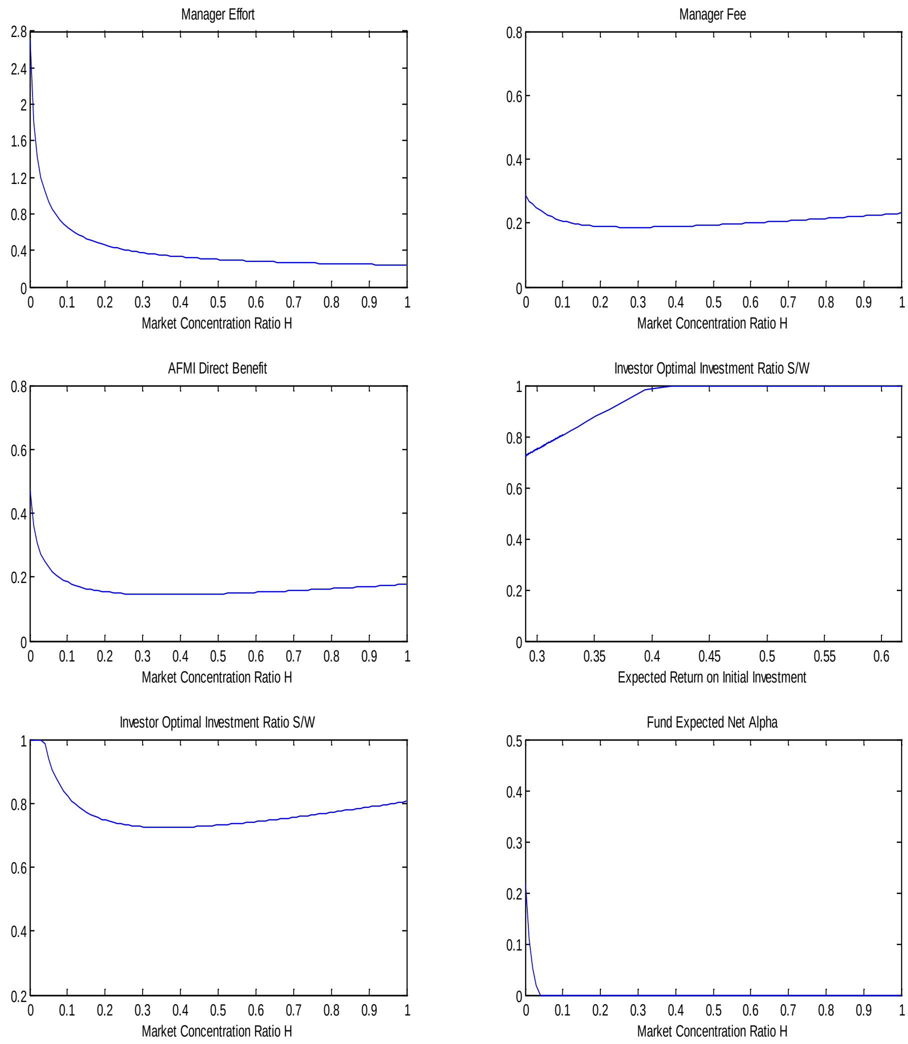
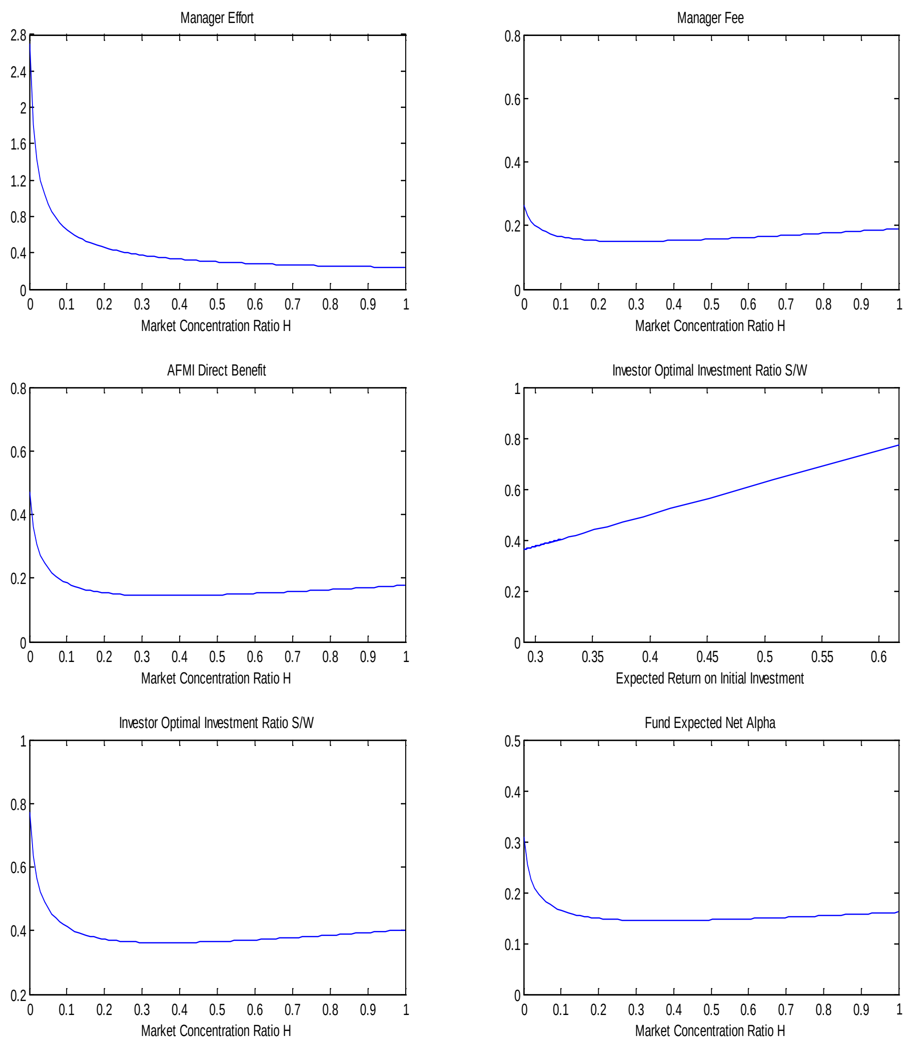


Figure A.2. A Single Large Risk-Neutral Investor—Comparative Statistics with Respect to Market Concentration

This figure presents the numerical results for the case of a single large risk-neutral investor. The two subplots on the top illustrate the equilibrium optimal management effort and management fees at each market concentration level. The two subplots in the middle reports $B(e_i^*, H)$ at each market concentration level and the equilibrium *S/W* ratio at each $X(e_i^*, H)$ level. The two subplots at the bottom show the equilibrium *S/W* ratio and the equilibrium fund expected net alphas at each market concentration level.



Agency Benefits Due to Market Concentration

We use the same settings as those in Section 2.4. We consider only the case where there are infinitely many risk-neutral investors. Managers are colluding to choose e_i to maximize their objective function (2.37), subject to the constraint (2.38). The first-order-condition gives

$$\sum_{i=1}^{M}\left\{\begin{array}{l} A_{e_i}(e_i^*;H)\left[f(H)-C^i(e_i^*,s_i;H)\right] \\ -\left[A(e_i^*;H)-f(H)\right]\left[\dfrac{W\left(\sum_{l=1}^{M}c_{1,l}^{-1}\right)^{-1}A_{e_i}(e_i^*;H)}{\hat{b}+\left(\sum_{j=1}^{M}c_{1,j}^{-1}\right)^{-1}}-c_{2e_i}(e_i^*;H)\right]\end{array}\right\}=0. \tag{A16}$$

The second-order condition with respect to e_i is satisfied, so e_i^* given by (A16) is a maximum point. Also, in equilibrium, where $S/W<1$, we have

$$\frac{S}{W}=\frac{\hat{a}+A(e_i^*;H)-f(H)}{\hat{b}}, \tag{A17}$$

$$\frac{d\left(S/W\right)}{dH}=\frac{A_{e_i}(e_i^*;H)e_i^{*}{}'(H)+A_H(e_i^*;H)-f'(H)}{\hat{b}}, \tag{A18}$$

$$\left.\frac{d\mathrm{E}(\alpha_i\mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=0\,. \tag{A19}$$

Thus, $d(S/W)/dH\geq 0(<0)$ if and only if $A_{e_i}(e_i^*;H)e_i^{*}{}'(H)+A_H(e_i^*;H)-f'(H)\geq 0(<0)$. In other words, if and only if higher concentration induces higher net alphas than agency benefits, investors are willing to invest more in funds, eventually driving equilibrium optimal expected net alphas to zero. Where $S/W=1$, managers choose e_i^* such that

$$\frac{S}{W}=\frac{\hat{a}+A(e_i^*;H)-f(H)}{\hat{b}}=1, \tag{A20}$$

so

$$\frac{d\left(S/W\right)}{dH}=0, \tag{A21}$$

and (A19) still holds because if managers can optimally induce investors to invest all their wealth in funds, they choose the minimum effort to do so. In this case, both S/W

and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}}$ are unaffected by H.

Mathematical Proof of Propositions and Corollaries

Proof of Proposition 1 and 2, and the Corollary

To maximize $\mathrm{E}(\alpha_i \mid D)$, manager i chooses the breakeven management fee,

$$f_i^* - C^i(e_i^*, s_i; H) = 0. \tag{A22}$$

We substitute it into $\mathrm{E}(\alpha_i \mid D)$. If $A_{e_i}(0;H) - c_{2e_i}(0;H) \le 0$, managers' optimal choice is to spend zero effort and to charge a proportional fee equal to $c_0 + c_{1,i}s_i$; in this case, our equilibrium will be similar to that in Pastor and Stambaugh (2012), where managerial effort is not modeled and managers do not charge fees above opportunity costs.

If $A_{e_i}(0;H) - c_{2e_i}(0;H) > 0$, the first-order-condition to maximize $\mathrm{E}(\alpha_i \mid D)$ becomes,

$$A_{e_i}(e_i^*;H) - c_{2e_i}(e_i^*;H) = B_{e_i}(e_i^*;H) = 0. \tag{A23}$$

The second-order condition, $A_{e_i,e_i}(e_i^*;H) - c_{2e_i,e_i}(e_i^*;H) = B_{e_i,e_i}(e_i^*;H) < 0$, is automatically satisfied, so e_i^* is a maximum point. Here we assume that e_i^* is finite and attainable.

Next, we can see that both f_i^* and e_i^* are functions of H, so

$$e_i^* = e_i^*(H), \text{ and} \tag{A24}$$

$$f_i^* = f_i^*(H). \tag{A25}$$

Completing differentiation of (A23) gives

$$e_i^{*\prime}(H) = -\frac{A_{e_i,H}(e_i^*;H) - c_{2e_i,H}(e_i^*;H)}{A_{e_i,e_i}(e_i^*;H) - c_{2e_i,e_i}(e_i^*;H)}, \tag{A26}$$

and so we have if $A_{e_i,H}(e_i^*;H) - c_{2e_i,H}(e_i^*;H) \ge 0 (< 0)$, then $e_i^{*\prime}(H) \ge 0 (< 0)$. Also, based on

$$
\begin{aligned}
f_i^* &= C^i(e_i^*, s_i; H) = c_0 + c_{1,i}s_i + c_2(e_i^*; H) \\
&= c_0 + c_{1,i}\frac{s_i}{S}\frac{S}{W}W + c_2(e_i^*; H) \\
&= c_0 + c_{1,i}W\left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}{}^{-1}\right)\right)^{-1}\frac{S}{W} + c_2(e_i^*; H),
\end{aligned}
\tag{A27}
$$

we have

$$
f_i^*{}'(H) = c_{1,i}W\left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}{}^{-1}\right)\right)^{-1}\frac{d(S/W)}{dH} + c_{2e_i}(e_i^*; H)e_i^*{}'(H) + c_{2H}(e_i^*; H)\,. \tag{A28}
$$

Complete differentiation of $B(e_i^*; H)$ with respect to H, gives

$$
B'(e_i^*; H) = B_{e_i}(e_i^*; H)e_i^*{}'(H) + A_H(e_i^*; H) - c_{2H}(e_i^*; H) = A_H(e_i^*; H) - c_{2H}(e_i^*; H), \tag{A29}
$$

so we have if $A_H(e_i^*; H) - c_{2H}(e_i^*; H) \geq 0(<0)$, then $B'(e_i^*; H) \geq 0(<0)$. In addition, we have

$$
\mathrm{E}(r_{F,i} \mid D) = \mathrm{E}(\alpha_i \mid D) + \mu_p\,. \tag{A30}
$$

Because $\mathrm{E}(\alpha_i \mid D)$'s are the same across funds, $\mathrm{E}(r_{F,i} \mid D)$'s are the same across funds. Moreover, we have

$$
\mathrm{Var}(r_{F,i} \mid D) = \sigma_p^2 + \sigma_a^2 + \sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_x^2 + \sigma_\varepsilon^2\,. \tag{A31}
$$

The source of fund returns' variance is the same across funds, so we have that $\mathrm{Var}(r_{F,i} \mid D)$'s are the same across funds. Combining (A30) and (A31), we have all managers offering the same market competitive Sharpe ratio.

Q.E.D.

Proof of Proposition 3

The optimal manager effort e_i^* is determined only by the functions $c_2(e_i; H)$, and $A(e_i; H)$, which are the same across funds. Thus, we have $e_i^* = e_j^*$ and $B(e_i^*; H) = B(e_j^*; H),\ \forall i, j$. Also, following the fifth point of Proposition 2, we have $A(e_i^*; H) - f_i^*$ for $i = 1, \ldots, M$ the same across funds; thus, we have $f_i^* = f_j^*,\ \forall i, j$. In addition, by (A22), we further have $C^i(e_i^*, s_i; H) = C^j(e_j^*, s_j; H),\ \forall i, j$; therefore, we have the following relationship between different funds' sizes, $c_{1,i}s_i = c_{1,j}s_j,\ \forall i, j$, inducing $s_i / s_j = c_{1,j} / c_{1,i}, \forall i, j$. Then we can sum s_i / s_j with respect to i, $i = 1, 2, ..., M$, reverse the sum, and replace the subscript j by i; and we have the expression

$$\frac{s_i}{S} = \left(c_{1,i} \sum_{j=1}^{M} \left(c_{1,j}{}^{-1} \right) \right)^{-1}, \ \forall i. \tag{A32}$$

Q.E.D.

Proof of Proposition RN1

$\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}$ is a Nash equilibrium for the following reasons:

1. Given other managers' optimal choices, manager i has no incentives to deviate from e_i^* and f_i^*. This is because increasing e_i^* or decreasing f_i^* generates negative economic profits, whereas decreasing e_i^* or increasing f_i^* lowers fund expected net alpha. Thus the manager receives no investments from investors.
2. Given managers' and other investors' optimal choices, an investor has no incentive to deviate from $\boldsymbol{\delta}_\mathbf{j}^*$ because where $N \to \infty$, changing allocations across funds or between the fund industry and the passive benchmark does not improve his or her portfolio's expected net returns; where $N = 1$, changing allocation across funds does not improve the portfolio's expected net returns, whereas shifting allocations between the fund industry and the passive benchmark lowers the portfolio's expected net returns.

$\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}$ is unique because

1. $\mathbf{e}^*$ is unique because for each fund, $B_{e_i}(e_i, H) = 0$ is uniquely solved by e_i^*;
2. $\mathbf{f}^*$ is unique because for each fund $f_i^* - C^i(e_i^*, s_i; H) = 0$, $C^i(e_i^*, s_i; H)$ is deterministic, and e_i^* is unique;
3. $\boldsymbol{\delta}^*$ is unique because allocations to funds maximize investor portfolios' expected net returns, driving fund expected net alphas to the same values; the uniqueness of $\mathbf{e}^*$ and $\mathbf{f}^*$ makes equilibrium allocations unique.

Q.E.D.

Proof of Proposition RN2, RN3, RN4, RN5 and the Corresponding Corollaries

Infinitely Many Small Risk-Neutral Investors

When observing positive fund expected net alphas, to maximize their portfolio expected net returns, investors will continue investing into funds if they have additional wealth, until fund expected net alphas are driven down to zero by their investments. In equilibrium, if investors have additional wealth to allocate but funds have exhausted the abilities to produce positive fund expected net alphas, then $S/W<1$ and $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \hat{a} - \hat{b}(S/W) + A(e_i^*;H) - f_i^* = 0,\ \forall i$. With conditions (A22), (A23) and (A32), we have

$$\frac{S}{W} = \frac{\hat{a} + A(e_i^*;H) - c_0 - c_2(e_i^*;H)}{\hat{b} + (c_{1,i}\dfrac{s_i}{S})W} = \frac{X(e_i^*,H)}{\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W}. \tag{A33}$$

Note that $X(e_i^*,H)$'s are equal across funds, as implied by equal e_i^* across funds. Based on (A33), we have

$$\frac{d(S/W)}{dX} = \frac{1}{\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} > 0, \tag{A34}$$

$$\frac{d(S/W)}{d\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} = \frac{-X(e_i^*,H)}{\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]^2} < 0, \tag{A35}$$

$$\frac{d(S/W)}{dH} = \frac{d(S/W)}{dX}\frac{dX(e_i^*,H)}{dH} = \frac{d(S/W)}{dX}\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right], \text{ and} \tag{A36}$$

$$\begin{aligned}\frac{d^2(S/W)}{dH^2} &= \frac{d^2(S/W)}{dX^2}\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]^2 + \frac{d(S/W)}{dX}\frac{dA_H(e_i^*;H)}{dH} \\ &= \frac{d(S/W)}{dX}\frac{d^2B(e_i^*;H)}{dH^2}.\end{aligned} \tag{A37}$$

As $\mathrm{E}(\alpha_i \mid D)|_{\{\mathrm{e}^*,\mathrm{f}^*,\delta^*\}} = 0$, we have

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0, \tag{A38}$$

$$\left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0. \tag{A39}$$

Also,

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,i}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0. \tag{A40}$$

As for each fund i, s_i / S and $c_{1,i}$ are negatively related to each other, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is unrelated to s_i / S.

In equilibrium, if investors have no additional wealth to allocate but funds are still able to produce positive fund expected net alphas, then $S/W = 1$ and

$$\frac{d\left(S/W\right)}{dX} = 0, \tag{A41}$$

$$\frac{d\left(S/W\right)}{d\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W\right]} = 0, \tag{A42}$$

$$\frac{d\left(S/W\right)}{dH} = 0, \quad \text{and} \tag{A43}$$

$$\frac{d^2\left(S/W\right)}{dH^2} = 0. \tag{A44}$$

Also,

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = X(e_i^*, H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W\right] \geq 0. \tag{A45}$$

Differentiating (A45) with respect to H, gives

$$\begin{aligned} \left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} &= A_H(e_i^*;H) - c_{2H}(e_i^*;H) + e_i^*{}'(H) \times \left(A_{e_i}(e_i^*;H) - C_{e_i}^i(e_i^*, s_i; H)\right) \\ &= A_H(e_i^*;H) - c_{2H}(e_i^*;H), \end{aligned} \tag{A46}$$

and

$$\left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{dA_H(e_i^*;H)}{dH} = \frac{d^2B(e_i^*;H)}{dH^2}. \tag{A47}$$

In this case, managers will spend more effort to increase fund expected net alphas in a more concentrated market, making the fund expected net alphas higher; but because investors cannot further increase their investments in the fund industry to make the fund expected net alphas lower, equilibrium fund expected net alphas increase with market concentration. On the other hand, the fact that fund expected net alphas are concave (convex) in the market concentration level is a necessary and sufficient condition for the direct benefit (i.e., $B(e_i^*;H)$) to be concave (convex) in the market concentration level. Moreover,

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,i}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{-W}{\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^2 c_{1,i}^2} < 0 . \tag{A48}$$

As s_i / S and $c_{1,i}$ are negatively related whereas $s_j / S, \forall j \neq i$ and $c_{1,i}$ are positively related, based on (A32) we will find a positive relationship between $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i / S , and a negative relationship between $\mathrm{E}(\alpha_j \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and $s_j / S, \forall j \neq i$, holding $c_{1,j}, \forall j \neq i$ unchanged.

One Single Large Risk-Neutral Investor

As fund expected net alphas are the same across funds in equilibrium, the weight of each fund relative to the fund industry is determined by (A32). Thus, the single investor just needs to determine the weight of the fund industry relative to the passive benchmark. In other words, he or she chooses S / W to maximize expected net portfolio return, and his objective function becomes:

$$\max_{s/W} \mathrm{E}(r_1 \mid D) = \mu_p + (\hat{a} - \hat{b}\frac{S}{W} + A(e_i^*; H) - f_i^*)\frac{S}{W}, \tag{A49}$$

subject to

$$0 \leq S / W \leq 1 \tag{A50}$$

and conditions (A22) and (A23). To maximize (A49), the first-order condition generates

$$\frac{S}{W} = \frac{\hat{a} + A(e_i^*; H) - c_0 - c_2(e_i^*; H)}{2\left[\hat{b} + (c_{1,i}\frac{s_i}{S})W\right]} = \frac{X(e_i^*, H)/2}{\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W}, \tag{A51}$$

if $S / W < 1$, and $S / W = 1$ if (A51) is larger than or equal to 1.

Where $S / W < 1$, based on (A51), we have

$$\frac{d\left(S / W\right)}{dX} = \frac{1}{2\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} > 0 , \tag{A52}$$

$$\frac{d\left(S / W\right)}{d\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} = \frac{-X(e_i^*, H)}{2\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]^2} < 0, \tag{A53}$$

$$\frac{d\left(S / W\right)}{dH} = \frac{d\left(S / W\right)}{dX}\frac{dX(e_i^*, H)}{dH} = \frac{d\left(S / W\right)}{dX}\left[A_H(e_i^*; H) - c_{2H}(e_i^*; H)\right], \text{ and} \tag{A54}$$

$$\begin{aligned}\frac{d^2(S/W)}{dH^2} &= \frac{d^2(S/W)}{dX^2}\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]^2+\frac{d(S/W)}{dX}\frac{dA_H(e_i^*;H)}{dH} \\ &= \frac{d(S/W)}{dX}\frac{d^2B(e_i^*;H)}{dH^2}.\end{aligned} \quad (A55)$$

Also, when we substitute (A51) into $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$, we have

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = X(e_i^*,H)/2>0; \quad (A56)$$

and complete differentiation of (A56) with respect to H gives

$$\begin{aligned}\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} &= \frac{1}{2}\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)+e_i^{*}{}'(H)\left(A_{e_i}(e_i^*;H)-C_{e_i}^i(e_i^*,s_i;H)\right)\right] \\ &= \frac{1}{2}\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right],\end{aligned} \quad (A57)$$

and

$$\left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{1}{2}\frac{d^2B(e_i^*;H)}{dH^2}. \quad (A58)$$

Also,

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,i}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0. \quad (A59)$$

As for each fund i, s_i/S and $c_{1,i}$ are negatively related to each other, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is unrelated to s_i/S.

Where $S/W=1$, then the results from (A41) to (A45) are still valid. Also, substituting $S/W=1$ into $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ we have

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = X(e_i^*,H)-\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]. \quad (A60)$$

Let $\overline{W}$ be a threshold such that

$$\left.\frac{S}{W}\right|_{W=\overline{W}} = \frac{X(e_i^*,H)/2}{\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}\overline{W}} = \frac{S}{\overline{W}} = 1. \quad (A61)$$

In other words, $\overline{W}$ is a threshold where the internal solution of S/W is achieved and is equal to 1. Thus, at $\overline{W}$ we have

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}\Big|_{W=\overline{W}} = X(e_i^*, H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} \overline{W}\right] = X(e_i^*, H)/2. \tag{A62}$$

Any additional wealth above $\overline{W}$ is optimally allocated to the passive benchmark; thus, it does not directly affect the fund industry. Therefore, if $W > \overline{W}$, $S/W < 1$ and

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}\Big|_{W>\overline{W}} = X(e_i^*, H)/2. \tag{A63}$$

Also, at wealth levels below $\overline{W}$, all wealth is optimally invested in the fund industry. Quantitatively, if $W < \overline{W}$, $S/W = 1$, and

$$\begin{aligned}
\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}\Big|_{W<\overline{W}} &= X(e_i^*, H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W\right] \\
&> X(e_i^*, H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} \overline{W}\right] \\
&= X(e_i^*, H)/2 \\
&= \mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}\Big|_{W>\overline{W}}.
\end{aligned} \tag{A64}$$

Thus, where $S/W = 1$, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$'s are higher than those where $S/W < 1$, except for a knife edge case $W = \overline{W}$, where they are equal. Based on this result, we can see that because of (A56), expression (A60) is also larger than zero. Moreover, complete differentiation of (A60) with respect to H gives the same results as in (A46) and (A47). In addition, complete differentiation of (A60) with respect to $c_{1,i}$, gives the same results as in (A48), and we have the same conclusion about the relationship between equilibrium funds' expected net alphas and funds' market shares.

Q.E.D.

Proof of Proposition RA1

$\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}$ is a Nash equilibrium for the following reasons.

1. Given other managers' optimal choices, a manager has no incentive to deviate from $\mathbf{e}^*$ and $\mathbf{f}^*$, and the reasons are the same as those in the proof of Proposition RN1.
2. Given managers' and other investors' optimal choices, an investor has no incentive to deviate from $\boldsymbol{\delta}_\mathbf{j}^*$ because, where $N \to \infty$, changing allocations across funds does

not improve his or her portfolio's Sharpe ratio, whereas changing allocations between the fund industry and the passive benchmark decreases the portfolio's Sharpe ratio.

$\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}$ is unique because

1. $\mathbf{e}^*$ and $\mathbf{f}^*$ are unique for the same reason in the proof of Proposition RN1;
2. $\boldsymbol{\delta}^*$ is unique because allocations to funds maximize investor portfolios' Sharpe ratios, driving fund expected net alphas to the same values; the uniqueness of $\mathbf{e}^*$ and $\mathbf{f}^*$ makes equilibrium allocations unique.

Q.E.D.

Proof of Proposition RA2, RA3, RA4, RA5, and the Corresponding Corollaries

Infinitely Many Small Mean-Variance Risk-Averse Investors

In this case, each investor is trying to maximize their portfolio Sharpe ratio without affecting the fund size of each fund (i.e., s_i, $i=1,\dots,M$) and S/W, subject to the constraints described in the paper. When we take the first-order condition and substitute the constraints $f_i^* - C^i(e_i^*, s_i; H) = 0,\ \forall i$ and $A_{e_i}(e_i^*; H) - C_{e_i}^i(e_i^*, s_i; H) = 0,\ \forall i$ into the first-order condition. we have

$$\begin{aligned}
&-\left(\mu_p/\sigma_p^2\right)\left[\sigma_a^2+\sigma_b^2\left(\frac{S}{W}\right)^2+\sigma_x^2\right]\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}*}\boldsymbol{\iota}_{\mathbf{M}}-\left[\hat{b}+(c_{1,i}\frac{s_i}{S})W\right]\frac{S}{W}+\hat{a}+A(e_i^*;H)-c_0-c_2(e_i^*;H)\\
&=-\left(\mu_p/\sigma_p^2\right)\left[\sigma_a^2+\sigma_b^2\left(\frac{S}{W}\right)^2+\sigma_x^2\right]\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}*}\boldsymbol{\iota}_{\mathbf{M}}-\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\frac{S}{W}+X(e_i^*;H)\\
&=0.
\end{aligned}\tag{A65}$$

Substituting $\gamma \triangleq \mu_p/\sigma_p^2$ and $S/W = \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}*}\boldsymbol{\iota}_{\mathbf{M}}$, we have

$$-\gamma\sigma_b^2\left(\frac{S}{W}\right)^3-\left[\gamma\sigma_a^2+\gamma\sigma_x^2+\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\frac{S}{W}+X(e_i^*;H)=0\,. \tag{A66}$$

If the constraint $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}} \le 1$ is not binding (i.e., $S/W < 1$), the equilibrium optimal S/W is a real positive solution of this cubic equation. The condition $X(e_i^*;H) > 0,\ \forall H$ and the coefficient of the highest order term $-\gamma\sigma_b^2 < 0$ guarantee that there is at least one positive real solution for S/W. Also, as each investor cannot affect the value of S/W, (A65) shows that the solution for $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}*}\boldsymbol{\iota}_{\mathbf{M}} = S/W$ is unique given the parameter values and the market S/W. Based on the characteristics of cubic equations, from (A66) we can see that there are three possibilities for the three roots of S/W, three real positive roots, one real positive root and two real negative roots, and one real positive root and two imaginary solutions. The way to determine which root is the solution of S/W is described as follows. Define $(S/W)_s$ and $(S/W)_l$ as the smallest and the largest positive real solutions of (A65). Note that they can be equal to each other. If $(S/W)_l < 1$, the equilibrium S/W is either $(S/W)_s$ or $(S/W)_l$, depending on which of them maximizes the portfolio Sharpe ratio. If $(S/W)_s < 1 \le (S/W)_l$, the equilibrium S/W is either $(S/W)_s$ or 1, depending on which of them maximizes the portfolio's Sharpe ratio. If $(S/W)_s \ge 1$, the equilibrium S/W is 1. In one special case, both $(S/W)_s < 1$ and $(S/W)_l \le 1$ (when they are not equal to each other) attain the maximum value of the portfolio's Sharpe ratio. Depending on the initial value of S/W, the equilibrium S/W will uniquely converge to either of them with a probability of one.

Where the equilibrium optimal $S/W < 1$, complete differentiation of (A66) gives

$$\frac{d(S/W)}{dX} = \frac{1}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W}. \tag{A67}$$

Based on the positive parameter values, we know that

$$\frac{d(S/W)}{dX} > 0. \tag{A68}$$

Also, complete differentiation of (A66) gives

$$\begin{aligned}\frac{d(S/W)}{d\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} &= \frac{-(S/W)}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} \\ &= -\frac{S}{W}\frac{d(S/W)}{dX} \\ &< 0.\end{aligned} \tag{A69}$$

Moreover, by chain rule,

$$\frac{d(S/W)}{dH}=\frac{d\left(S/W\right)}{dX}\frac{dX(e_i^*,H)}{dH}=\frac{d(S/W)}{dX}\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]. \tag{A70}$$

In addition,

$$\begin{aligned}\frac{d^2(S/W)}{dH^2}&=\frac{d\left(S/W\right)}{dX}\frac{d^2B(e_i^*;H)}{dH^2}+\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]^2\frac{d^2\left(S/W\right)}{dX^2}\\&=\frac{d\left(S/W\right)}{dX}\frac{d^2B(e_i^*;H)}{dH^2}\\&\quad-\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]^2\left[\frac{d\left(S/W\right)}{dX}\right]^3\gamma\left[6\sigma_b^2\frac{S}{W}\right].\end{aligned} \tag{A71}$$

Based on the outcome of (A68), $d^2B(e_i^*;H)/dH^2<0$ implies $d^2(S/W)/dH^2<0$, whereas $d^2(S/W)/dH^2>0$ implies $d^2B(e_i^*;H)/dH^2>0$.

In addition, where $S/W<1$, we substitute

$$\begin{aligned}\mathrm{E}(\alpha_i\mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}&=\hat{a}-\hat{b}\left(S/W\right)+A(e_i^*;H)-f_i^*\\&=-\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}^{-1}\right)^{-1}W\right]\left(S/W\right)+X(e_i^*;H)\end{aligned} \tag{A72}$$

into (A66); and after some simple transformation, we have

$$\mathrm{E}(\alpha_i\mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=\frac{\gamma\left(\sigma_\alpha^2+\sigma_x^2\right)X(e_i^*;H)}{\gamma\left(\sigma_\alpha^2+\sigma_x^2\right)+\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}^{-1}\right)^{-1}W}, \tag{A73}$$

where

$$\sigma_\alpha^2\triangleq\mathrm{Var}(\alpha_j\mid D)=\sigma_b^2\left(\frac{S}{W}\right)^2+\sigma_a^2. \tag{A74}$$

As all the terms of (A73) are positive, $\mathrm{E}(\alpha_i\mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is positive. The intuition is as follows. By investors' portfolio variance formulas, we can easily see that a portfolio with allocations to both funds and the passive benchmark is always riskier than a portfolio with allocations only to the passive benchmark. If $\mathrm{E}(\alpha_i\mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=0$, because of a sufficiently large amount of investment in funds, investors can always improve their portfolio Sharpe ratios (in particular, reduce their portfolio risks) by shifting wealth allocation from funds to the passive benchmark. Differentiating (A72) with respect to H, we have

$$\left.\frac{d\mathrm{E}(\alpha_i\mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}=\left\{1-\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\frac{d(S/W)}{dX}\right\}\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right], \tag{A75}$$

and

$$\begin{aligned}\left.\frac{d^2\mathrm{E}(\alpha_i\mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}&=\frac{d^2B(e_i^*;H)}{dH^2}\left\{1-\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\frac{d(S/W)}{dX}\right\}\\&+\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]^2\\&\times\left[\frac{d\left(S/W\right)}{dX}\right]^3\gamma\left[6\sigma_b^2\frac{S}{W}\right].\end{aligned} \tag{A76}$$

Therefore, $d^2\mathrm{E}(\alpha_i\mid D)/dH^2\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}<0$ implies $d^2B(e_i^*;H)/dH^2<0$, whereas $d^2B(e_i^*;H)/dH^2>0$ implies $d^2\mathrm{E}(\alpha_i\mid D)/dH^2\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}>0$.

Also, complete differentiation of (A65) with respect to $c_{1,i}$ gives

$$\frac{d(S/W)}{dc_{1,i}}=\frac{-\left(S/W\right)W}{\left\{\gamma\left[3\sigma_b^2\left(S/W\right)^2+\sigma_a^2+\sigma_x^2\right]+\hat{b}+\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^{-1}W\right\}\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^2c_{1,i}^2}<0\,. \tag{A77}$$

Thus,

$$\begin{aligned}\left.\frac{d\mathrm{E}(\alpha_i\mid D)}{dc_{1,i}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}&=-\left[\hat{b}+\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^{-1}W\right]\frac{d(S/W)}{dc_{1,i}}-\frac{\left(S/W\right)W}{\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^2c_{1,i}^2}\\&=\frac{-\gamma\left[3\sigma_b^2\left(S/W\right)^2+\sigma_a^2+\sigma_x^2\right]\left(S/W\right)W}{\left\{\gamma\left[3\sigma_b^2\left(S/W\right)^2+\sigma_a^2+\sigma_x^2\right]+\hat{b}+\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^{-1}W\right\}\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^2c_{1,i}^2}.\end{aligned}$$

(A78)

Based on (A78), $d\mathrm{E}(\alpha_i\mid D)/dc_{1,i}\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}<0$ and $d\mathrm{E}(\alpha_j\mid D)/dc_{1,i}\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}<0,\ \forall j\neq i$ (found in a similar way). As s_i/S and $c_{1,i}$ are negatively related whereas $s_j/S,\ \forall j\neq i$ and $c_{1,i}$ are positively related based on (A32), we will find a positive relationship between $\mathrm{E}(\alpha_i\mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i/S, and a negative relationship between $\mathrm{E}(\alpha_j\mid D)\mid_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and $s_j/S,\ \forall j\neq i$.

Where $S/W=1$, we also have $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] + X(e_i^*; H) > 0$. The proof of all other results where there are infinitely many mean-variance risk-averse investors and $S/W=1$ is the same as the proof of the results where there are infinitely many risk-neutral investors and $S/W=1$.

One Single Large Mean-Variance Risk-Averse Investor

Where there is only one investor and $S/W=1$, all the proof of results are the same as that of the results of the counterpart risk-neutral situation. Where there is a single investor and $S/W<1$, we required numerical solutions.

Q.E.D.

B. Appendix of Chapter 3

U.S. Active Equity Mutual Funds from Morningstar

We begin by downloading the U.S. mutual fund data from Morningstar. Morningstar provides fund share class level data. The datasets we download include “United States Mutual Funds” (30,782 share classes), “United States Closed-End Funds” (565 share class), and “Unit Investment Trust” (14,881 share classes). However, all funds in the “Unit Investment Trust” dataset have neither long enough lives nor records of net asset values, so they are screened out by the criteria in the later steps described in the next sections. Our sample period is from January 1979 to December 2014, and we use monthly observations.

Secondly, we screen out share classes from funds of funds and index funds. We drop a fund share class if it has a value of “Yes” in any of the variables “Fund of Funds”, “In House FOF”, “Enhanced Index”, and “Index Fund”.

Thirdly, to screen out share classes of non-equity funds, we drop the funds if their “Global Broad Category Group” value is not “Equity”. We classify the remaining funds in our sample based on their “Morningstar Category” into the following categories:

Equity Funds	International Funds	Real Estate Funds	Sector Funds
US OE Small Blend	US OE World Stock	US CE Global Real Estate	US OE Financial
US OE Mid-Cap Growth	US OE Foreign Large Growth	US CE Real Estate	US OE Technology
US OE Small Growth	US OE Diversified Emerging Mkts	US OE Global Real Estate	US OE Natural Resources
US OE Large Blend	US OE Foreign Large Blend	US OE Real Estate	US OE Energy Limited Partnership
US OE Large Growth	US OE Foreign Large Value	US UIT Real Estate	US OE Health
US OE Mid-Cap Value	US OE Pacific/Asia ex-Japan Stk	US UIT Global Real Estate	US OE Equity Precious Metals
US OE Large Value	US OE China Region		US OE Utilities
US OE Small Value	US OE Europe Stock		US OE Equity Energy
EAA OE US Large-Cap Growth Equity	US OE Foreign Small/Mid Growth		US OE Communications
EAA OE US Mid-Cap Equity	US OE Latin America Stock		US OE Consumer Cyclical
US OE Mid-Cap Blend	US OE Foreign Small/Mid Value		US OE Consumer Defensive
US CE Large Blend	US OE India Equity		US OE Industrials
US CE Large Value	US OE Foreign Small/Mid Blend		US OE Miscellaneous Sector
US CE Mid-Cap Blend	US OE Miscellaneous Region		US CE Equity Energy
US CE Large Growth	US OE Diversified Pacific/Asia		US CE Equity Precious Metals
US CE Mid-Cap Growth	US OE Japan Stock		US CE Health
US CE Small Blend	EAA OE Islamic Global Equity		US CE Natural Resources
US UIT Mid-Cap Blend	US CE Miscellaneous Region		US CE Technology
US UIT Large Blend	US CE Diversified Emerging Mkts		US CE Utilities
US UIT Large Growth	US CE China Region		US CE Energy Limited Partnership
US UIT Large Value	US CE Japan Stock		US CE Financial
US UIT Mid-Cap Value	US CE Latin America Stock		US CE Industrials
US UIT Mid-Cap Growth	US CE World Stock		US UIT Industrials
US UIT Small Value	US CE Pacific/Asia ex-Japan Stk		US UIT Financial
US UIT Small Blend	US CE Foreign Large Blend		US UIT Energy Limited Partnership
US UIT Small Growth	US CE Europe Stock		US UIT Equity Energy
	US CE Foreign Small/Mid Blend		US UIT Natural Resources
	US CE India Equity		US UIT Equity Precious Metals
	US CE Diversified Pacific/Asia		US UIT Technology
	US CE Foreign Small/Mid Value		US UIT Health
	US CE Foreign Large Value		US UIT Utilities
	US UIT Diversified Emerging Mkts		US UIT Consumer Cyclical
	US UIT Europe Stock		US UIT Communications
	US UIT World Stock		US UIT Miscellaneous Sector
	US UIT Foreign Large Value		
	US UIT Foreign Large Blend		
	US UIT Foreign Large Growth		
	US UIT Japan Stock		

We only keep the funds in the "Equity Funds" category because these funds invest only in the domestic equity market. We do not need to use the "Primary Prospectus Benchmark" to screen out non-equity funds as do Pastor, Stambaugh and Taylor (2015) (PST hereafter) because the funds with a value of "Equity" in the "Global Broad Category Group" variable do not contain any keywords in "Primary Prospectus Benchmark" that would be defined as non-equity funds in PST.

Fourthly, we aggregate share class level data to fund level data by using the fund identifier, the "Fund ID" variable. The first step is to fill in any missing fund net asset value observations (the variable "Net Asset Share Class (Monthly)" in Morningstar) by using the last available net asset value observation. A new variable "Fund Asset" is defined as the sum of the net asset values of share classes with the same "Fund ID" value in a particular month. If "Fund Asset" is zero, we regard it as missing.

All the monthly share class returns are net of administrative and management fees and other costs taken out of fund assets, and they are in percentage. To aggregate share class returns to fund returns, we use the net asset value weighted average share class return as the fund return. We use the following algorithm:

1. define a variable “NA” equal to “Net Asset Share Class (Monthly)”;
2. in each month, if the observation of the “Monthly Return” variable (share class return variable) is not missing, but “NA” is missing, then set “NA” to $1;
3. in each month, if the observation of the “Monthly Return” variable is missing, set this month’s “NA” to missing;
4. in each month, calculate a variable “Total Return” as the sum-product of “NA” and “Monthly Return” of share classes with the same “Fund ID” value;
5. in each month calculate a variable “TA” as the sum of “NA” of share classes with the same “Fund ID” value;
6. calculate the variable “Fund Return” as “Total Return” divided by “TA”.

This algorithm can ensure the following: in each month, if a share class’ return is missing, its net asset value is not used in calculating the asset value weighted average return. If for a fund, all share classes have return observations but all their net asset values are missing, the fund returns are equally weighted averages of all the share classes’ returns. If for a fund, all share classes have return observations but some have missing net asset values, then almost all the weights will be allocated to the share classes with net asset value observations.

The fund data sample after this step is defined as our U.S. active equity fund sample.

Fund Market Concentration Measures

We use all the fund net asset value data in our U.S. active equity fund sample to calculate fund market concentration measures.

First, in each month, we calculate fund “Market Share” variable as the fund’s “Fund Asset”, divided by the sum of all the funds’ “Fund Asset” in this month. Next, in each month, we count the number of “Fund Asset” observations, and use it as the value of the variable “Number of Funds”. The fund market concentration measures are calculated as follows:

1. in each month, “H Index” is calculated as the sum of all the funds’ “Market Share” squared;
2. in each month, “Normalized H Index” is calculated as (“Number of Funds”×“H Index”-1)/(“Number of Funds”-1);
3. in each month, “5 Fund Index” is calculated as the sum of the first five largest funds’ “Market Share”.

We have 36 years, or 432 monthly observations of all these fund market concentration measures.

Fund Net Alphas and Principal Components Adjusted Fund Net Alphas

We use our U.S. active equity fund sample, and further require a fund to include at least 10 years' observations with missing observations in between no more than 5 years. The reason is that in our recursive rolling window style-matching model, we use 5 years (60 monthly observations) as our estimating window, so a fund with 10 years' observations, with a gap of no more than 5 years of missing observations, will have reasonably large number of observations in fund net alphas, mitigating the measurement error issue. The funds in the "Unit Investment Trust" dataset are all screened out. In our sample, we have 1,374 funds' net alphas.

In choosing the factors in the style-matching model, we use index funds and risk-free return as our factors. We keep the share class observations with the value "Yes" in the "Index Fund" variable and drop the funds with a value "Yes" in any of the variables "Enhanced Index", "Fund of Funds" and "In House FOF". Then we aggregate the share class returns into fund returns following the same algorithm discussed in Section 2. We further require an index fund to have non-missing observations in our sample period to make our style-matching procedure meaning and stable. The index funds in our sample include

1. Vanguard Small Cap Index, defined as "Small Core" in the "Morningstar Institutional Category" variable,
2. Vanguard 500 Index, defined as "S&P 500 Tracking" in the "Morningstar Institutional Category" variable, and
3. EQ/Common Stock Index Portfolio, defined as "Large Core" in the "Morningstar Institutional Category" variable.

The monthly risk-free return is the Fama-French risk-free rate downloaded from the CRSP dataset, and it is transformed into percentage return.

Our algorithm of running the style-matching model is as follows:

1. use the data of the previous 60 months (from $t-60$ to $t-1$) to run the style-matching model, minimizing the variance of the regression residuals and calculating the R-squared of the model as 1 minus variance of the residuals divided by the variance of the fund return;
2. predict the fund return at month t;

3. in each month, calculate the variable “Fund Net Alpha” as the “Fund Net Return” minus the predicted value of fund return.

As our style-matching model might miss some relevant but unobservable factors, we also calculate principal component adjusted fund net alphas (“PC-Adjusted Fund Net Alphas”) by using the following method. First, we follow Connor and Korajczyk (1988) (CK) to calculate principal components of fund net alphas. CK’s method is originally applied on balanced samples, samples without missing observations. As our sample of fund net alphas contains missing values, we need to do some adjustment. We define $\boldsymbol{\alpha}$ as a $M \times T$ matrix of the fund net alphas in market, whose row indices represent funds and column indices represent time. Next, we calculate the $T \times T$ cross-product matrix $\boldsymbol{\Omega}$, with $\boldsymbol{\Omega}(i,j) = \sum_{t=1}^{T} \boldsymbol{\alpha}(i,t) \times \boldsymbol{\alpha}(j,t) / m_{ij}$, where m_{ij} is the number of non-missing terms of $\boldsymbol{\alpha}(i,t) \times \boldsymbol{\alpha}(j,t)$ at time t. We estimate the eigenvectors of $\boldsymbol{\Omega}$ and define it by $\mathbf{G}$. Matlab automatically outputs the first 6 eigenvectors, so $\mathbf{G}$ is a $T \times 6$ matrix. Then we regress $\boldsymbol{\alpha}^{\mathbf{T}}$ on $\mathbf{G}$, with a constant term, and calculate the $M \times M$ pair-wise residual variance matrix $\mathbf{V}$. We define the $M \times M$ diagonal matrix $\mathbf{DIAGV}$, whose diagonal elements are the same as those in $\mathbf{V}$. After that, we calculate $\boldsymbol{\Omega}^{*}$, with $\boldsymbol{\Omega}^{*}(i,j) = \sum_{t=1}^{T} \boldsymbol{\alpha}(i,t) \times \mathbf{DIAGV}^{-1}(i,j) \times \boldsymbol{\alpha}(j,t) / m_{ij}$. Then we calculate the eigenvectors of $\boldsymbol{\Omega}^{*}$ and define it as $\mathbf{G}^{*}$. Matlab automatically outputs the first 6 eigenvectors, so $\mathbf{G}^{*}$ is also a $T \times 6$ matrix, and each column in $\mathbf{G}^{*}$ is a principal component. We use the first two principal components to calculate the “PC-Adjusted Fund Net Alphas”. For each fund, we regress its time-series of fund net alphas on the first two principal components, without a constant term, and the estimated residuals are regarded as “PC-Adjusted Fund Net Alphas”.

Stock Market Capitalization, Fund Industry Size, and Fund Size

We use the U.S. stock data in CRSP Monthly Stock File to do our analyses. We keep the stock observations with a share code “shrcd” equal to 10 or 11. For each stock in each month, the stock’s market capitalization is calculated as its share price “prc” multiplied by total shares outstanding “shrout” multiplied by 1,000, since the unit of the variable “shrout” is thousand shares. For each month, the stock market capitalization is the sum of each stock’s market capitalization in this month.

The “Industry Size” variable in our analyses represents the active equity mutual fund industry size relative to stock market capitalization. For each month, “Industry Size” is equal to the sum of all the funds’ “Fund Asset” in our U.S. active equity fund sample, divided by the stock market capitalization.

The “Fund Size” variable in our analyses represents each fund’s size in December 2014 dollars. For each month, “Fund Size” is equal to “Fund Asset” divided by stock market capitalization in this month, then multiplied by the stock market capitalization in December 2014.

C. Appendix of Chapter 4

Proof of Equivalence of the Two Managers' Problems

Because of competition, if funds offer higher (lower) expected net alphas, investments shift into (out of) it. Thus, in equilibrium all funds offer the same expected net alphas. We show in Proposition 2 and 3 that the equilibrium expected net alpha is the highest one that each manager can achieve at zero profit, and that equilibrium fund sizes are determined by managers' costs (which can be viewed as a reflection of their skills).

Suppose that the market expected net alpha is $\bar{\alpha}$, where $\bar{\alpha}$ is below the highest level of fund expected net alpha that mangers can produce (implying positive profits). While producing expected net alpha of $\bar{\alpha}$, manager i maximizes profits by choosing optimal efforts $e_i^{11^*}$ and $e_i^{12^*}$ that maximize the fund expected net alpha (i.e., the condition in Proposition 3, *ii*.a holds), and then charges a fee f_i^1 such that his or her fund expected net alpha is exactly $\bar{\alpha}$. Then, the current managerial fee becomes

$$f_i^1 = \widehat{a^1} - \widehat{b^1}\frac{S^{1^*}}{W^1} + A^{11}\left(e_i^{11^*};\, H^1, H^2\right) + A^{12}\left(e_i^{12^*};\, H^1, H^2\right) - \bar{\alpha}. \tag{C1}$$

Define the *profit rate* of manager i, pro_i^1 as $pro_i^1 \triangleq f_i^1 - C_i^1\left(e_i^{11^*}, e_i^{12^*};\, s_i^1, H^1, H^2\right)$; then from the last definition and the equation above, we have

$$\begin{aligned}\bar{\alpha} = \widehat{a^1} - \widehat{b^1}\frac{S^{1^*}}{W^1} + A^{11}\left(e_i^{11^*};\, H^1, H^2\right) + A^{12}\left(e_i^{12^*};\, H^1, H^2\right) - \\ pro_i^1 - c_0^1 - c_{1,i}^1 s_i^1 - c_2^{11}\left(e_i^{11^*};\, H^1, H^2\right) - c_2^{12}\left(e_i^{12^*};\, H^1, H^2\right).\end{aligned} \tag{C2}$$

As all managers produce the same level of expected net alphas, Equation (C2) implies an equilibrium condition,

$$pro_i^1 + c_{1,i}^1 s_i^1 = pro_j^1 + c_{1,j}^1 s_j^1,\ \forall i, j. \tag{C3}$$

Next, we consider manager i's profit function

$$s_i^1\left[f_i^1 - c_0^1 - c_{1,i}^1 s_i^1 - c_2^{11}\left(e_i^{11^*};\, H^1, H^2\right) - c_2^{12}\left(e_i^{12^*};\, H^1, H^2\right)\right] \tag{C4}$$

and by the first-order condition, the optimal fund size given manager i's profit level is

$$\begin{aligned}s_i^{1^{opt}} &= \frac{f_i^1 - c_0^1 - c_2^{11}\left(e_i^{11^*};\, H^1, H^2\right) - c_2^{12}\left(e_i^{12^*};\, H^1, H^2\right)}{2c_{1,i}^1} \\ &= \frac{pro_i^1}{2c_{1,i}^1} + \frac{s_i^1}{2}.\end{aligned} \tag{C5}$$

The latter equality is useful in presenting the optimal size relative to current size. Note that if manager i maximizes his or her fund's expected net alpha, the profit rate

$pro_i^1 = 0$, and the condition in (C5) for $s_i^{1\,opt}$ does not exist. For a particular manager $j, j \neq i$, it is possible that pro_j^1 is so high that $s_j^1 < s_j^{1\,opt}$. In other words, although manager i does not observe other managers' cost functions and profit rates, he or she knows that it is possible that some other manager(s) might have incentive to lower down the profit rate to attract investments, increase their fund size, and increase their fund's profits.

Following this argument, we analyze a simple game between manager i and other managers, grouped as an entity "$-i$". If manager i improves his or her fund expected net alpha infinitesimally, then profit will also be changed by an infinitesimal amount ε_i, and other managers will receive no investments and get zero profit. If any of the other managers increases fund expected net alpha infinitesimally, then this manager's profit will be changed by ε_{-i}, and manager i will receive no investment and get zero profit. If all managers produce the same level of fund expected net alphas, then they can make profit. Notice that ε_i (ε_{-i}) can be positive or negative, depending on whether manager i's ($-i$'s) fund size is below or above optimal. Assume manager $-i$'s strategy is to improve the fund expected net alpha infinitesimally with probability p and maintain $\bar{\alpha}$ with probability $1-p$. This does not mean that manager $-i$ randomly chooses an action. Instead, it means that manager i knows that it is possible that some other manager(s) want to improve fund expected net alpha to attract investments in order to improve fund profit, and this probability p is nontrivial. Manager i's strategy is to improve his or her fund expected net alpha infinitesimally with probability θ and maintain $\bar{\alpha}$ with probability $1-\theta$. The payoffs of the game are illustrated in the following table, with the row (column) representing manager i's ($-i$'s) action, and with manager i's ($-i$'s) payoffs in the first (second) figures in the brackets.

		Maintain $\bar{\alpha}$	Improve Infinitesimally
		$1-p$	p
Maintain $\bar{\alpha}$	$1-\theta$	$(pro_i^1 s_i^1, pro_{-i}^1 s_{-i}^1)$	$(0, pro_{-i}^1 s_{-i}^1 + \varepsilon_{-i})$
Improve Infinitesimally	θ	$(pro_i^1 s_i^1 + \varepsilon_i, 0)$	$(pro_i^1 s_i^1 + \varepsilon_i, pro_{-i}^1 s_{-i}^1 + \varepsilon_{-i})$

The expected payoff of manager i is

$$\pi_i^1 = (1-p)[(1-\theta)pro_i^1 s_i^1 + \theta(pro_i^1 s_i^1 + \varepsilon_i)] + p\theta(pro_i^1 s_i^1 + \varepsilon_i). \tag{C6}$$

The first-order condition is

$$\frac{d\pi_i^1}{d\theta} = \varepsilon_i + p * pro_i^1 s_i^1 \tag{C7}$$

With $\varepsilon_i \to 0$, $d\pi_i^1/d\theta > 0$. Thus, manager i chooses $\theta = 1$ to maximize π_i^1. Notice that if $\bar{\alpha}$ is the maximum fund expected net alpha defined by Proposition I1, managers' profit rates are zero, and ε_i and ε_{-i} are negative. Then, in this case, the unique Nash equilibrium is (Maintain $\bar{\alpha}$, Maintain $\bar{\alpha}$).

Therefore, each manager will improve his or her fund expected net alpha as long as it is below the maximum fund expected net alpha, thus, managers' problems of maximizing profits is equivalent to maximizing their funds' expected net alphas.

Proof of Proposition I1

$\{\mathbf{e^{11}}^*, \mathbf{e^{12}}^*, \mathbf{f^1}^*, \boldsymbol{\delta^1}^*\}$ is a Nash Equilibrium because

1. Managers have to maximize fund expected net alphas to attract investments, and they are earning zero economic profits in equilibrium. Given other Country 1 managers' optimal choices, manager i has no incentive to deviate from $\mathbf{e^{11}}^*$, $\mathbf{e^{12}}^*$, and $\mathbf{f^1}^*$. This is because increasing $\mathbf{e^{11}}^*$ and $\mathbf{e^{12}}^*$ or decreasing $\mathbf{f^1}^*$ generates negative economic profit, whereas decreasing $\mathbf{e^{11}}^*$ and $\mathbf{e^{12}}^*$ or increasing $\mathbf{f^1}^*$ reduces fund expected net alpha; thus, this manager receives no investments from investors.
2. Given Country 1 managers' and other Country 1 investors' optimal choices, a Country 1 investor has no incentive to deviate from $\boldsymbol{\delta^1}^*$, changing allocations across funds or between the fund industry and the international passive benchmark does not improve his portfolio's expected net returns.

$\{\mathbf{e^{11}}^*, \mathbf{e^{12}}^*, \mathbf{f^1}^*, \boldsymbol{\delta^1}^*\}$ is unique because

1. $\mathbf{e^{11}}^*$ is unique as for each fund, $B^{11}_{e_i^{11}}\left(e_i^{11^*};\ H^1, H^2\right) = 0$ is uniquely solved by $e_i^{11^*}$;
2. $\mathbf{e^{12}}^*$ is unique as for each fund, $B^{12}_{e_i^{12}}\left(e_i^{12^*};\ H^1, H^2\right) = 0$ is uniquely solved by $e_i^{12^*}$;
3. $\mathbf{f^1}^*$ is unique as for each fund, as for each fund, fees are break-even fees, i.e., $f_i^{1^*} - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) = 0$, $C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right)$ is deterministic, and $\mathbf{e^{11}}^*$ and $\mathbf{e^{12}}^*$ are unique;
4. $\boldsymbol{\delta^1}^*$ is unique as allocations to funds maximize investor portfolios' Sharpe ratios, driving fund expected net alphas to the same values; the uniqueness of $\mathbf{e^{11}}^*$, $\mathbf{e^{12}}^*$ and $\mathbf{f^1}^*$ makes $\boldsymbol{\delta^1}^*$ unique.

In equilibrium, all funds' expected net alphas are the same, i.e., $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ is the same across for all funds. This is because, given other managers' alphas, if a fund manager cannot produce the AFMI highest expected net alpha, even for an infinitesimal fund size, investments continue to shift out of his or her fund. It lowers the fund costs due to decreasing returns to scale, and allows the manager to charge lower fee to improve alpha. Consequently, fund expected net returns $\mathrm{E}(r_{F,i}^1|D)|_{\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\}} = \mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\}} + \mu_p$ are the same in equilibrium. In addition, as funds have the same expected net alphas, they have the same expected net returns. The source of fund returns' variance is the same across funds, and $\mathrm{Var}(r_{F,i}^1|D)|_{\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\}} = \sigma_p^2 + \sigma_{a^1}^2 + \left(\frac{S^{1\,*}}{W^1}\right)^2 \sigma_{b^1}^2 + \sigma_x^2 + \sigma_\varepsilon^2, \forall i$. That is, the fund return variance is the same across funds. Combining these results, we conclude that all managers offer the same competitive Sharpe ratio.

Q.E.D.

Proof of Proposition I2 and I3.

To maximize fund net alphas, manager i chooses optimal fees such that $f_i^{1^*} - C_i^1(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2) = 0$. Substitute this in to $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$, regard $\left(\frac{S^{1\,*}}{W^1}\right)^2$ and $s_i^{1^*}$ as given, and take the first-order condition on $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ with respect to e_i^{11}. We, then, have

$$A_{e_i^{11}}^{11}(e_i^{11^*};\ H^1, H^2) - c_{2\,e_i^{11}}^{11}(e_i^{11^*};\ H^1, H^2) = B_{e_i^{11}}^{11}(e_i^{11^*};\ H^1, H^2) = 0. \tag{C8}$$

Here if $A_{e_i^{11}}^{11}(0;\ H^1, H^2) - c_{2\,e_i^{11}}^{11}(0;\ H^1, H^2) \le 0$, or $e_i^{11^*} \le 0$, manager i chooses $e_i^{11^*} = 0$. The second-order condition is satisfied by our functional assumptions.

$$A_{e_i^{11},e_i^{11}}^{11}(e_i^{11^*};\ H^1, H^2) - c_{2\,e_i^{11},e_i^{11}}^{11}(e_i^{11^*};\ H^1, H^2) < 0. \tag{C9}$$

Here we assume that $e_i^{11^*}$ is finite and attainable. Since the functional form of $A^{11}(e_i^{11};\ H^1, H^2)$ and $c_2^{11}(e_i^{11};\ H^1, H^2)$ are the same across funds, based on (C8), $e_i^{11^*}$ is the same across funds. Fully differentiating (C8) with respect to H^1 and H^2, we have

$$\frac{de_i^{11^*}}{dH^1} = -\frac{A^{11}_{e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right)}{A^{11}_{e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)} \tag{C10}$$

$$\frac{de_i^{11^*}}{dH^2} = -\frac{A^{11}_{e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right)}{A^{11}_{e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)} \tag{C11}$$

Thus, the sign of $de_i^{11^*}/dH^1$ ($de_i^{11^*}/dH^2$) depends on the sign of $A^{11}_{e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right)$ ($A^{11}_{e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right)$).

Also, as $e_i^{11^*}$ is the same across funds, $B^{11}\left(e_i^{11^*};\ H^1,H^2\right) \triangleq A^{11}\left(e_i^{11^*};\ H^1,H^2\right) - c_2^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ is the same across funds. Fully differentiate $B^{11}\left(e_i^{11^*};\ H^1,H^2\right)$ with respect to H^1 and H^2, and we have

$$\begin{aligned}\frac{dB^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^1} &\\ &= B^{11}_{e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)\frac{de_i^{11^*}}{dH^1} + A^{11}_{H^1}\left(e_i^{11^*};\ H^1,H^2\right) \\ &\quad - c^{11}_{2\ H^1}\left(e_i^{11^*};\ H^1,H^2\right) \\ &= A^{11}_{H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ H^1}\left(e_i^{11^*};\ H^1,H^2\right)\end{aligned} \tag{C12}$$

and

$$\begin{aligned}\frac{dB^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^2} &\\ &= B^{11}_{e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)\frac{de_i^{11^*}}{dH^2} + A^{11}_{H^2}\left(e_i^{11^*};\ H^1,H^2\right) \\ &\quad - c^{11}_{2\ H^2}\left(e_i^{11^*};\ H^1,H^2\right) \\ &= A^{11}_{H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ H^2}\left(e_i^{11^*};\ H^1,H^2\right)\end{aligned} \tag{C13}$$

The proof of the results regarding $e_i^{12^*}$ is similar to the proof above.

As $e_i^{11^*}$, $e_i^{12^*}$, and $\mathrm{E}(\alpha_i^1 | D)\big|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{\mathbf{1}^*}\}}$ are the same across funds, we have $c^1_{1,i}s_i^{1^*} = c^1_{1,j}s_j^{1^*}$, $\forall i,j$, thus $s_i^{1^*}/s_j^{1^*} = c^1_{1,j}/c^1_{1,i}$, $\forall i,j$. Using the fact that $S^{1^*} = \sum_{i=1}^{M^1} s_i^{1^*}$, we have $(s_i^1/S^1)^* = \left[c^1_{1,i}\sum_{j=1}^{M^1}\left(c^1_{1,i}\right)^{-1}\right]^{-1}$, $\forall i$.

Moreover, in equilibrium, we have

$$
\begin{aligned}
f_i^{1^*} &= C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) \\
&= c_0^1 + c_{1,i}^1 s_i^{1^*} + c_2^{11}\left(e_i^{11^*};\ H^1, H^2\right) + c_2^{12}\left(e_i^{12^*};\ H^1, H^2\right) \\
&= c_0^1 + c_{1,i}^1\left(\frac{s_i^1}{S^1}\right)^*\left(\frac{S^{1}}{W^1}^{*}\right)W^1 + c_2^{11}\left(e_i^{11^*};\ H^1, H^2\right) \\
&\quad + c_2^{12}\left(e_i^{12^*};\ H^1, H^2\right) \\
&= c_0^1 + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\left(\frac{S^{1}}{W^1}^{*}\right) + c_2^{11}\left(e_i^{11^*};\ H^1, H^2\right) \\
&\quad + c_2^{12}\left(e_i^{12^*};\ H^1, H^2\right).
\end{aligned} \tag{C14}
$$

Fully differentiate $f_i^{1^*}$ with respect to H^1 and H^2, and we have

$$
\begin{aligned}
\frac{df_i^{1^*}}{dH^1} &= \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\frac{d(S^1/W^1)^*}{dH^1} \\
&\quad + c_{2\,e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right)\frac{de_i^{11^*}}{dH^1} \\
&\quad + c_{2\,e_i^{12}}^{12}\left(e_i^{12^*};\ H^1, H^2\right)\frac{de_i^{12^*}}{dH^1}
\end{aligned} \tag{C15}
$$

$$
\begin{aligned}
\frac{df_i^{1^*}}{dH^2} &= \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\frac{d(S^1/W^1)^*}{dH^2} \\
&\quad + c_{2\,e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right)\frac{de_i^{11^*}}{dH^2} \\
&\quad + c_{2\,e_i^{12}}^{12}\left(e_i^{12^*};\ H^1, H^2\right)\frac{de_i^{12^*}}{dH^2}.
\end{aligned} \tag{C16}
$$

Thus, the sign of $df_i^{1^*}/dH^1$ depends on the signs of $d(S^1/W^1)^*/dH^1$, $de_i^{11^*}/dH^1$, and $de_i^{12^*}/dH^1$; and the sign of $df_i^{1^*}/dH^2$ depends on the signs of $d(S^1/W^1)^*/dH^2$, $de_i^{11^*}/dH^2$, and $de_i^{12^*}/dH^2$

Q.E.D.

Proof of Proposition I4 to I7.

Investor j's portfolio Sharpe ratio is

$$\frac{\mathrm{E}\left(r_j^1 \middle| D\right)}{\sqrt{\mathrm{Var}\left(r_j^1 \middle| D\right)}}$$

$$= \frac{\mu_p + \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} \left[\widehat{a^1} - \widehat{b^1}\frac{S^{1\,*}}{W^1} + A^{11}\left(e_i^{11^*};\ H^1, H^2\right) + A^{12}\left(e_i^{12^*};\ H^1, H^2\right) - f_i^{1^*}\right]}{\sqrt{\sigma_p^2 + \left[\sigma_{a^1}^2 + \sigma_x^2 + \left(\frac{S^{1\,*}}{W^1}\right)^2 \sigma_{b^1}^2\right]\left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}\right)^2 + \sigma_\varepsilon^2 \left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}\right)}} \quad \text{(C17)}$$

$$= \frac{\mu_p + \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} \left\{\widehat{a^1} - \frac{S^{1\,*}}{W^1}\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1 + \widehat{b^1}\right\} + X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)\right.}{\sqrt{\sigma_p^2 + \left[\sigma_{a^1}^2 + \sigma_x^2 + \left(\frac{S^{1\,*}}{W^1}\right)^2 \sigma_{b^1}^2\right]\left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}\right)^2 + \sigma_\varepsilon^2 \left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}\right)}}$$

We assume that the marginal diversification benefits of investing in one more fund is trivial, so we set $\sigma_\varepsilon^2 \left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}\right) \to \infty$ when solving the problem. When maximizing investor j 's portfolio Sharpe ratio, we substitute $A_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_{2\,e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) = 0$, $A_{e_i^{12}}^{12}\left(e_i^{12^*};\ H^1, H^2\right) - c_{2\,e_i^{12}}^{12}\left(e_i^{12^*};\ H^1, H^2\right) = 0$, and $f_i^{1^*} - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) = 0$ into (4.14). Taking the first-order condition with respect to $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}$, we have

$$\begin{gathered}\frac{\mu_p}{\sigma_p^2}\left[\sigma_{a^1}^2 + \sigma_{b^1}^2\left(\frac{S^{1\,*}}{W^1}\right)^2 + \sigma_x^2\right]\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} - \left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1 + \right. \\ \left.\widehat{b^1}\right\}\frac{S^{1\,*}}{W^1} + X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) = 0.\end{gathered} \quad \text{(C18)}$$

Notice small investors regard $(S^1/W^1)^*$ as given, since each of them cannot affect this ratio. Substitute $\gamma \triangleq \mu_p/\sigma_p^2$, $(\gamma > 0)$ and symmetric equilibrium condition $(S^1/W^1)^* = \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}$ into (C18), and we have

$$\begin{gathered}-\gamma\sigma_{b^1}^2\left(\frac{S^{1\,*}}{W^1}\right)^3 - \left\{\gamma\sigma_{a^1}^2 + \gamma\sigma_x^2 + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{S^{1\,*}}{W^1} + \\ X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) = 0.\end{gathered} \quad \text{(C19)}$$

The assumption that $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) > 0,\ \forall\, H^1, H^2$ and the coefficient of the third-order term $-\gamma\sigma_{b^1}^2 < 0$ together guarantee that this cubic equation of $(S^1/W^1)^*$ has at least one real positive solution (there are three cases: three real positive roots, one real positive root and two real negative roots, and one real positive root and two imaginary solutions). Notice that if the solution of (C19) $(S^1/W^1)^* > 1$, then $(S^1/W^1)^* = 1$ and there is a corner solution in this case. There is one special case where there are two different solutions of $(S^1/W^1)^*$ that are both smaller than one and both maximize (4.14). As each small investor regards $(S^1/W^1)^*$ as given when solving (C18), the market will end up with one of these two solutions with probability one.

Where $(S^1/W^1)^* < 1$, fully differentiate (C19) with respect to $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$ and $\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1$, and we have

$$\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)} = \frac{1}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2 + \sigma_{a^1}^2 + \sigma_x^2\right] + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1} > 0, \tag{C20}$$

$$\frac{d\left(S^1/W^1\right)^*}{d\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}} = \frac{-\left(S^1/W^1\right)^*}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2 + \sigma_{a^1}^2 + \sigma_x^2\right] + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1} < 0. \tag{C21}$$

Also, where $(S^1/W^1)^* < 1$, fully differentiate (C19) with respect to H^1, and we have

$$\begin{aligned}
\frac{d(S^1/W^1)^*}{dH^1} &= \frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)} \frac{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)}{dH^1} \\
&= \frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)} \Big[A_{H^1}^{11}\left(e_i^{11^*};\ H^1, H^2\right) + A_{H^1}^{12}\left(e_i^{12^*};\ H^1, H^2\right) \\
&\qquad - c_{2\,H^1}^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_{2\,H^1}^{12}\left(e_i^{12^*};\ H^1, H^2\right)\Big] \\
&= \frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)} \left[\frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^1}\right].
\end{aligned} \tag{C22}$$

We know that $\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)} > 0$. Thus, we know that $\frac{d\left(S^1/W^1\right)^*}{dH^1} \geq 0\ (< 0)$ if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} \geq 0\ (< 0)$. Fully differentiate it with respect to H^1 again, and we have

$$\frac{d^2\left(S^1/W^1\right)^*}{d{H^1}^2} = \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*}; H^1, H^2\right)}{d{H^1}^2} + \frac{d^2B^{12}\left(e_i^{12^*}; H^1, H^2\right)}{d{H^1}^2}\right] + \tag{C23}$$

$$\frac{d^2(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)^2}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]$$

$$=\frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\right]-$$

$$6\gamma\sigma_{b^1}^2\frac{S^{1^*}}{W^1}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]^2\left[\frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3.$$

As $6\gamma^1\sigma_{b^1}^2(S^1/W^1)^*>0$, if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\le 0$ then $\frac{d^2(S^1/W^1)^*}{dH^{1^2}}\le 0$, and if $\frac{d^2(S^1/W^1)^*}{dH^{1^2}}\ge 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\ge 0$. Similarly, we can prove the results of $\frac{d(S^1/W^1)^*}{dH^2}$, $\frac{d^2(S^1/W^1)^*}{dH^{2^2}}$ and $\frac{d^2(S^1/W^1)^*}{dH^1dH^2}$ where$(S^1/W^1)^*<1$. Where $(S^1/W^1)^*=1$, $(S^1/W^1)^*$ does not depend on H^1 or H^2.

Moreover, we know that

$$\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}}=\widehat{a^1}-\frac{S^{1^*}}{W^1}\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1+\widehat{b^1}\right\}+ X\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right). \tag{C24}$$

We know that $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}}>0$. This is because

$$\mathrm{Var}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}}=\sigma_{a^1}^2+\sigma_x^2+\left(\frac{S^{1^*}}{W^1}\right)^2\sigma_{b^1}^2, \tag{C25}$$

so $\mathrm{Var}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}}$ increases with $(S^1/W^1)^*$. Thus, a portfolio with allocations to both funds and the passive benchmark is always riskier than a portfolio with allocations only to the passive benchmark, and we should have $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}}>0$ to induce investments to funds.

Where $(S^1/W^1)^*<1$, fully differentiate (C24) with respect to H^1, and we have

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}} =-\frac{d(S^1/W^1)^*}{dH^1}\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1+\widehat{b^1}\right\} \tag{C26}$$

$$+\left[\frac{dB^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};\ H^1,H^2\right)}{dH^1}\right]$$

$$= \left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]\left\{1-\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right\}.$$

We know that based on (C21) and the parameter assumptions such that $\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_x^2\right]>0$, then $\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}<1$. Thus, $\left.\frac{d\mathrm{E}\left(\alpha_i^1|D\right)}{dH^1}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\right\}}\geq 0\ (<0)$ if and only if $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\geq 0\ (<0)$. Also,

$$\left.\frac{d^2\mathrm{E}\left(\alpha_i^1|D\right)}{dH^{1^2}}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\right\}}= \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\right]\left\{1-\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right\}-\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d^2\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)^2} \quad \text{(C27)}$$

$$= \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\right]\left\{1-\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right\}+6\gamma\sigma_{b^1}^2\frac{S^{1^*}}{W^1}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3.$$

As $6\gamma\sigma_{b^1}^2\left(S^1/W^1\right)^*>0$, if $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1|D\right)}{dH^{1^2}}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\right\}}\leq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\leq 0$, and if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\geq 0$, then $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1|D\right)}{dH^{1^2}}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\right\}}\geq 0$.

Where $\left(S^1/W^1\right)^*=1$, fully differentiate (C24) with respect to H^1, and we have

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = \frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^1} \tag{C28}$$

and

$$\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = \frac{d^2B^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^{1^2}}. \tag{C29}$$

Similarly, we can prove the results of $\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^2}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$, $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$, and $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^1dH^2}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$.

In addition, where $(S^1/W^1)^* < 1$, if we fully differentiate $(S^1/W^1)^*$ with respect to $c_{1,i}^1$, we have

$$\frac{d\left(S^1/W^1\right)^*}{dc_{1,i}^1} = \frac{-\left(S^1/W^1\right)^* W^1}{\left\{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^1}{W^1}^*\right)^2 + \sigma_{a^1}^2 + \sigma_x^2\right] + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} < 0. \tag{C30}$$

Also, if we fully differentiate $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$ with respect to $c_{1,i}^1$, we have

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dc_{1,i}^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = -\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dc_{1,i}^1} - \frac{-\left(S^1/W^1\right)^* W^1}{\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} < 0. \tag{C31}$$

Where $(S^1/W^1)^* = 1$, $\frac{d\left(S^1/W^1\right)^*}{dc_{1,i}^1} = 0$, and

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dc_{1,i}^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} = -\frac{-\left(S^1/W^1\right)^* W^1}{\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} < 0. \tag{C32}$$

The result of $\left.\frac{d\mathrm{E}\left(\alpha_j^1|D\right)}{dc_{1,i}^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$, $\forall j \neq i$ are similar. Thus, we can see that both $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$ and $\mathrm{E}\left(\alpha_j^1|D\right)\big|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$, $\forall j \neq i$ are negatively related to $c_{1,i}^1$, whether $(S^1/W^1)^* < 1$ or $(S^1/W^1)^* = 1$. We also know that $(s_i^1/S^1)^*$ and

$c_{1,i}^{1}$ are negatively related, whereas $\left(s_j^1/S^1\right)^*, \forall j \neq i$ and $c_{1,i}^{1}$ are positively related. Then we have the results in Proposition I7.

Q.E.D.

Data Set Development

Global Active Equity Mutual Funds from Morningstar

We begin by downloading the global mutual fund data from the Global Databases of Morningstar Direct. Morningstar provides fund share class level data. Due to data availability, we download the data from 30 global markets and the U.S. market. We do not include any data of restricted funds in our sample. All the fund returns and asset under management values are in U.S. dollar.

Secondly, in each market, we screen out share classes from funds of funds and index funds. We drop a fund share class if it has a value of “Yes” in any of the variables “Fund of Funds”, “In House FOF”, “Enhanced Index”, and “Index Fund” where these indicators are available.

Thirdly, in each market, to screen out share classes of non-equity funds, we drop the funds if their “Global Broad Category Group” value is not “Equity”. We classify the remaining funds in our sample as “Active Equity funds”, based on their “Morningstar Category”. We only keep the funds in the “Active Equity Funds” category as these funds invest only in the domestic equity market. In each market, the Morningstar Category values that we use to define “Active Equity Funds” are shown in the following table.

Global Market	Morningstar Category
Australia	Australia OE Equity Australia Large Blend
	Australia OE Equity Australia Large Geared
	Australia OE Equity Australia Large Growth
	Australia OE Equity Australia Large Value
	Australia OE Equity Australia Mid/Small Blend
	Australia OE Equity Australia Mid/Small Growth
	Australia OE Equity Australia Mid/Small Value
	Australia OE Equity Australia Other
Austria	EAA OE Austria Equity
Belgium	EAA OE Belgium Equity
Brazil	Brazil OE Brazil All Cap Equity
	Brazil OE Brazil Large-Cap Equity
	Brazil OE Brazil Mid & Small Cap Equity
	Brazil OE Other Equity
Canada	Canada Canadian Focused Small/Mid Cap Equity
	Canada Canadian Equity
	Canada Canadian Focused Equity
	Canada Canadian Small/Mid Cap Equity
	Canada Canadian Dividend & Income Equity
Chile	Chile OE Chile Equity
China (Mainland)	China OE QDII Greater China Equity
	EAA OE China Equity
Denmark	EAA OE Denmark Equity
Finland	EAA OE Finland Equity
France	EAA OE France Large-Cap Equity
	EAA OE France Small/Mid-Cap Equity
Germany	EAA OE Germany Large-Cap Equity
	EAA OE Germany Small/Mid-Cap Equity
Greece	EAA OE Greece Equity
Hong Kong	EAA OE Hong Kong Equity
India	India OE Flexicap
	India OE Large-Cap
	India OE Small/Mid-Cap
Israel	EAA OE Israel Small-Cap Equity
	EAA OE Israel Large/Mid-Cap Equity
Italy	EAA OE Italy Equity
Japan	EAA OE Japan Equity - Currency Hedged
	EAA OE Japan Large-Cap Equity
	EAA OE Japan Small/Mid-Cap Equity
	Japan OE Japan Equity Large-Cap Blend
	Japan OE Japan Equity Large-Cap Growth
	Japan OE Japan Equity Large-Cap Value
	Japan OE Japan Equity Mid-Cap Blend
	Japan OE Japan Equity Mid-Cap Growth
	Japan OE Japan Equity Mid-Cap Value
	Japan OE Japan Equity Small-Cap Blend
	Japan OE Japan Equity Small-Cap Growth
	Japan OE Japan Equity Small-Cap Value
Korea	EAA OE Korea Equity
	Korea OE Korea Large-Cap Equity
	Korea OE Korea Small/Mid-Cap Equity
Mexico	Mexico OE Mexico Equity
	Mexico OE Small/Mid-Cap Equity
Netherlands	EAA OE Netherlands Equity
Norway	EAA OE Norway Equity
Portugal	EAA OE Portugal Equity
Singapore	EAA OE Singapore Equity
South Africa	EAA OE South Africa & Namibia Equity
	EAA OE South Africa & Namibia Small-Cap Equity
Spain	EAA OE Spain Equity
Sweden	EAA OE Sweden Large-Cap Equity
	EAA OE Sweden Small/Mid-Cap Equity
Switzerland	EAA OE Switzerland Large-Cap Equity
	EAA OE Switzerland Small/Mid-Cap Equity
Taiwan	EAA OE Taiwan Large-Cap Equity
	EAA OE Taiwan Small/Mid-Cap Equity
Thailand	Thailand OE Equity Fix Term
	Thailand OE Equity Large-Cap
	Thailand OE Equity Small/Mid-Cap
United Kingdom	EAA OE UK Equity Income
	EAA OE UK Flex-Cap Equity
	EAA OE UK Large-Cap Blend Equity
	EAA OE UK Large-Cap Growth Equity
	EAA OE UK Large-Cap Value Equity
	EAA OE UK Mid-Cap Equity
	EAA OE UK Small-Cap Equity
United States	EAA OE US Large-Cap Blend Equity
	EAA OE US Large-Cap Growth Equity
	EAA OE US Mid-Cap Equity
	EAA OE US Small-Cap Equity
	US OE Large Blend
	US OE Large Growth
	US OE Large Value
	US OE Mid-Cap Blend
	US OE Mid-Cap Growth
	US OE Mid-Cap Value
	US OE Small Blend
	US OE Small Growth
	US OE Small Value

Fourthly, in each market, we aggregate share class level data to fund level data by using the fund identifier, the "Fund ID" variable. The first step is to fill in any missing fund net asset value observations (the variable “Net Asset Share Class (Monthly)” in Morningstar) by using the last available net asset value observation. A new variable “Fund Asset” is defined as the sum of the net asset values of share classes with the same “Fund ID” value in a particular month. If “Fund Asset” is zero, we regard it as missing.

All the monthly share class returns are net of administrative and management fees and other costs taken out of fund assets, and they are in percentage. To aggregate share class returns to fund returns, we use the net asset value weighted average share class return as the fund return. We use the following algorithm:

7. define a variable “NA” equal to “Net Asset Share Class (Monthly)”;
8. in each month, if the observation of the “Monthly Return” variable (share class return variable) is not missing, but “NA” is missing, then set “NA” to $1;
9. in each month, if the observation of the “Monthly Return” variable is missing, set this month’s “NA” to missing;
10. in each month, calculate a variable “Total Return” as the sum-product of “NA” and “Monthly Return” of share classes with the same “Fund ID” value;
11. in each month calculate a variable “TA” as the sum of “NA” of share classes with the same “Fund ID” value;
12. calculate the variable “Fund Return” as “Total Return” divided by “TA”.

This algorithm can ensure the following: in each month, if a share class’ return is missing, its net asset value is not used in calculating the asset value weighted average return. If for a fund, all share classes have return observations but all their net asset values are missing, the fund returns are equally weighted averages of all the share classes’ returns. If for a fund, all share classes have return observations but some have missing net asset values, then almost all the weights will be allocated to the share classes with net asset value observations.

Fund Market Concentration Measures

In each market, we use all the fund level data in our “Active Equity Funds” sample to calculate fund market concentration measures.

First, in each market, in each month, we calculate fund “Market Share” variable as the fund’s “Fund Asset”, divided by the sum of all the funds’ “Fund Asset” in this month. Next, in each month, we count the number of “Fund Asset” observations, and use it as the value of the variable “Number of Funds”. The fund market concentration measures are calculated as follows:

4. in each month, HHI is calculated as the sum of all the funds’ “Market Share” squared;
5. in each month, NHHI is calculated as (“Number of Funds”×HHI-1)/(“Number of Funds”-1);
6. in each month, “5-Fund-Index” is calculated as the sum of the first five largest funds’ “Market Share”.

Fund Net Alphas

In each global market, we use our "Active Equity Funds" sample, and further require a fund to include at least 10 years' observations with missing observations in between no more than 5 years. The reason is that in our recursive rolling window style-matching model, we use 5 years (60 monthly observations) as our estimating window, so a fund with 10 years' observations, with a gap of no more than 5 years of missing observations, will have reasonably large number of observations in fund net alphas, mitigating the measurement error issue.

In choosing the factors in the style-matching model, we use local index fund returns, a U.S. large-cap index fund return, and risk-free return as factors. We keep the share class observations with the value "Yes" in the "Index Fund" variable and drop the funds with a value "Yes" in any of the variables "Enhanced Index", "Fund of Funds" and "In House FOF". We also require their "Global Broad Category Group" value is "Equity". Then we aggregate the share class returns into fund returns following the same algorithm discussed in Section 2. We further require an index fund to have non-missing observations in our sample period to make our style-matching procedure meaning and stable. The name and "Morningstar Category" of the index funds in each market are shown in the following table. Notice that in Chile and Italy, there is no equity index funds that have return data for the whole sample period of these countries, so we combine two index funds into one: use the first index fund data return, and after its data terminates, then use the second index fund return data.

The monthly risk-free returns of the 30 Global markets are from the International Financial Statistics on the official website of International Monetary Fund (IMF). We use the Treasury Bill Rate in each market to proxy the risk-free return. If Treasury Bill Rate is not available, we use Money Market Rate of the market. We adjust all the risk-free returns into U.S. Dollar returns using the Exchange Rate information from this website. Some markets, such as mainland China, Taiwan and India, do not have Treasury Bill Rate or Money Market Rate information on this website. Then I collect the term deposit rates of mainland China and India from the official website of the major banks and transform them into U.S. dollar returns. I collect the T-bill rate and exchange rate of the Taiwan market from Datastream.

Global Market	Morningstar Category	Name
Australia	Australia OE Equity Australia Large Blend	BlackRock Australian Share
Austria	EAA OE Austria Equity	EMIF-Austria Index
Belgium	EAA OE Belgium Equity	KBC Multi Track Belgium
Brazil	Brazil OE Brazil All Cap Equity	Itaú Timing FIA
Canada	Canada Canadian Equity	TD Canadian Index
Chile	Chile OE Chile Equity	Security Index Fund Chile Im Trust Acciones Índice Chile
China (Mainland)	China OE Equity Funds	Wanjia 180 Index Fd
Denmark	EAA OE Denmark Equity	Danske Invest Danmark Indeks
Finland	EAA OE Finland Equity	Seligson & Co Finland Index
France	EAA OE France Large-Cap Equity	BNP Paribas Indice France
Germany	EAA OE Germany Large-Cap Equity	Pioneer Inv Akt Deutschland
Greece	EAA OE Greece Equity	ALPHA Athens Index
Hong Kong	EAA OE Hong Kong Equity	Hang Seng Index
India	India OE Large-Cap	Principal Index Nifty Div
Israel	EAA OE Israel Large/Mid-Cap Equity	I.B.I Tel Aviv 25 Basket
Italy	EAA OE Italy Equity	SSgA Italy Index Equity Fund Eurizon EasyFund Eq Italy LTE
Japan	Japan OE Japan Equity Large-Cap Growth Japan OE Japan Equity Large-Cap Blend	Nikko Index Fund 225 Nomura TOPIX Index Open
Korea	Korea OE Korea Large-Cap Equity	Samsung Index Premium Equity-Deriv
Mexico	Mexico OE Mexico Equity	GBMIPC
Netherlands	EAA OE Netherlands Equity	BNP Paribas AEX Index
Norway	EAA OE Norway Equity	Carnegie Norge Indeks
Portugal	EAA OE Portugal Equity	BBVA PPA Índice PSI 20 FIMAA
Singapore	EAA OE Singapore Equity	Singapore Index
South Africa	EAA OE South Africa & Namibia Equity	STANLIB Index
Spain	EAA OE Spain Equity	BBVA Bolsa Índice FI
Sweden	EAA OE Sweden Large-Cap Equity	Handelsbanken Sverigefond Index
Switzerland	EAA OE Switzerland Large-Cap Equity	UBS 100 Index-Switzerland
Taiwan	EAA OE Taiwan Large-Cap Equity	Yuanta/P-shares TAIEX Index Fund
Thailand	Thailand OE Equity Large-Cap	SCB SET Index
United Kingdom	EAA OE UK Large-Cap Blend Equity EAA OE UK Mid-Cap Equity	L&G UK Index HSBC FTSE 250 Index
United States	US OE Large Blend	Vanguard 500 Index

Our algorithm of running the style-matching model is as follows:

4. use the data of the previous 60 months (from $t-60$ to $t-1$) to run the style-matching model, minimizing the variance of the regression residuals and calculating the R-squared of the model as 1 minus variance of the residuals divided by the variance of the fund return;
5. predict the fund return at month t;
6. in each month, calculate the variable "Fund Net Alpha" as the "Fund Net Return" minus the predicted value of fund return.

Stock Market Capitalization and AFMI Size

We use the global stock data in Global Databases of Morningstar Direct to do our analyses. We use "Exchange Country" in the database to define the stock of a particular market. Then we screen out the observations with a "Security Type" that is not equal to "Common Stock". For each month, the stock market capitalization is the sum of each stock's market capitalization in this month. All the market capitalization values are in U.S. dollar.

The "AFMI Size" variable in our analyses represents the active equity mutual fund industry size relative to stock market capitalization. For each month, "Industry Size" is equal to the sum of all the funds' "Fund Asset" in our "Active Equity Funds" sample, divided by the stock market capitalization.