Economic growth and financial instability

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Publication Date:
1997

DOI:
https://doi.org/10.26190/unsworks/8855

The University of New South Wales

School of Economics

Economic Growth And Financial Instability

A thesis submitted for the degree of

Doctor of Philosophy

by

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May 1997

Certificate of Originality

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of a university or other institute of higher learning, except where due acknowledgment is made in the text.

Acknowledgments

My primary thanks go to my wife, Patsy McDonnell, for her her love and support throughout this process, and for her forbearance in her role as a Thesis Widow for the past four years.

My supervisors Geoffrey Fishburn and Peter Kriesler deserve thanks for their guidance, friendship, and willingness to let me explore the issues developed in this thesis. Bill Junor, now of Macquarie University, played a pivotal role in this thesis by first introducing me to the ideas of Hyman Minsky. Bill Rao, the Director of PhD studies for the School of Economics, fulfilled his responsibilities well, and his personal support for my work is much appreciated.

I gratefully acknowledge the guidance I have received from the referees who have reviewed papers derived from this thesis for the *Journal of Post Keynesian Economy*, *Economies et Societes*, *Nonlinear Dynamics, Psychology and Life Sciences*, the *Economic Journal*, the *Review of Political Economy*, and the forthcoming *Encyclopaedic Dictionary of Political Economy*. Though most of these are unknown to me, five that are known are J. Barkley Rosser, Ted Winslow, the late Richard Goodwin, Marc Levoie, and the overall editor of the *Encyclopaedic Dictionary of Political Economy*, Phil O'Hara of Curtin University.

Thanks are due to many members of the Mathematics Department at the University of New South Wales, but especially to Bruce Henry. Bruce allowed me to attend his class on Chaos Theory, and gave excellent instruction in the foundations of differential equation analysis in his introductory course on Ordinary Differential Equations. His door was always open when I wished to discuss the application of these concepts to economics. Colin Rogers extended my understanding of differential equations in his Second Year Honours course. Elvin Moore gave me a firm footing in difference equations in his course on Discrete Dynamics. Mike Hirschhorn reintroduced me to Calculus and Algebra in his excellent First Year courses, and David Tacon extended my appreciation of Linear Algebra in Second Year.

Colleagues from the History of Economic Thought Society of Australia, and the Centre for the Study of the History of Economic Thought, are to be thanked for suggesting some of the issues covered in this thesis, for providing a venue in which parts of the analysis could be presented, and for their personal support for my work. Chief amongst these are Phil O'Hara, John Lodewijks, John Nightingale, and Peter Groenewegen.

I would like to thank Ross Milbourne in his capacity as the then Head of the School of Economics at the University of New South Wales, for supporting my research with a Supplementary Scholarship to the Australian Postgraduate Award.

I am pleased to acknowledge the support provided to me by the Australian Government via the award of a Priority Rate APA Scholarship.

Finally I must thank my mother, my family and friends for tolerating my distracted state for the past four years. I hope that they and all the others mentioned above find the product worthwhile.

In Memoriam

Richard C. Goodwin August 6th 1996

Hyman P. Minsky October 24th 1996

This thesis develops and integrates the ideas of Hyman Minsky and Richard Goodwin, and is based to some extent on suggestions made in Blatt (1983). All three were pioneers in the area of economic dynamics. Though Goodwin and Minsky did at one stage work together, they took different approaches to the relevance of money to economic analysis.[1] Blatt was a strong advocate of Goodwin's approach to economic modelling (1983: 204:216), and believed that Minsky's work on finance could be fruitfully used to extend the analysis of economic dynamics (1983: 161, 310-314). I hope that this attempt to integrate their analyses does justice to their memory.

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1 Personal correspondence from Richard Goodwin, 25.11.93

Abstract

Minsky's "Financial Instability Hypothesis" argues that an advanced capitalist economy with sophisticated financial institutions is prone to debt deflations, that this tendency can be contained by government counter-cyclical policies, and that the trade cycle is largely driven by debt-induced cycles in finance and their interaction with government regulatory behaviour. The evolution of this Hypothesis is detailed, from the initial influence of Kalecki's "Principle of Increasing Risk", through Fisher's concept of a debt deflation, to Minsky's eventual assimilation of Keynes's *General Theory*.

Minsky's failure to develop a mathematical model of the Financial Instability Hypothesis (FIH) is traced to his reliance on Hansen-Samuelson multiplier-accelerator models, which are shown to be unsound because of the confusion of intended investment with actual investment. A corrected third order linear difference equation model is derived which generates both cycles and growth, and which confirm many of Keynes's insights in the *General Theory*, including the primacy of capitalist "animal spirits" in determining the rate of growth, and the deleterious impact of raising the savings ratio on the rate of growth.

It is established that only a nonlinear mathematical model can capture the essence of the FIH. Goodwin's predator-prey model is

generalised to include a nonlinear investment function, variable capacity utilisation and endogenous technical change, as a prelude to introducing a finance sector into the model.

The introduction of a finance sector results in a system which confirms the Fisherian essence of the FIH, that a capitalist economy can undergo a debt deflation. Numerical simulations establish that only the simplest model with a low rate of interest does not undergo a debt deflation.

Minsky's claim that Big Government can contain this tendency towards deflation was examined by introducing a stylised government sector which increases taxes and decreases subsides during booms, and vice versa during slumps. In all versions of the model, this resulted in a system which, though cyclical, did not undergo a debt deflation. This superior performance of the mixed economy over a pure capitalist one is the primary justification for government intervention in the market system.

While the preceding single sector models can explore the concept of a debt deflation, they cannot capture the price dynamics of the FIH. A multi-sectoral model therefore appeared justified, but this ran counter to the analysis of the dominant school of thought in multisectoral modelling, the Sraffians. Steedman's argument that, in the long run, multi-dimensional dynamics gives the same results as multi-dimensional statics, is shown to be in general incorrect. His critique of Kaleckian markup pricing is also found to be based on an inadequate understanding of Kalecki. After these preliminaries, an initial multisectoral model of an economy with finance is developed, though not simulated.

Finally, given the cogency of the FIH as an explanation for the current state of modern capitalism, the current economic policy fetish for balanced budgets and the maintenance of low commodity price inflation at all costs are criticised. Given these misguided policies, the thesis concludes that the outlook for capitalism in the 21st century is relatively bleak.

There are three Appendices and one Glossary. Appendix A provides more detail on the derivation of various equations used in this thesis. Appendix B establishes that the two multi-sectoral price setting models of Chapter Seven are marginally unstable. Appendix C provides the text of proposals to enforce a balanced Budget in the United States, Europe, and Australia. The Glossary lists all terms used in the equations in this thesis, in Chapter order.

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1 Introduction

On Monday October 19th, 1987, the New York Stock Exchange NYSA Index fell by 19.17 per cent,[1] the largest one-day fall in the Exchange's history. This brought to an end a bull run which had seen share values more than double in a 42 month period, and the severity of The Crash led many to ask the question "Can 'It' Happen Again?"

"It" was the Great Depression, the most severe and prolonged downturn in the history of capitalism. In the popular consciousness, "It" was ushered in by the Wall Street Crash of 1929, when a now apparently modest 10 per cent had been shaved off the value of America's stocks in one day. The question itself is the title of an essay published over twenty years before the 1987 Crash by Hyman Minsky (1963; 1982: 3-13), in which he began the development of the *Financial Instability Hypothesis* (FIH).[2] This Hypothesis contends that a capitalist economy with sophisticated financial institutions is fundamentally unstable, and liable to fall into a depression induced by debt-financed over-investment during a period of speculative excess.

"It" did not recur after 1987, which was consistent with Minsky's conclusion that the fiscal and monetary institutions of the post-Depression era make the economy considerably more robust. These changes have not eliminated financial instability, however: according to Minsky, though a debt deflation is no longer feasible, endogenously generated financial disturbances will nonetheless shape the cyclical and long term development of the modern capitalist economy. The key to these disturbances is the level of corporate debt: the higher the corporate debt to output ratio, the more the economy is prone to downturns in economic activity. Minsky asserts that the trade cycle itself is largely driven by cycles in corporate reliance upon debt finance, and that the most severe downturns, Depressions, are the product of a deflationary process initiated by an excessive level of debt.

Minsky began the development of the Financial Instability Hypothesis in the 1960s and had largely completed it by the late 1970s, when he declared that it was, in his judgment, a far more legitimate expression of Keynes's analysis than the Keynesian-neoclassical synthesis of Hicks and Samuelson (Minsky 1977; 1982: 68-69). Yet during the same period, the Keynesian orthodoxy of the 1950s and 1960s had given way to a resurgent neoclassicism which saw little justification for government intervention in any markets, let alone the financial ones.

The global wave of deregulation which characterised the 1980s was in part ushered in by this confidence in the self-equilibrating properties of unfettered markets, and it unwittingly provided a fertile testing ground for Minsky's theory of systemic risk. By 1990, the major economies of the Western world, and in particular America and Japan, had experienced a stock market boom and bust, the development and collapse of a real estate bubble, a wave of bankruptcies, and the collapse of economic growth. Seven years later, Japan has still not escaped from the debt burden of the 1980s, while, somewhat the wiser after 1987, a resurgent America stands bemused but no longer besotted by the speculative antics of Wall Street.

It can be argued that the events of this "decade of greed" vindicated Minsky's perspective, but it has also been said that the experience of the 1980s was simply a temporary aberration of over-leveraging, which the markets themselves will correct (Miller, 1990, 1991: 104). This attitude derives from the mainstream analysis of the role of finance in a capitalist economy, which originated in the work of Sharpe (1975), Modigliani and Miller (1958, 1961). This body of theory, broadly characterised as the *Capital Assets Pricing Model* (CAPM), maintains that the manner in which a firm finances its investment has no impact on the performance of the firm, and by extension, the level of debt has no impact on the macroeconomic performance of the economy.

1.1 Objectives of this thesis

Minsky's analysis is clearly a challenge to the theoretical orthodoxy in finance. But in one sense this challenge is incomplete, since Minsky's FIH has to date lacked that most persuasive element in the lexicon of economics, a mathematical model. In contrast, the CAPM has a sophisticated static mathematical model (though at the microeconomic level only), and despite widespread dissatisfaction with CAPM amongst finance practitioners, it continues to hold sway as *the* mathematical theory of finance. The primary objective of this thesis is to bring some balance to this theoretical contest, by providing a dynamic mathematical rendition of Minsky's theory.

The secondary objective is to champion the development of dynamic economic models, in contrast to the profession's long history of reliance upon static analysis. The founders of modern economic analysis made this choice of method, not because it was appropriate, but because it was (relatively) easy. Jevons's explanation for this choice of method was one of immediacy:

> If we wished to have a complete solution ... we should have to treat it as a problem of dynamics. But it would surely be absurd to attempt the more difficult question when the more easy one is yet so imperfectly within our power. (Jevons 1871, 1911: 93)

Marshall's explanation was that the static method was a useful pedagogical precursor to the development of dynamical analysis, and static results a first approximation to the results of dynamics:

> The modern mathematician is familiar with the notion that dynamics includes statics. If he can solve a problem dynamically, he seldom cares to solve it statically also... But the statical solution has claims of its own. It is simpler than the dynamical; it may afford useful preparation and training for the more difficult dynamical solution; and it may be the first step towards a provisional and partial solution in problems so complex that a complete dynamical solution is beyond our attainment. (Marshall, 1907 in Groenewegen 1996: 432)

As Blatt puts it, such sentiments "did carry a great deal of conviction ... when the basic ideas of the science of economics were being formulated for the first time" (Blatt 1983: 5),[3] but even by Marshall's day, this excuse for employing static in preference to dynamic methodology was wearing thin. At the end of the 20th century, this excuse is inexcusable, yet just a decade ago

Blatt could dryly and rightly observe that "capitalism as a social system may disappear before its dynamics are understood by economists" (1983: 5).

The situation is somewhat improved today. Though the vast majority of economists are still practitioners of statics, there are now numerous economists working to develop dynamic models of various aspects of capitalism, and this thesis is a contribution to that endeavour in the area of finance. As is often the case, its findings contradict those derived from static methodology: whereas CAPM denies the possibility of a debt deflation (unless it is precipitated by mistakes made by Central Bankers)[4], the dynamic model in this paper suggests that, in the absence of a government sector, debt deflations are an inevitable symptom of the debt financing of investment. This conflict between the conclusions of static and dynamic methodology is commonplace, and gives the lie to Marshall's hope that static reasoning would be "the first step" towards a dynamic understanding of the issue. This is because, to quote Blatt again, the economy appears to belong to that class of systems in which

> the important and interesting features of the system are "essentially dynamic", in the sense that they are not just small perturbations around some equilibrium state, perturbations which can be understood by starting from a study of the equilibrium state and tacking on the dynamics as an afterthought. If it should be true that a competitive market system is of that kind...[then] No progress can then be made by continuing along the road that economists have been following for two hundred years. The study

> of economic equilibrium is then little more than a waste of time.
>
> (Blatt 1983: 5-6)

I therefore make no apology for the fact that the tools used in this thesis—systems of nonlinear ordinary differential equations and difference equations, eigenvalue stability analysis, and numerical simulations—may be rather unfamiliar to many economists. These are the stuff of dynamics, and if economics is ever to develop beyond its historic but misguided reliance upon static analysis, they must become the stuff of economics.

1.2 Choice of method

The main dynamic method used in this thesis is numerical simulation: nonlinear differential equation models of a capitalist economy with finance and government are derived, simulated and analysed graphically. Many other techniques of dynamical analysis could have been applied to these models, but for two reasons they have not been.

Firstly, while the basic two equation Goodwin model (on which my model of the FIH is based) is a conservative and analytically soluble system, the addition of an equation for the rate of change of the debt ratio makes the system dissipative, analytically insoluble, and capable of displaying chaotic behaviour. The behaviour of my model of the FIH can therefore only be analysed by numerical means. As Costanza (1993: 33) emphasises, the vast majority of mathematical functions fall into this category:

Table 3.1 The limits of analytical methods in solving mathematical problems (from von Bertalanffy, 1968). The thick solid line divides the range of problems that are solvable with analytical methods from those that are very difficult or impossible using analytical methods and require numerical methods and computers to solve. Almost all 'systems' problems fall in the range that requires numerical methods

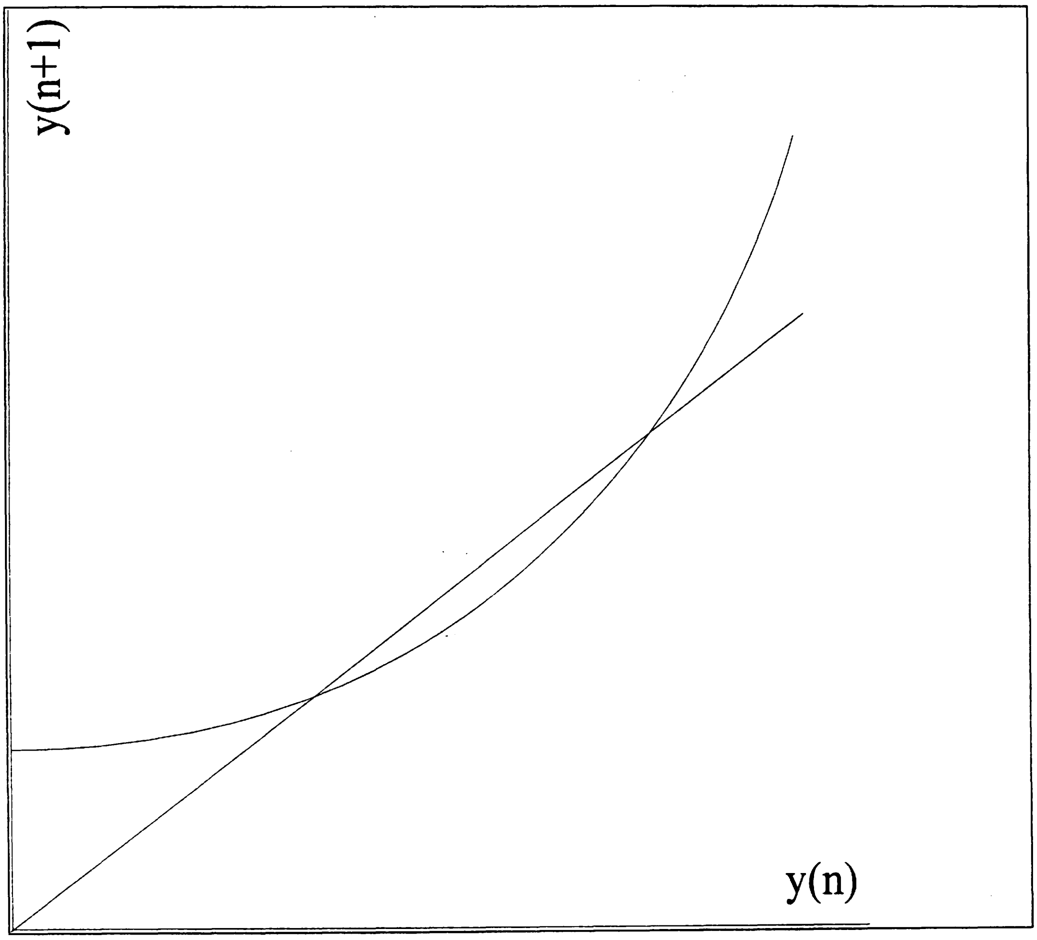
	Linear			Non-linear		
Equations	One equation	Several equations	Many equations	One equation	Several equations	Many equations
Algebraic	trivial	easy	essentially impossible	very difficult	very difficult	impossible
Ordinary Differential	easy	difficult	essentially impossible	very difficult	impossible	impossible
Partial Differential	difficult	essentially impossible	impossible	impossible	impossible	impossible

The models in this thesis are systems of several to many nonlinear ordinary differential equations, and are therefore impossible to solve analytically.

Secondly, some systems of nonlinear differential equations can be partly characterised by analysis of their stability properties about points of equilibrium, or by the calculation of Lyapunov exponents and similar measures of chaotic behaviour. However, again for two reasons, these techniques have not been applied here.

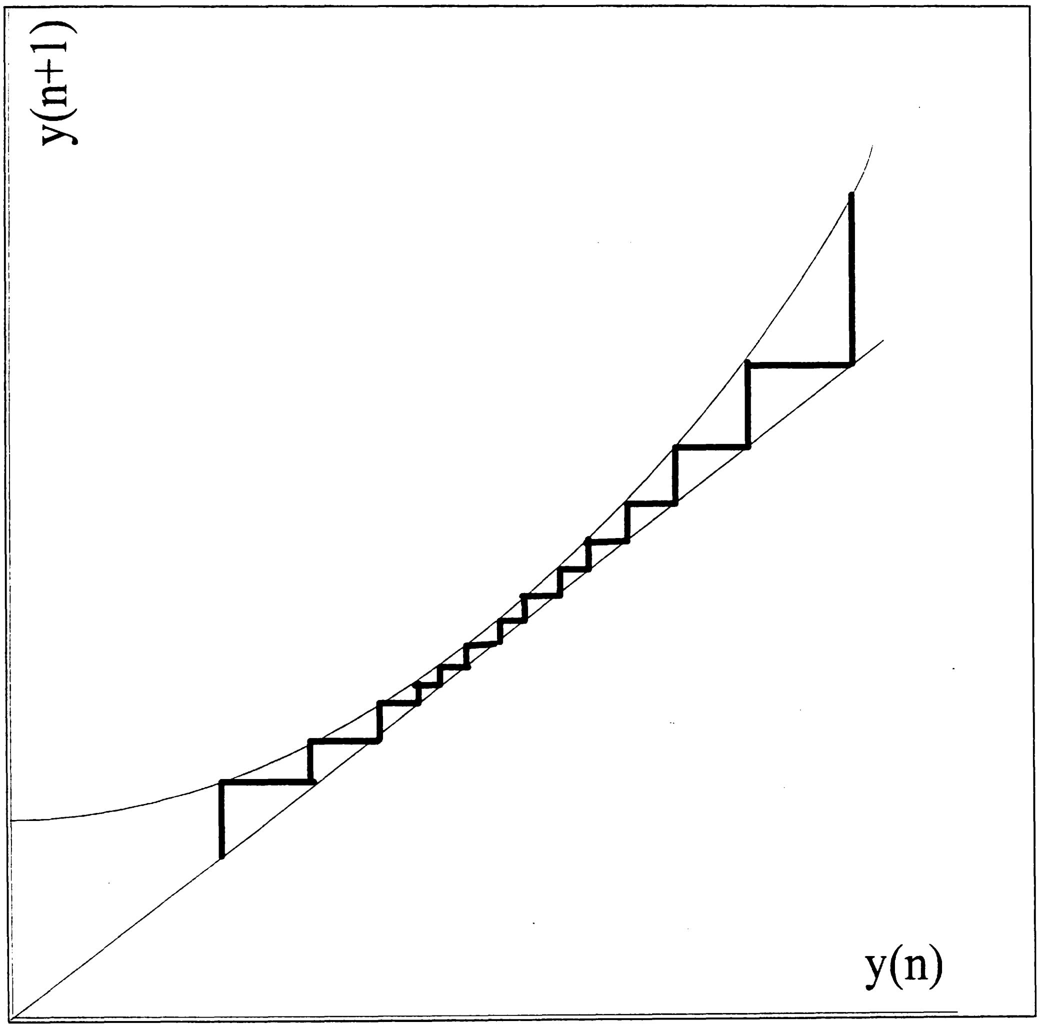
Firstly, the base model of this thesis belongs to a class known as "inverse tangent" chaotic models, which was first identified in the study of the transition from laminar to turbulent flow in fluids (Pomeau and Manneville 1980). A peculiarity of this class of models is that an equilibrium exists and is a stable attractor for some parameter values, but when parameter values which generate chaos are used, no equilibrium exists. Pomeau and Manneville illustrate this with a stylised system $y_{n+1} = f(y_n, r)$, where *f()* is a nonlinear function and *r* a key parameter of that function (see Figure 1). The dynamic process stops when $y_{n-1}=y_n$, which obviously occurs when the line in Figure 1 crosses the curve:

Figure 1 The Intermittent Route to Chaos (Pomeau & Manneville 1980: 191)



As the parameter r is changed, the location of the curve $f(y_n,r)$ also changes, until eventually the curve and the line no longer intersect. The dynamic process will thus start with large steps and appear to converge, only to subsequently start increasing again as it moves through the "corridor" between the line and the function. This process can be represented as follows (Figure 3 in Pomeau and Manneville 1980):

Figure 2 The Intermittent Route to Chaos (Pomeau & Manneville 1980: 192)



Viewing each iteration from y_n to y_{n-1} as a cycle, the cycles initially diminish, but eventually begin to increase in size. In an actual model (such as Lorenz's model of two-dimensional fluid flow under the influence of a heat differential), the cyclical behaviour in the corridor is, for some parameter values, characterised by the irregular and apparently random cycles which are the hallmark of chaotic systems (Pomeau & Manneville 1980: 190).

Analysis of the properties of the system in the vicinity of an equilibrium point is thus rather pointless in this class of models, since their behaviour is more properly characterised by the existence or non-existence of an equilibrium. If the parameter values are such that an equilibrium exists, then it will be stable; if they are such that an equilibrium does not exist, then the system will be unstable, and may or may not display chaotic behaviour in the transition from the converging to the diverging state. Both the equilibrium and nonequilibrium states of such models are equally valid, and have manifestations in physical systems, such as the models of fluid flow which first re-ignited interest in nonlinear analysis. The model in this thesis extend the application of this class of models to the social phenomenon of debt-deflation, confirming Fisher's and Minsky's intuitions that events such as the Great Depression and the Stock Market Crash of 1987 are fundamentally nonequilibrium phenomena.

Secondly, numerical simulation of the two ultimate models in this thesis (respectively, a pure market economy and a mixed economy with endogenous technical change and variable capacity utilisation) clearly indicate that they display far from equilibrium behaviour for a wide range of parameter values. Lyapunov exponents and other diagnostic techniques would have confirmed these numerical findings. However for readers unfamiliar with these techniques, the application and discussion of these methods would have detracted from understanding the economic insights behind these dynamical models. I have instead chosen to focus upon the economic interpretation of the models, via discussion of the equations and graphical analysis of their behaviour.

1.3 Structure of this thesis

For Minsky, as for Keynes before him, the development of his analysis was "a long process of escape from habitual modes of thought and expression" (Keynes 1936: viii). Chapter One tracks this process, which started with Kalecki and Fisher and concluded with Minsky's assimilation of Keynes's original writings, and rejection of the "neoclassical synthesis" interpretation of Keynes.

Chapter Two outlines Minsky's original unsuccessful attempts to produce a mathematical portrait of finance-driven endogenous cycles, using the then common tool of the multiplier-accelerator model. The effective failure of this attempt is traced to the unsoundness of the tool itself, as it is shown that multiplier-accelerator models are based on a simple economic mistake. In the process, a linear model of endogenous growth with cycles is developed from a combination of Keynes's perspective on investment, Harrod's growth model, and Hicks's lags. The practice of divorcing models of cyclical behaviour from models of growth is criticised.

Chapter Three outlines the case that, with the sole exception of the model developed in Chapter Two, models of endogenous cyclical growth must be nonlinear. Goodwin's 1967 predator-prey model of the economy is outlined, and three extensions are made to the model: a nonlinear investment function replaces Goodwin's linear relation; a variable level of capacity utilisation replaces the fixed accelerator relation between capital stock and output; and a combination of endogenous and exogenous technical change replaces the assumption of constant technical progress. These alterations make the model more realistic, and convert its behaviour from that of a closed curve in phase space to an open cycle with an unstable equilibrium. The primary purpose behind these generalisations of Goodwin's model is to convert it into a form in which the Fisherian essence of Minsky's Hypothesis can be explored: the argument that a capitalist economy with finance can undergo a debt deflation.

The dynamics of debt deflation are explored in Chapter Four, which adds a banking sector to Goodwin's model. The impact of a finance sector is evaluated with respect to four permutations of the Goodwin model (all of which are augmented with a nonlinear investment function): the basic model with a constant rate of interest; the basic model with a variable rate of interest; and the general model, firstly with a constant, and then with a variable rate of interest. It is shown that in the basic model, the occurrence of a debt deflation is dependent upon the rate of interest. However in the general model, a debt deflation is inevitable at any rate of interest greater than zero. This is the first of the two key findings of this thesis: that in the absence of price dynamics (which can only be properly considered in a multi-commodity model), a capitalist economy without a government sector can, and perhaps must, periodically experience a debt deflation.

Chapter Five completes the single commodity model of the FIH with the introduction of a welfare state which follows "Minskian" guidelines. The behaviour of this model confirms Minsky's argument that a debt-deflation is impossible in an economy with a large government sector which practices countercyclical policies. This chapter introduces the welfare state into the basic and the general versions of the model developed in Chapter Four, and in both cases it is clear that the government sector prevents the occurrence of a debt deflation, regardless of the rate of interest. This is the second key finding of this thesis: the presence of a government sector converts a model in which a debt deflation was inevitable, into one in which a debt deflation is impossible. This is a far stronger case for the mixed economy than is made by conventional "Keynesian" macroeconomics, in which the government's role is seen to be simply to reduce the size of the trade cycle's perturbations.

As noted above, the conclusion that capitalism is inherently prone to debt deflations is conditional only, since price dynamics are not explored in the single commodity models of Chapters Four and Five. Minsky argues that price movements can defeat this tendency towards Depression, but in a fashion which completely contradicts the conventional economic analysis of the price system. In Minsky's model, there are two price levels, one for normal commodities and another for capital assets (and capital equipment). A high rate of commodity price inflation can prevent a Depression, by enabling the debts accumulated during a boom to be repaid during a slump as the commodity price level rises and the asset price level falls. On the other hand, a low rate of commodity inflation can prolong a downturn while falling prices are part of the causal mechanism behind a true Depression. Contrary to conventional economic analysis therefore, inflation is not always and everywhere a "bad thing". This aspect of Minsky's theory cannot be captured in a single commodity model, and requires the construction of a multi-commodity dynamic model of capitalism.

To do justice to Minsky's analysis, such a multi-commodity model must be based upon the Kaleckian theory of markup pricing. This theory has been largely ignored by mainstream economists, and has been criticised by members of the Sraffian school of economics, who have also argued that multisectoral analysis does not require dynamics. Chapter Six considers these arguments, and establishes that the Sraffian critique of Kaleckian markup pricing is invalid. It is also shown that multisectoral dynamic analysis contradicts the conclusions of multisectoral statics, and that once the nonlinear nature of the markup setting process is taken into account, the static equilibrium price vector may not even be an attractor of a multisectoral dynamic model.

Chapter Seven builds on the foundations of Chapter Six to provide an initial multisectoral dynamic circulating capital model of an economy with finance. The mathematical complexity of this conceptually simple economy is such that, to date, it has not proven possible to simulate it. Nonetheless, the derivation of the model enables two interesting insights which confirm the Post Keynesian perspective on the endogeneity of the money supply, and support the assertion that multi-sectoral dynamic analysis necessarily results in nonlinear models, even when behavioural functions are assumed to be linear.

Chapter Eight concludes the thesis, and includes critical observations on the current fad for balanced budgets and the maintenance of low inflation.

There are three Appendices, and a Glossary. Appendix A provides more detail on the derivation of various equations used in this thesis. Appendix B establishes that the two multi-sectoral price setting models of Chapter Seven are marginally unstable. Appendix C contains extracts from European, American and Australian documents which show the extent to which current government policy is driven by a fetish for zero public sector deficits. The Glossary lists all the terms used in the equations of this thesis, on a Chapter by Chapter basis.

2 The Intellectual Foundations of the Financial Instability Hypothesis

There are five key pillars on which Minsky constructed the Financial Instability Hypothesis: Kalecki's principle of increasing risk (Kalecki 1937); Fisher's concept of a debt deflation (Fisher 1933); Keynes's numerous contributions on uncertainty, capitalist expectations, the finance demand for money, and price levels in capitalism (Keynes 1936, 1937a, 1937b); the Post Keynesian position that the money supply is endogenous; and Minsky's exhaustive life-long study of the actual mechanics of finance. Though retrospectively Minsky has portrayed the FIH as an interpretation of Keynes (Minsky 1975; 1977, 1982: 59-70), in fact Keynes was the last of these influences to be assimilated.

The first was Kalecki, whose "Principle of Increasing Risk" provided a means by which Minsky provided a finance-induced turning point to the upward movement of a multiplier-accelerator model of the trade cycle (Minsky 1957a ,1982: 243).

2.1 Kalecki and Increasing Risk

Minsky's model was the typical Hansen-Samuelson construct:

(1) $$Y_t = (\alpha + \beta) \cdot Y_{t-1} - \beta \cdot Y_{t-2}$$

where Y is output, α the average propensity to consume, and β the desired incremental stock to output ratio (or the marginal propensity to invest). The accelerator coefficient β was significantly greater than one, which generated a monotonic and extremely rapid increase in the magnitude of Y. Minsky argued that this mechanism could be used to explain the trade cycle if one posited "that the value of β , the accelerator coefficient, depends on money-market conditions and the balance sheets of firms" (Minsky 1957, 1982: 233). Minsky found the mechanism he needed in Kalecki's argument that the size of an investment a firm would undertake was limited by increasing risk as the proportion of debt financing rose (Kalecki 1937, 1990: 285-293).

Kalecki considered an entrepreneur with capital k to invest, a given method of production, an expected stream of returns and hence an expected internal rate of return ε . These could be combined to yield the prospective gross profit $\pi = k \cdot \varepsilon$. To work out the prospective net gain g, the entrepreneur must deduct both the market rate of return ρ , and an allowance for the riskiness of the project, σ. This results in the formula

(2) $g = \pi - (\rho + \sigma) \cdot k$[5]

Under the conventional assumption that expected gain is a function of capital employed and has a single maximum turning point, the second differential of g with respect to k is negative. The zero of the first differential therefore tells us the optimal amount of capital to invest, so that the condition for the maximum prospective gain is:

(3) $\frac{d\pi}{dk} = \rho + \sigma + k \cdot \left(\frac{d\rho}{dk} + \frac{d\sigma}{dk} \right)$ [6]

The conventional proposition is that expected profit is also an increasing but diminishing function of capital employed, while there is no relationship between capital and the interest rate or capital and risk, so that $\frac{d\rho}{dk} = \frac{d\sigma}{dk} = 0$. Thus, since $\frac{d\pi}{dk} > 0$, profit diminishes as a direct consequence of increasing capital until such time as $\frac{d\pi}{dk} = \rho + \sigma$. The argument goes that the amount of investment is constrained either by diseconomies of scale as k rises, or by imperfect competition (or similar real barriers to expansion).

Kalecki rejects this, arguing against the former that "True, every machine has an optimum size, but why not have 10 (or more) machines of this type?", and against the latter that it is not a relevant objection when an investor can put his funds into a portfolio of projects across a range of industries. $\frac{d\pi}{dk}$ is therefore a constant, and there is no effective real (non-monetary) barrier to the optimal size of k for a single investor (Kalecki 1937: 286-287). Instead Kalecki argues that to expand k, the entrepreneur must borrow, and the more he borrows the greater is his risk σ. If the entrepreneur is not cautious, then "it is the creditor who imposes on his calculation the burden of increasing risk, charging the successive portions of credits above a certain amount with a rising rate of interest" (1937: 288). Thus either $\frac{d\sigma}{dk} > 0$ or $\frac{d\rho}{dk} > 0$, so that it not declining profitability but increasing risk which restrains the size of an individual capitalist's investment.

Minsky employed this mechanism (at the macro level) to explain a decline in β —and hence a change in the character of the model from monotonic explosive to cyclical. As an expansion occurs in a model with an infinitely elastic money supply, firms add bank debt to their portfolios, and the debt to equity ratio rises. This leads to rising borrowers' risk, which attenuates the desire of firms to invest, thus reducing the value of β "which in turn lowers the rate of increase of income. This continues until the accelerator coefficient falls sufficiently to replace the explosive by a cyclical time series, in which there eventually occurs a fall in income." (Minsky 1957a, 1982: 243)

At this stage, Minsky's approach to modelling the trade cycle was already distinguished by its integration of financial and real forces, but it was still a long way removed from the Financial Instability Hypothesis. Two illustrations of this are his analysis of the cyclical characteristics of corporate debt, and the manner in which Minsky analysed the impact of bankruptcy in this model.

Minsky presumed that, once the upward turning point had been encountered, corporate debt would then start to fall at the same time that income was falling, as "the excess of ex ante saving over induced investment will be utilised to reduce bank debt" (Minsky 1957a, 1982: 243). The cumulative effects of bankruptcy were also ignored in the statement that "the failure of some firms which have relied heavily upon debt financing will result in the substitution of equity for debt in balance sheets." The action of debt on investment and profits was thus seen as entirely symmetrical:

> The endogenous limits to an explosive accelerator process, in the absence of restrictions on the money supply, are the deterioration of firms' balance sheets due to debt-financing of investment on the upswing; and the reverse circumstances during the liquidation process on the downswing. (Minsky 1957a, 1982: 243-244)

Clearly, Minsky was not then cognisant of the proposition that an overaccumulation of debt could occur and lead to the decidedly asymmetrical experience of a debt-deflation. Thus while this early model was capable of demonstrating a trade cycle in which finance played an integral part, it was not a model of the Financial Instability Hypothesis. However, as Wray (1992: 162) points out, that same year Minsky published a paper (1957b, 1982: 162-178) which began the development of his theory of endogenous money, with its rich appreciation of the structure of the banking system. Here Minsky first raises the possibility that insolvency for even a single large corporation can set in train a "chain reaction" which affects "the solvency or liquidity of many organisations" (Minsky 1957a, 1982: 173), and argues that the evolutionary changes in banking can increase "the vulnerability of money-market assets to a fall in value" (1957a, 1982: 174). The former issue presaged the issue of debt deflation, on which Fisher had made the seminal contribution to date; the latter was something which would strike a chord with Minsky when he later turned his attention to the disputed intellectual legacy of John Maynard Keynes.

2.2 Fisher and Debt Deflation

In 1930, Fisher published The Theory of Interest, which set out the neoclassical argument that the interest rate "expresses a price in the exchange between present and future goods" (Fisher 1930: 61). This encompasses the subjective preferences of each individual for present goods over future goods on the one hand, the objective possibilities for profitable investment on the other, and their reconciliation via a market mechanism for loanable funds. This argument is an extension of the conventional neoclassical treatment of the equilibrium price and quantity of a commodity, with time preference equating to subjective marginal utility, and investment opportunity equating to marginal cost.

From the subjective perspective, the lender of money is one who has a low time preference for present over future goods, and the act of borrowing is thus a means by which those with a high preference for present goods acquire the funds they need now, at the expense of later income. These preferences themselves depend in part upon the income flow that an individual anticipates, so that a wealthy individual, or someone who expects income to decline in the future, is likely to be a lender, whereas a poor individual, or one who expects income to rise in the future, is likely to be a borrower (Fisher 1930: 71-80).

From the objective perspective, the rate of interest reflects the marginal productivity of investment or "marginal return over cost" (1930: 182). The market mechanism brings these two forces into harmony, so that, Fisher muses, if "the rate of return over cost is fixed immutably at 10 per cent, the rates of impatience must conform thereto and the rate of interest can only be 10 per cent" (1930: 182) . This coincidence of time preference and marginal rate of return depends upon a market mechanism which is assumed to achieve two outcomes: "(A) The market must be cleared—and cleared with respect to every interval of time. (B) The debts must be paid" (1930: 495).

There is no little irony in the fact that this argument was published in the first full year of the Great Depression. To his credit, Fisher's response was worthy of Keynes's apocryphal statement that "when the facts prove me wrong, I change my mind". Inspired by the pressing circumstance of the time, Fisher abandoned the static reasoning of his previous treatise to consider the dynamic forces which could have caused the Great Depression, countenancing the case he previously dismissed, that debts can fail to be repaid.

He ventured the opinion that the "two dominant factors" which cause depressions are "over-indebtedness to start with and deflation following soon after". Though other factors are important, the combination of these factors with debt—the entry into a contractual obligation to repay principal with interest—and price level disturbances is crucial:

> Thus over-investment and over-speculation are often important; but they would have far less serious results were they not conducted with borrowed money. That is, over-indebtedness may lend importance to over-investment or to over-speculation. The same is true as to over-confidence. I fancy that over-confidence seldom does any great harm except when, as, and if, it beguiles its victims into debt. (Fisher 1933: 341)

This insight of Fisher's can be related to Kalecki's concept of increasing risk. Kalecki's analysis considered the level of capital which generated the optimum gain g from invested capital k against a background of rising risk σ (see equation 2); here Fisher argues that overconfidence may lead investors to overestimate the prospective gain from investment, or to underestimate the risks, and thus commit themselves to an unsustainable level of debt, so that in either case the investor commits funds well beyond the level which returns an optimum gain. Such overconfidence is an inevitability in the real world because, while the static model of 1930 is always in equilibrium, any real world equilibrium would be short-lived since "New disturbances are, humanly speaking, sure to occur, so that, in actual fact, any variable is almost always above or below the ideal equilibrium" (1933: 339). When that overconfidence leads to over-indebtedness, the following chain of events will flow from it:

> (1) Debt liquidation leads to distress selling and to (2) Contraction of deposit currency, as bank loans are paid off, and to a slowing down of velocity of circulation. This contraction of deposits and of their velocity, precipitated by distress selling, causes (3) A fall in the level of prices, in other words, a swelling of the dollar. Assuming, as above stated, that this fall of prices is not interfered with by reflation or otherwise, there must be (4) A still greater fall in the net worths of business, precipitating bankruptcies and (5) A like fall in profits, which in a "capitalistic," that is, a private-profit society, leads the concerns which are running at a loss to make (6) A reduction in output, in trade and in employment of labor. These losses, bankruptcies, and unemployment, lead to (7) Pessimism and loss of confidence, which in turn lead to (8) Hoarding and slowing down still more the velocity of circulation. The above eight changes cause (9) Complicated disturbances in the rates of interest, in particular, a fall in the nominal, or money, rates and a rise in the real, or commodity, rates of interest." (1933: 342)

Underlying this litany of a debt deflation was the perception that the deflation was endogenously generated. This perspective on economic disequilibrium differed substantially from the exogenous shock perspective of Frisch's paper of the same year, which came to dominate the practice of economic dynamics.[7] Whereas Frisch focused on external shocks as the cause of cycles, Fisher argued that the important cause of cycles was "not forced from outside, but self-generating" (1933: 338); whereas to Frisch the underlying economic mechanism was stable, to Fisher it could have an equilibrium which, "though stable, is so delicately poised that, after departure from it beyond certain limits, instability ensues" (1933: 339). While any of a multitude of factors can, according to Fisher, push the system away from equilibrium, the crucial ingredient needed to turn this limited instability into a catastrophic collapse into Depression is an excessive level of debt, where "the breaking of many debtors constitutes a 'crash', after which there is no coming back to the original equilibrium" (1933: 339).

Before this paper, Fisher had been a major contributor to neoclassical static equilibrium reasoning. By arguing that the economy is always in a non-equilibrium state, and that the equilibrium itself may be unstable, so that divergences from equilibrium are not self-correcting, Fisher provided a major challenge to the orthodoxy he had helped develop. This challenge remained largely unmet and ignored by orthodox and critic alike until 1963, when Minsky attempted to explain the 25% fall in the Dow Jones Index which occurred in the hundred days from mid-March till mid-June 1962. Though Minsky's paper clearly bears the stamp of Fisher's analysis, it is substantially enriched by Minsky's appreciation of the macroeconomic balances which are needed to sustain a process of debt accumulation during a boom.

Minsky commences with the basic Keynesian ex-post identity for a closed economy that

$$(S - I) + (T - G) \equiv 0 \tag{4}$$

(where S is savings, I investment, T taxation and G government spending) and observes that "If income is to grow, the financial markets ... must generate an aggregate demand that, aside from brief intervals, is ever rising" (Minsky 1963, 1982: 6). Thus for growth to occur in the context of a zero sum for ex-post expenditure and a generally stable or increasing price level, debt must be issued, since

> it is necessary that current spending plans, summed over all sectors, be greater than current received income and that some market technique exist by which aggregate spending in excess of aggregate anticipated income can be financed. It follows that over a period during which economic growth takes place, at least some sectors finance a part of their spending by emitting debt or selling assets. (1963, 1982: 6)

Similarly, as increasing output requires an increase in debt, the increase in debt requires "the creation of new money" (1963, 1982: 6). A tight link is thus welded between the phenomena of economic growth, debt, and endogenous money.

Since growth at the macroeconomic level requires an increase in debt, at the microeconomic level some entities will experience a rise in their debt to income ratio. Without at this stage having an explanation for how a rational entity might get itself in this situation, Minsky observes that as the debt to income ratio rises, a stage can be reached at which a small decline in income can "make it difficult, if not impossible, for the unit to meet the payment commitments stated on its debt" (1963, 1982: 6). In contrast to the 1957 paper, there is now an explicit appreciation of the asymmetrical impact of debt:

> However, for any unit, capital losses and gains are not symmetrical: there is a ceiling to the capital losses a unit can take and still fulfil its commitments. Any loss beyond this limit is passed on to its creditors by way of default or refinancing of the contracts. Such induced capital losses result in a further contraction of consumption and investment beyond that due to the initiating decline in income. This can result in a recursive debt-deflation process. (1963, 1982: 6-7)

At this point in the development of his hypothesis, a debt-induced deflation is a possibility, but not yet a cyclical inevitability. This argument is developed in Minsky (1964), where he argues that the financial structure of the economy—defined as "the assets owned and liabilities emitted by ultimate units, financial intermediaries and governments" (Minsky 1964: 326)—evolves in response to profit opportunities in a manner which leads to increasing fragility. For this case to be made, Minsky needed to explain how the expectations of profit-motivated entities would inevitably induce behaviour which pushed the overall economy from a position of financial security and tranquillity into one of crisis. He found the elements of such an explanation in Keynes, but only after he had discarded the conventional IS-LM interpretation of the General Theory.

2.3 Keynes: Hamlet with the Prince

The conventional interpretation of Keynes emanated from John Hicks's 1937 review of the *General Theory*, "Mr Keynes and the Classics". In this paper, Hicks argued that the Keynes's "special" analysis could be encapsulated by the three equations

$$\begin{aligned} &M = L(i) \\ (5)\quad &I_x = C(i) \\ &I_x = S(I) \end{aligned}$$

(where M is the exogenous money supply, I income, i the rate of interest, L is the demand for liquidity function, C the investment function, and S the savings function). It is clearly this rendition of Keynes that Minsky has in mind in his second attempt to generate a model of cyclical growth with finance, when he says that

> If we make the Keynesian assumption that consumption demand is independent of interest rates, but assume that investment demand, and hence the β coefficient, depends on interest rates, then a rising set of interest rates will lower the β coefficient. (Minsky 1965, 1982: 262)

No mention is made of expectations in this interpretation of Keynes, and as Minsky was later to remark that "Keynes without uncertainty is something like Hamlet without the Prince" (1975: 57), it appears that at this stage in his intellectual development he had not yet acquainted himself with the original.[8] Keynes qua Keynes is first cited by Minsky in his comment upon Friedman and Schwartz's argument that business cycles are caused by exogenous changes in the rate of change of the money supply (Minsky 1963: 68). However the reference itself is a cosmetic one, portraying Keynes as merely qualifying orthodox economics, rather than challenging it. At this stage, Fisher's debt-deflation hypothesis remains the dominant critical influence on Minsky's thinking. His perception of Keynes was still clearly a secondhand one when in late 1967/early 1968 he argued that "the successful application of Keynesian policy may result in an economy that is inherently unstable" (Minsky 1968: 331).

The realisation that orthodoxy was based, not on an accurate rendition of Keynes, but on a particularly questionable interpretation, first occurred to Minsky in 1968 when, for the first time, Keynes's 1937 paper "The General Theory of Employment" appears in Minsky's list of references (Minsky 1969a, 1982: 190). In this paper, "Alternative theories of the rate of interest" (Keynes 1937b), and Chapter 17 of the *General Theory*, Keynes developed a perspective on the motive for investment which differs radically from the "Marginal Efficiency of Capital" argument which became the basis of the conventional analysis of investment. There are three key facets to Keynes's distinctly expectations-based explanation of investment in these three works: a dual price level; a volatile basis for the formation of expectations, which determines the desire to invest; and a finance-based demand for money,[9] in addition to the traditional triad of transactions, precautionary and speculative demand.

In Chapter 17, Keynes argued that investment is motivated by the desire to produce "those assets of which the normal supply-price is less than the demand price" (Keynes 1936: 228), where the demand price was determined by the influences of prospective yields, depreciation and liquidity preference. This insight was further refined in "The General Theory of Employment", where Keynes talks of the progress towards equilibrium between different prospective investments leading to "shifts in the money-prices of capital assets relative to the prices of money-loans." The concept of two price levels and the focus on capital appreciation as the motive for investment are even more evident in the observation that the scale of production of capital assets "depends, of course, on the relation between their costs of production and the prices which they are expected to realise in the market." (Keynes 1937a: 217.)

Keynes' discussion of uncertainty in this article is allied to an increased use of the concept of asset prices, and a much diminished status for the marginal efficiency of capital. Keynes associates the latter with the view that uncertainty can be reduced "to the same calculable status as that of certainty itself" via a "Benthamite calculus", whereas the kind of uncertainty that matters in investment is that about which "there is no scientific basis on which to form any calculable probability whatever. We simply do not know" (Keynes 1937a: 213, 214). Keynes argues that in the midst of this incalculable uncertainty, investors form fragile expectations about the future, which are crystallised in the prices they place upon capital assets, and these prices are therefore subject to sudden and violent change—with equally sudden and violent consequences for the propensity to invest. Seen in this light, the marginal efficiency of capital is simply the ratio of the yield from an asset to its current demand price. There is therefore a different "marginal efficiency of capital" for every different state of capitalist expectations and its consequent level of asset prices (1937a: 222).

Keynes' explanation for the formation of expectations under true uncertainty had three components: a convention shared by investors that "the present is a much more serviceable guide to the future than a candid examination of past experience would show it to have been hitherto"; the belief that "the existing state of opinion as expressed in prices and the character of existing output is based on a correct summing up of future prospects"; and a reliance on mass sentiment: "we endeavour to fall back on the judgment of the rest of the world which is perhaps better informed" (Keynes 1936: 214). The fundamental effect of shifts in expectations is to change the importance attributed to liquidity, thus shifting the apportionment of funds between assets embodying varying degrees of liquidity, with volatile consequences for the level and composition of investment.

Keynes strengthens this increasingly financial focus with the observation that there exists a finance demand for money, which must be exercised and fulfilled before investment is undertaken. Having neglected this concept in the General Theory, he argues here that "it is, to an important extent, the 'financial' facilities which regulate the pace of new investment". It is therefore not a lack of savings which inhibits investment, but a lack of finance consequent upon "too great a press of uncompleted investment" (Keynes 1937b: 247).

Though these observations are potent, they are not as systematic as those in the *General Theory* on the marginal efficiency of capital, let alone as structured as the model in "Mr Keynes and the Classics" (Hicks 1937), which led to IS-LM analysis with its static treatment of expectations (Hicks 1982). Since Keynes adhered to the concept of diminishing returns in the short run, it was difficult for him to explain how the two price levels could diverge. There was no explanation for the state of expectations at any given time, nor for why shifts might occur; nor was there any integration of the question of expectations with the question of supply of finance, and the notion of an endogenously variable supply of finance sits uneasily with the exogenous view of the supply of money presented in the *General Theory*. It is therefore little wonder that these insights were not developed in the conventional literature. However, with his foundations in Kalecki's theory of investment under uncertainty and Fisher's picture of a debt deflation driven by the collapse of asset prices, Minsky readily assimilated and improved upon Keynes's vision.

Uncertainty then entered the cast of Minsky's capitalist drama, with its first role being as a counterpoint to the fragility of business confidence, under the direction of the central bank. Minsky argued that a central bank emphasis upon "orderly conditions in financial markets" and the function of a lender of last resort will "act upon 'confidence' and thus uncertainty", increasing the former and decreasing the latter. However, deluded both by economic theory which misunderstood the significance of uncertainty and by the passage of time since the Great Depression, central banks had instead come to believe that "active monetary and fiscal policy [can be] used to 'fine tune' the economy... As a result monetary policy is being carried out without the constraints ... that might result from prospects of serious business depressions." This altered behaviour of central banks meant that "instead of acting as an insurer (substituting certainty for uncertainty) central banking has taken on some aspects of a casino (substituting uncertainty for certainty)" (1982: 180).[10]

Uncertainty's second and more crucial role—from the point of view of the development of the Financial Instability Hypothesis—involves a linkage with business confidence and asset management behaviour over the course of the trade cycle. Minsky argues that "A protracted period of rising prosperity ... breeds a view in ordinary business corporations and financial institutions which allows them to raise their short-term payment commitments as a ratio, for example, to their expected cash flows from operations" (1982: 186-187). This behaviour is predicated on the belief that, should some short-term refinancing or asset sales be required, they can be undertaken at little cost. Yet this behaviour breeds the very conditions in which these costs are likely to be large.

By 1969, Minsky had firmed in the belief that his perspective and that of Keynes were consistent visions of capitalism. He christened his analysis "unreconstructed Keynesian" (Minsky 1969b: 224), and it was in essence that

> capitalism is inherently flawed, being prone to booms, crises and depressions. This instability, in my view, is due to characteristics the financial system must posses if it is to be consistent with full-blown capitalism. Such a financial system will be capable of both generating signals that induce an accelerating

desire to invest and of financing that accelerating investment. (224)[11]

This paper began the integration of Keynes's perspectives—on uncertainty, the formation of capitalist expectations and the motives for investment, the finance demand for money, and the dual price structure of capitalism—with Fisher's debt-deflation analysis and Kalecki's analysis of the micro aspects of the financing of investment. Two additional factors were needed to provide a foundation for Minsky's model—a theory of prices which allows for two price levels and the development of non-correcting divergence between them in the medium term; and a perspective on the supply of money which is consistent with variations in finance affecting the level of investment. Both had, to varying degrees, been present in Minsky's thinking since his interest in finance and economic instability began, but they were strengthened as his position moved further towards the Post Keynesian camp.

As a Post Keynesian, Minsky argues that the prices of most (end-consumer) commodities are set by a markup on prime cost.[12] He portrays changes in the price level for "current goods" as mainly a consequence of cost pressures (largely from wages and raw materials) and changes to markups (though there is also a poorly developed monetary component to his explanation of inflation). The largely independent price level of assets—broadly defined as items whose ownership gives rise to claims to a stream of future cash flows—is based, not on the original cost of production of the assets, but on the net present value of anticipated cash flows. These in turn depend on the general state of expectations, which vary systematically over the financial cycle, lagging behind current prices in a slump, running ahead of them in a recovery and boom. Though asset prices must eventually return to some kind of harmony[13] with current prices over the very long term, this perspective allows for significant divergence between the two price levels as expectations rise and fall over the medium term. The price system thus displays far-from-equilibrium dynamics, according to Minsky, in contrast to the neoclassical argument that the price system is *the* stabilising force in a capitalist economy.

Minsky argues that the supply of money is essentially endogenously determined, and provides two reasons why the controls of a regulated system do not make it strictly exogenous. Firstly, if the current regulatory regime limits the supply of finance for investment to less than that desired by the private sector, then intermediation will occur and innovative financial products will be developed, increasing velocity. Secondly, if a financial institution gets into difficulties, the authorities will normally guarantee its deposits to prevent a "run"; in this case, either the money base will be expanded, or the credit multiplier will be increased. In other words, in times of potential financial crisis, the conventional money equation works backwards, from the supply of money to the base and multiplier. The resulting endogenous increase in the money stock then persists through time.

In a deregulated system, where the Central Bank has influence over only the monetary base and the rediscount rate, expansion of the money supply can occur much more easily, through both increased willingness of banks to lend—which increases the credit multiplier—and through financial innovation.

The more difficult and slower path of intermediation is no longer required (though it can still be practised).

The fundamental elements of Minsky's hypothesis can thus be found in the work of his predecessors Kalecki, Fisher and Keynes, and in the work of his Post Keynesian, institutional and evolutionary counterparts. Minsky's contribution has been to codify these insights, with the aim of developing a theory of investment consistent with the occurrence of periodic economic disturbances of the kind experienced in the 1930s—and, arguably, in the 1980s. It could thus be argued, as it was with Keynes, that there is "nothing new" in the Financial Instability Hypothesis. However this would be an unfortunate and inaccurate evaluation, belittling the art which Minsky practised so well of "standing on the shoulders of giants"—in a profession which has been more notable for the practice of standing on their toes. It would also be a very "linear" summary of the process of theory formation, when it is in fact a very "nonlinear" process in which the whole can be far greater than the sum of the parts.

2.4 The basic Minsky model

Minsky's analysis of a financial cycle begins at a time when the economy is doing well (the rate of economic growth equals or exceeds that needed to reduce unemployment), but firms are conservative in their portfolio management (debt to equity ratios are low and profit to interest cover is high), and this conservatism is shared by banks, who are only willing to fund cash-flow shortfalls or low-risk investments. The cause of this high and universally practised risk aversion is the memory of a not too distant system-wide financial failure, when many investment projects foundered, many firms could not finance their borrowings, and many banks had to write off bad debts. Because of this recent experience, both sides of the borrowing relationship prefer extremely conservative estimates of prospective cash flows: their risk premiums are very high.

However, the combination of a growing economy and conservatively financed investment means that most projects succeed. Two things gradually become evident to managers and bankers: "Existing debts are easily validated and units that were heavily in debt prospered: it pays to lever." (Minsky 1977, 1982: 65). As a result, both managers and bankers come to regard the previously accepted risk premium as excessive. Investment projects are evaluated using less conservative estimates of prospective cash flows, so that with these rising expectations go rising investment and asset prices. The general decline in risk aversion thus sets off both growth in investment and exponential growth in the price level of assets, which is the foundation both of the boom and its eventual collapse.

More external finance is needed to fund the increased level of investment and the speculative purchase of assets, and these external funds are forthcoming because the banking sector shares the increased optimism of investors (Minsky 1970, 1982: 121). This results in an increase in the accepted debt to equity ratio and falling profit to interest cover,[14] a decrease in liquidity, and accelerated growth of credit. In a regulated environment this will manifest itself via the growth of non-bank financial intermediaries; in a deregulated environment, there will simply be a rapid increase in the money supply.

Depending upon how the money supply is measured, these changes will be recorded as either an increase in the relevant measure of the money supply, or in an increase in the velocity of money circulation.

This marks the beginning of what Minsky calls "the euphoric economy" (Minsky 1970, 1982: 120-124.), where both lenders and borrowers believe that the future is assured, and therefore that most investments will succeed. Asset prices are revalued upward as previous valuations are perceived to be based on mistakenly conservative grounds. Highly liquid, low-yielding financial instruments are devalued, leading to a rise in the interest rates offered by them as their purveyors fight to retain market share.

Financial institutions now accept liability structures both for themselves and their customers "that, in a more sober expectational climate, they would have rejected" (Minsky 1970, 1982: 123.). The liquidity of firms is simultaneously reduced by the rise in debt to equity ratios, making firms more susceptible to increased interest rates. The general decrease in liquidity and the rise in interest paid on highly liquid instruments triggers a market-based increase in the interest rate, even without any attempt by monetary authorities to control the boom. However the increased cost of credit does little to temper the boom, since the rate of asset price inflation results in anticipated yields from speculative investments which far exceed prevailing interest rates. The elasticity of demand for credit with respect to interest rates therefore falls.

The condition of euphoria also permits the development of an important actor in Minsky's drama, the Ponzi financier.[15] These capitalists profit by trading assets on a rising market, and incur significant debt in the process. The servicing costs for Ponzi debtors exceed the cash flows of the businesses they own, but the capital appreciation they anticipate far exceeds the interest bill. They therefore play an important role in pushing up the market interest rate, and an equally important role in increasing the fragility of the system to a reversal in the growth of asset values.

Rising interest rates and increasing debt to equity ratios eventually affect the viability of many business activities, reducing the interest rate cover, turning projects which were originally conservatively funded into speculative ones, and making ones which were speculative “Ponzi”. Such businesses will find themselves having to sell assets to finance their debt servicing—and this entry of new sellers into the market for assets pricks the exponential growth of asset prices. With the price boom checked, Ponzi financiers now find themselves with assets which can no longer be traded at a profit, and levels of debt which cannot be serviced from the cash flows of the businesses they now control. Banks which financed these assets purchases now find that their leading customers can no longer pay their debts—and this realisation leads initially to a further bank-driven increase in interest rates. Liquidity is suddenly much more highly prized, holders of illiquid assets attempt to sell them in return for liquidity. The asset market becomes flooded and the euphoria becomes a panic, the boom becomes a slump.

As the boom collapses, the fundamental problem facing the economy is one of excessive divergence between the debts incurred to purchase assets, and the cash flows generated by them—with those cash flows depending both upon the level of investment and the rate of inflation. The level of investment has collapsed in the aftermath of the boom, leaving only two forces which can bring asset prices and cash flows back into harmony: asset price deflation, or current price inflation. This dilemma is the foundation of Minsky's iconoclastic perception of the role of inflation, and his explanation for the stagflation of the 70s and early 80s.

Minsky argues that if the rate of inflation is high at the time of the crisis, then though the collapse of the boom causes investment to slump and economic growth to falter, rising cash flows rapidly enable the repayment of debt incurred during the boom. The economy can thus emerge from the crisis with diminished growth and high inflation, but few bankruptcies and a sustained decrease in liquidity. Thus though this course involves the twin "bads" of inflation and initially low growth, it is a self-correcting mechanism in that a prolonged slump is avoided. However the conditions are soon re-established for the cycle to repeat itself, and the avoidance of a true calamity is likely to lead to a secular decrease in liquidity preference.

If the rate of inflation is low at the time of the crisis, then cash flows will remain inadequate relative to the debt structures in place. Firms whose interest bills exceed their cash flows will be forced to undertake extreme measures: they will have to sell assets, attempt to increase their cash flows at the expense of their competitors, or go bankrupt. In contrast to the inflationary course, all three classes of action tend to further depress the current price level, thus at least partially exacerbating the original imbalance.

If assets are sold as going concerns, then those who buy them face a lower cost of capital, and can undercut their rivals in the current goods market. If assets are broken up, then the producer goods and commodities that constitute them compete at reduced prices with new output from competitors. If firms attempt to increase cash flows by reducing their markups, they will be matched by other competitors in similar circumstances, thus leading if anything to further reduced cash flows and a general reduction in the price level. If firms go bankrupt, their stocks and assets will be sold into depressed markets, thus further reducing current prices. The asset price deflation route is therefore not self-correcting but rather self-reinforcing, and is Minsky's explanation of a depression.

Thus while Minsky still sees inflation as having deleterious effects during a relatively stable period of economic growth, he perceives it in quite a different light during a time of crisis. The fundamental problem during a financial crisis is the imbalance between the debts incurred to purchase assets, and the cash flows those assets generate. A high rate of inflation during a crisis enables debts which were based on unrealistic expectations to be nonetheless validated, albeit over a longer period than planned and with far less real gain to the investors. A low rate of inflation will mean that those debts cannot be met, with consequent "domino" effects even for investments which were not unrealistic.

The above sketch basically describes Minsky's perception of an economy in the absence of a government sector. With Big Government, the picture changes in two ways, due to the impact of fiscal deficits and Reserve Bank interventions on corporate cash flows. With a developed social security system, the collapse in cash flows which occurs when a boom becomes a panic will be at least partly ameliorated by a fall in tax revenues and a rise in government spending—the classic "automatic stabilisers", though this time seen in a more monetary light. The collapse in credit can also be tempered or even reversed by rapid action by the Reserve Bank to increase liquidity. With both these forces operating in all Western economies since WWII, Minsky expected the conventional cycle to be marked by "chronic and ... accelerating inflation" (Minsky 1980b, 1982: 85). However by the end of the 1980s, the cost pressures which coincided with the slump of the early 70s had long since been eliminated, by 15 years of high unemployment, and the diminution of OPEC's cartel power. The crisis of the late 80s thus occurred in a milieu of low inflation, raising the spectre of a debt deflation.

Minsky's perspective casts a radically different light on post-WWII financial history. Rather than being seen as the normal functioning of a free market economy, the long period of economic stability from the end of WWII till the end of the 1960s is largely attributed to the conservative financial attitudes and structures established in the aftermath of the Great Depression. The stagflation of the 1970s is explained using Keynesian foundations, rather than Monetarist ones. The boom and bust experience of the 80s and early 90s is conceived as a manifestation of normal cyclical behaviour in a financial economy, rather than an aberration. Deregulation of the financial sector, while not portrayed as the sole cause of the instability during the 1980s, is seen as contributing to its severity by removing some of the limited constraints to cyclical behaviour which exist in a regulated system. The present "achievements" on inflation are seen as manifestations of a debt-deflationary process, indicative of the problems of a cyclical economy, rather than a foundation for recovery. Given the profound differences between Minsky's interpretation of the current state of capitalism and those of conventional economic analysis, his hypothesis warrants careful examination.

2.5 Modelling and Simulating the Financial Instability Hypothesis

Though the roots of the Financial Instability Hypothesis can be traced to Minsky's attempts to build a mathematical model of the trade cycle which integrated the financial sector with the real economy, as he refined the FIH, Minsky made fewer attempts to build comprehensive mathematical models. As the next Chapter indicates, this declining interest in model building may reflect the inappropriateness of the methods chosen by Minsky, rather than the worthwhileness of the endeavour itself. Chapter Three outlines the models Minsky did develop, based on the multiplier-accelerator principle, and shows that the multiplier-accelerator concept is based on a simple but profound economic mistake. Correcting this error leads to a linear model of endogenous cyclical growth which, while limited by its linearity, is a far more cogent form of dynamic analysis than that developed by Hansen, Samuelson and Hicks.

3 Linear Models of cyclical growth

3.1 Minsky's mathematical models

Minsky's first attempt to incorporate finance into a model of the trade cycle employed Hicks's rendition[16] of the conventional Hansen-Samuelson multiplier-accelerator model. In this model, a second order difference equation is derived for output at time t by combining a relation for investment and a relation for consumption. The derivation begins with the division of output Y into consumption C and investment I:

(6) $$Y_t = C_t + I_t$$

Consumption is portrayed as a lagged linear function of income:

(7) $$C_t = \alpha \cdot Y_{t-1}$$

Investment is modelled as a lagged linear function of the rate of change of income:

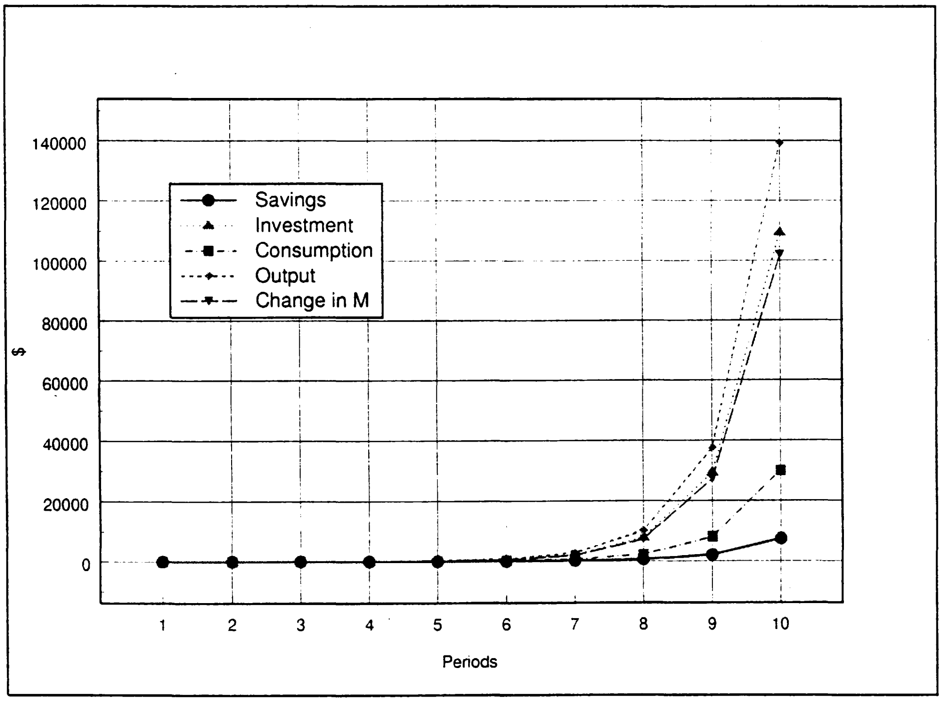
(8) $$I_t = \beta \cdot (Y_{t-1} - Y_{t-2})$$

These are substituted to yield a second-order linear difference equation for output:

(9) $$Y_t = (\alpha + \beta) \cdot Y_{t-1} - \beta \cdot Y_{t-2}$$

The characteristics of this model when α and β are constants are well-known. Minsky's innovations were to argue that β is a function which depends upon financial conditions, and to consider that when the economy "hit" the ceiling, the result was the creation of new "initial conditions" which would reset the difference equation process. With respect to the former, he examined a number of different financial regimes, including one in which the money supply was infinitely elastic. He argued that since β reflected the propensity to invest, this would fall as indebtedness rose, leading to the model switching from its monotonically explosive to its cyclical range. Indebtedness rose because the difference between ex-ante savings and ex-ante investment (defined respectively as $Y_{t-1} - C_t = (1-\alpha) \cdot Y_{t-1}$ and $It = \beta \cdot (Y_{t-1} - Y_{t-2})$; 1959: 133; 1957a, 1982: 236) was financed by borrowing. Borrowing occurred when ex-ante investment exceeded ex-ante savings, and the money supply increased to match the change in income. With values of .8 for α , 4 for β, and 100 and 110 for the first two values of Y, and keeping β constant, this generated the system plotted in Figure 3.[17]

Figure 3 Minsky's Explosive Money Supply Extension to Hicks's Second Order Difference Equation



Minsky argued that this would result in a rapidly growing component of investment being financed by debt, and in keeping with Kalecki (1937), a tendency for capitalists to attenuate future investment in response to the increase in risk. This would lead to a decline in the β coefficient, converting the system from an explosive one into a cyclical one, and thus explaining the upwards turning point of the model. While Minsky did not attempt to provide an explicit functional form for β , either in this paper or his subsequent attempt to reconcile models of growth and cycles (Minsky 1965), this analysis of the β coefficient made him aware of the fact that "the accelerator coefficient ... is in part based on the productive efficiency of investment, but it is also related to the willingness of investors to take risks and the terms in which investors can finance their endeavours..." (1965: 261)

While Minsky's argument is intuitively appealing, the deficiencies of the underlying multiplier-accelerator model overwhelm the intuitive case, since as Figure 3 indicates, the rate of growth predicted by the model is far too high to be taken seriously. Even if we define each period as the horizon for planning large-scale investment (say seven years), this implies output increasing ten thousandfold in ten such periods, in the absence of the finance constraint. This far exceeds achieved growth rates, and thus implies that the finance constraint would be invoked far more often than even Minsky would argue it has been. The model thus displays far more instability than the real world in fact possesses.

In part, the reason for this was best put by Kaldor: since linear models of cycles imply either "dangerous instabilities" or "more stability than the real world appears, in fact, to possess", the functions "cannot both be linear" (Kaldor 1940: 80-81) and the fault therefore lies with the assumption of linearity. However, part of the reason can be slated back to a simple but profound economic mistake in the derivation of the multiplier-accelerator model itself. This mistake is most apparent when Hicks's reinterpretation of Harrod's model of unstable growth is analysed.

3.2 Harrod

Though Harrod's "An Essay in Dynamic Theory" (Harrod 1939) has been conventionally interpreted solely as an attempt to construct a theory of growth, his aim was to devise a theory which explained both growth and cycles.[18] Harrod began the *Essay* by criticising models which relied solely upon lags to generate oscillatory behaviour, noting that "there is some doubt as to the nature of the trend on which the oscillation is superimposed", and implying that in these models the trend and the cycle are logically distinct, whereas he proposed to show that "the trend of growth may itself generate forces making for oscillation" (Harrod 1939: 14-15). What he later described as his "basic antinomy" (Harrod 1951: 262) is the set of propositions "(1) that the level of a community's income is the most important determinant of its supply of saving; (2) that the rate of increase of its income is an important determinant of its demand for saving, and (3) that demand is equal to supply" (Harrod 1939: 14). Expressing this in equations, Harrod's key relations are:

$$\begin{aligned} &S = S(Y) \\ (10)\quad &I = I\left(\tfrac{dY}{dt}\right) \\ &S = I \end{aligned}$$

Harrod's derivation of his dynamics is rather cumbersome. It is rather easier to reach the same result by dynamising the *General Theory* equality of ex-post savings and investment, and then making the analogy, as Harrod did, between an expression for actual outcomes and one for desired outcomes:

$$\begin{array}{lll} & I = S & \text{Divide both sides by Y} \\ & \frac{I}{Y} = \frac{S}{Y} & \text{Introduce the increment to output} \\ (11) & \frac{I}{Y} \cdot \frac{\partial Y}{\partial Y} = \frac{S}{Y} & \text{Rearrange terms} \\ & \frac{I}{\partial Y} \cdot \frac{\partial Y}{Y} = s & \text{Defining terms} \\ & C_p \cdot G = s & \end{array}$$

C_p is the actual incremental capital to output ratio (*ICOR*), G the actual rate of growth, and s the average propensity to save. By analogy (1939: 17-18), Harrod put forward his "Fundamental Equation" (1939: 17) $G_W = \frac{s}{C}$, where C is the desired *ICOR* and G_w the warranted rate of growth. In this case, G_w is a derived quantity: given s and C, G_w is determined. The instability of dynamic equilibrium—Harrod's "knife-edge", which, his later protestations notwithstanding, is an accurate characterisation of his early contributions[19]—results from the interaction of the equations for G and G_w. If $G>G_w$ then $C_p<C$, and the actual increment to stocks will be less than the desired increment, thus leading capitalists to increase orders. This will increase the rate of growth, further increasing the gap between G and G_w. Similarly, if $G<G_w$, actual stocks will exceed desired stocks, orders will fall and growth will falter.

Harrod added one further concept, the unfortunately named "natural" rate of growth, which he defined as "the maximum rate of growth allowed by the increase of population, increase of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense." (Harrod 1939: 30) The relationship of the warranted rate G_w to this maximum feasible rate of growth G_m determines whether the economy is prone to slumps or booms, and it involves a paradox. If $G_m > G_w$, then it is feasible for actual growth G to lie between the two. If so, then G will rise (the knife-edge) until it hits G_m—a boom. However, if $G_m < G_w$, then it is not possible for actual growth to lie between the two. G must therefore be less than G_w, and the economy will fall—a slump. The paradox is that an economy inhabited by conservative capitalists—which therefore has a low G_w—is likely to experience boom conditions in general, whereas an economy inhabited by aggressive capitalists—with a high G_w—is likely to experience slumps.

3.3 Hicks

Of the many commentators upon Harrod, the most influential was undoubtedly Hicks, and he objected strenuously to the mathematical instability of Harrod's dynamic equilibrium. Harrod, he notes,

> welcomes the instability of his system, because he believes it to be an explanation of the tendency to fluctuation which exists in the real world... But a mathematical instability does not in itself elucidate fluctuation. A mathematically unstable system does not fluctuate; it just breaks down. (Hicks 1949: 252, reviewing Harrod 1949)

This incorrect proposition[20] was the cue for Hicks to embark upon a reinterpretation of Harrod, which had as its major objective providing a stable but fluctuating alternative to Harrod's analysis, using the mathematical techniques available at that time. The technique he employed was the introduction of lags.

Hicks observed that "It is not generally realised (Mr Harrod has certainly failed to realise it) that the great function of lags ... in dynamic theory is to impart just that measure of stability ... as is required to make the movements of the system economically determinate" (Hicks 1949: 253). He began to introduce them by recasting Harrod's δY as $\delta Y = Y_n - Y_{n-1}$. Harrod's equation for actual growth[21] $G \times C = s$, thus became (when both sides were multiplied by Y_n) $c \times (Y_n - Y_{n-1}) = s \times Y_n$. Hicks commented that in this form the equation looked "decidedly queer. It is not really reasonable to assume that current investment should depend on the increment of output in the same period..." He instead proposed to base investment "upon the increment of income in the preceding period, and consumption upon the income of the preceding period." (1949: 254)

At this point he did something which Minsky was careful not to do (1965: 261), which is to confuse actual with desired investment. Hicks's statement that investment should not depend upon the increment to output in the current period is clearly directed at intended investment, since actual investment is properly defined as the increment to the capital stock:

(12) $I_t \equiv K_t - K_{t-1}$

Since capital stock can in turn be related to output via the accelerator ($K_t = v \times Y_t$) at this aggregate level, it is thus quite clear that actual investment in period t is related to output in the same period:

(13) $I_t = v \times (Y_t - Y_{t-1})$ [22]

On the other hand, Hicks's relation

(14) $I_n = c \times (Y_{n-1} - Y_{n-2})$

clearly refers to intended investment, rather than actual investment. His lagged relation for consumption

(15) $C_t = (1 - s) \cdot Y_{t-1}$

refers to actual consumption, so that his expression for savings

(16) $S_t = Y_t - C_t = Y_t - (1 - s) \cdot Y_{t-1}$

likewise refers to actual savings. Hicks substituted equations 14 and 15 into the Keynesian identity for actual output of

(17) $Y_t \equiv C_t + I_t$

to yield the second order difference equation

(18) $Y_t = (1 - s + c) \cdot Y_{t-1} - c \cdot Y_{t-2}$

which he then used as the basis of his model of the trade cycle. But this expression, which is functionally identical to the Hansen-Samuelson multiplier-accelerator models which predated it, is clearly meaningless, since I_t in identity 17 must be *actual* investment, not intended investment. The expression in equation 18, which purports to be an expansion of 17, is in fact the sum of *actual* consumption and *intended* investment, which is a meaningless quantity.

In contrast, the addition of a lagged expression for actual consumption to the proper expression for actual investment yields a first order difference equation,

(19) $$\begin{aligned} Y_t &= (1 - s) \cdot Y_{t-1} + v \times (Y_t - Y_{t-1}) \\ &= \tfrac{v-1+s}{v-1} Y_{t-1} \end{aligned}$$

This reduces to the following expression for the rate of growth:

(20) $\frac{Y_t - Y_{t-1}}{Y_{t-1}} = g = \frac{s}{v-1}$

Hicks's justification for using a lagged formulation for savings was that there is more reason to lag savings than there is to lag investment:

> "It is not really reasonable to assume that current investment should depend upon the increment of output in the same period (especially if periods are short), and still less is it reasonable to assume that savings depends wholly upon the increment of income in the preceding period" (Hicks 1949: 110).

In fact, the introduction of a lag for consumption was quite spurious, since the time frame for investment is substantially longer than the time frame for consumption, so that in this model consumption is best treated as depending upon income in the same period. The more likely motivation for Hicks to lag investment was that equating actual unlagged savings to Hicks's lagged desired investment generates a clearly meaningless equation. This indicates that an unlagged expression for actual consumption should be combined with the definition of actual investment to derive a relation for actual output. This yields the equation:

$$(21)\quad \begin{aligned} Y_t &= (1-s)\cdot Y_t + v\times(Y_t - Y_{t-1}) \\ &= \tfrac{v}{v-s} Y_{t-1} \end{aligned}$$

which reduces to the following expression for the rate of growth of output:

$$(22)\quad \frac{Y_t - Y_{t-1}}{Y_{t-1}} = g = \frac{s}{v-s}$$

Both equation 20 and equation 22 are first order expressions, which can only generate growth, not cycles. Unlike the Hansen-Samuelson-Hicks models, however, these give realistic results for realistic parameter values. If one substitutes feasible values for v and s of 3 and $.1$ respectively into equation 22, it yields the equally feasible value for g of 3.4% *p.a.*

There is thus a fundamental flaw in the Hansen-Samuelson approach to modelling the trade cycle. This class of difference equations were derived by adding expressions for consumption and investment to generate an expression for output, but to derive a model for actual output, the terms for consumption and investment must likewise be terms for actual values. There is some latitude in the definition for consumption, since it is an empirical question as to whether consumption is or is not based exclusively on income in the current period. However no such latitude exists for actual investment, which by definition is the increment to the capital stock in the relevant period.

That the class of models derived by adding actual consumption to intended investment has an equilibrium value of zero output is hardly surprising, since this approach amounts to equating intended investment to actual savings. But there is no basis in either pre or post Keynesian economic theory for the proposition that intended investment and actual savings should be equal in any given time period. Since both are functions of output in this simple model, the only level of output which guarantees their equality is zero, and equation 18 is merely an oscillatory method of reaching this trivial solution. There is, therefore, no economic rationale to Hicks's model, nor to the Hansen-Samuelson family of models to which it belongs, and it is little wonder that Minsky was unable to use it as an effective foundation for modelling the impact of finance on economic stability.

3.4 A third-order resolution

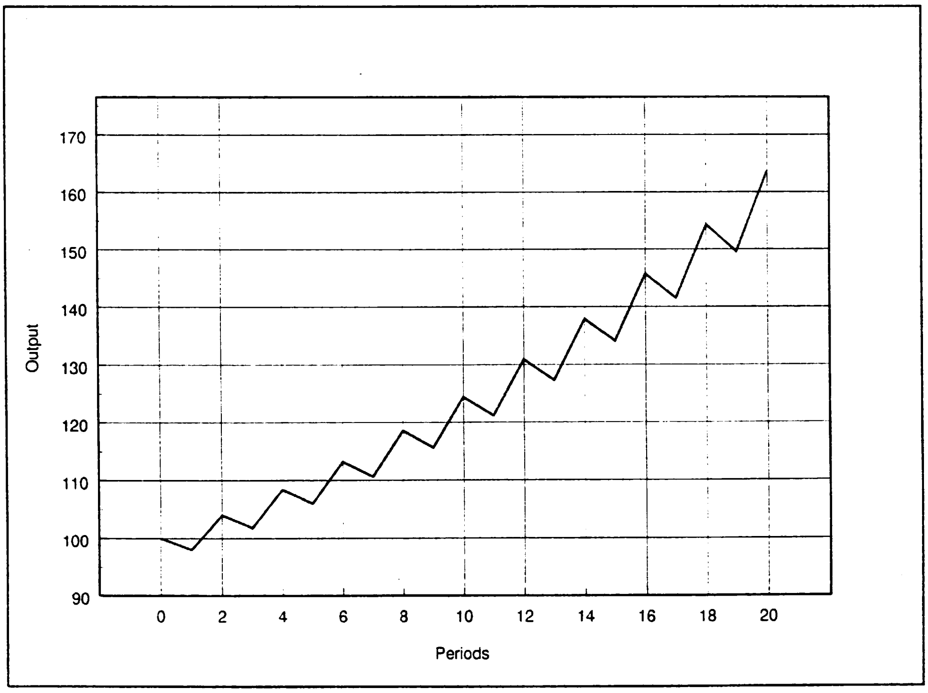
The foregoing analysis in effect dismisses the entire family of second order difference equation models which dominated analysis of the trade cycle in the 50s and 60s. However Hicks's key insight that lags provide "just that measure of stability ... as is required to make the movements of the system economically determinate" can be combined with Harrod's vision of dynamics, and the key *General Theory* contribution that investment determines the level of income, to yield a third order model which generates both growth and cycles.

Following Harrod, we argue that capitalists have a desired ratio between changes in output and changes in the capital stock: the c in what follows is definitely a behavioural variable. Following Hicks, we model this desired investment as a lagged response to changes in output. Following Keynes, we draw the causal chain from investment to savings via the level of output: capitalists carry out their investment plans, so that planned investment becomes actual investment, which adds to the capital stock. The altered capital stock determines this period's output via the accelerator, thus closing the model:

(23)
$$\begin{gathered} I_t = c \times (Y_{t-1} - Y_{t-2}) \\ K_t = K_{t-1} + I_{t-1} \\ Y_t = \tfrac{1}{v} \times K_t \\ Y_t = \tfrac{1}{v} \times (v \times Y_{t-1} + c \times (Y_{t-2} - Y_{t-3})) \\ Y_t = Y_{t-1} + \tfrac{c}{v} \times (Y_{t-2} - Y_{t-3}) \end{gathered}$$

This model reconciles the apparently disparate agendas of Harrod, Hicks and Keynes. Harrod's intuitions about growth and cycles are confirmed since this model's equilibrium can be shown to be unstable, and trend and cycle are interdependent (since if the three initial values for Y are identical, neither growth nor cycles will occur). Figure 4 plots the final equation in equation block 23,[23] and shows that Hicks is correct to argue that the introduction of lags will provide "just that measure of stability ... as is required to make the movements of the system economically determinate", since the instability of this model leads not to complete breakdown, but to cycles about a trend rate of growth.

Figure 4 Cyclical Growth From a Third Order Difference Equation



Finally, Keynes's insights that investment determines output, and that increasing the propensity to save can decrease the level of output, which were posed in static terms in the *General Theory*, are easily given dynamic expression in this simple difference equation. As the next section shows, the trend rate of growth itself is dependent upon the capitalist propensity to invest, and, when the propensity to save is introduced into the model, an increase in the propensity to save lowers the rate of growth.

3.4.1 Analysis of the model

This third order difference equation converts into the following system of first order equations:

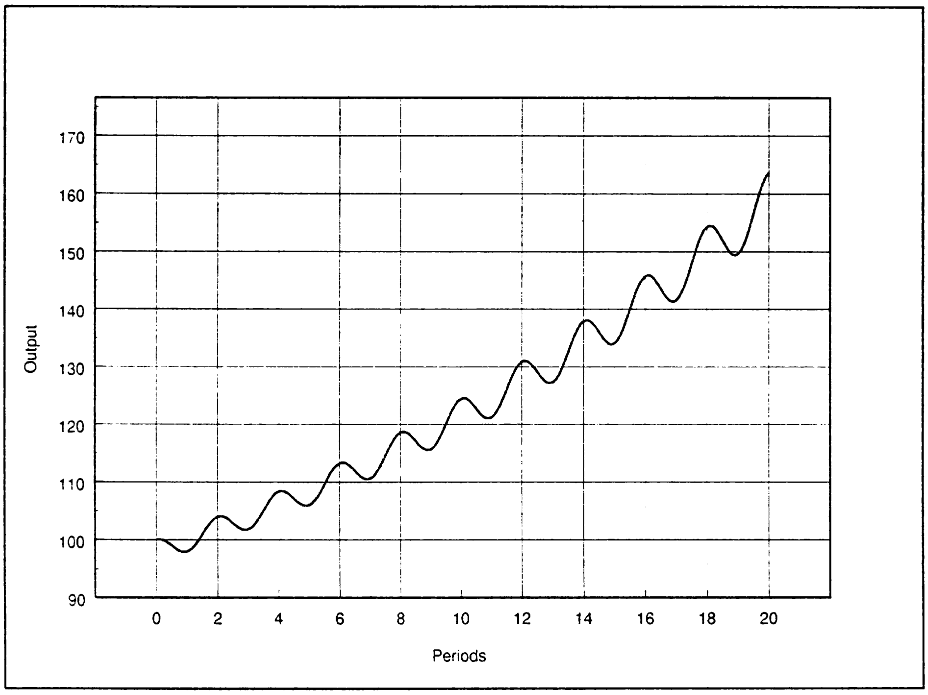
$$(24)\quad \begin{pmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{c}{v} & \frac{c}{v} & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$ [24]

The matrix is singular, which allows for solutions other than the trivial solution, and its eigenvalues are $\left(1, \sqrt{\frac{c}{v}}, -\sqrt{\frac{c}{v}}\right)$. The eigenvalue of one means that the system has no unique equilibrium—any sustained level of output is an equilibrium—and that each equilibria is necessarily marginally unstable. The other two eigenvalues indicate that, after a disturbance, output will converge to an equilibrium value if $c<v$, and that it will fluctuate around a trend rate of growth for $c \geq v$. If $c>v$, then the trend rate of growth is exponential; if $c=v$ then the trend rate of growth is linear. In the case where $\frac{c}{v} \neq 1$, the solution in terms of Y_t and three initial values of Y can be shown to be

$$(25)\quad Y_t = \left(\begin{array}{c} \dfrac{v\times Y_2 - c\times Y_0}{v-c} + \\ \dfrac{1}{2} \times \left(\dfrac{\sqrt{v\times c}\times(Y_1-Y_0)+v\times(Y_2-Y_1)}{c-\sqrt{v\times c}} \right) \times \sqrt{\frac{c}{v}}^{\,t} \\ -\dfrac{1}{2} \times \left(\dfrac{\sqrt{v\times c}\times(Y_1-Y_0)-v\times(Y_2-Y_1)}{c+\sqrt{v\times c}} \right) \times \left(-\sqrt{\frac{c}{v}}\right)^t \end{array} \right)$$

As Figure 5 indicates, this model generates both growth and cycles in the case in which $c>v$. Figure 5 was generated from equation 25 with values for Y_0, Y_1 and Y_2 of *100, 98 & 104* respectively, a c value of *3.3*, and v equal to *3*:

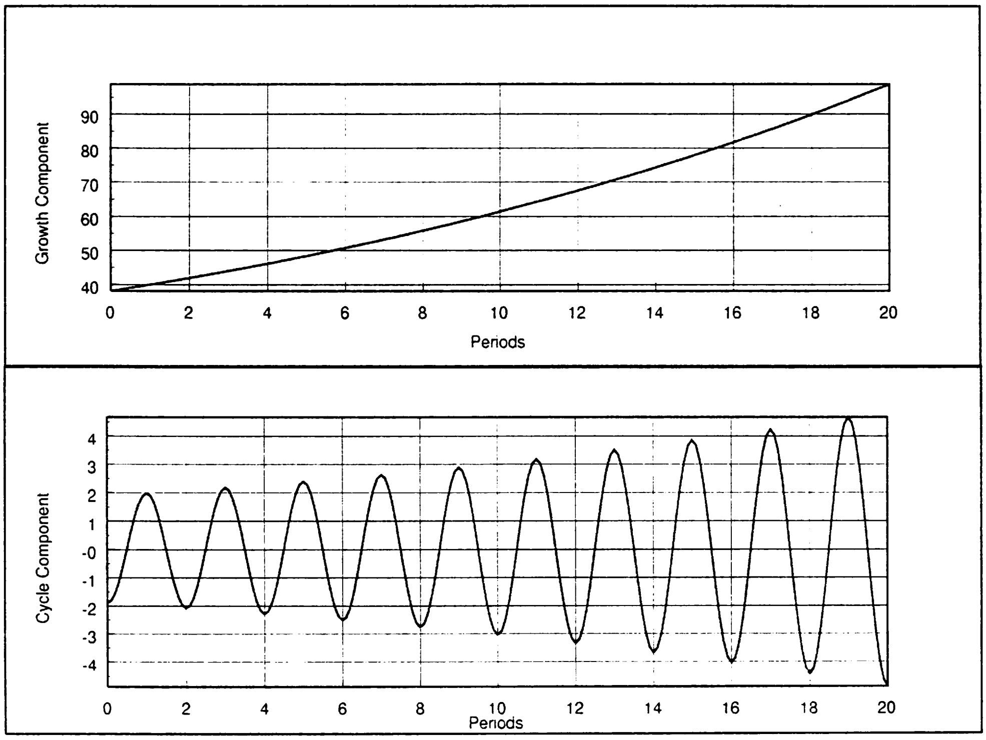
Figure 5 Plot of Polynomial Solution to Third Order Difference Equation



The expression can also be meaningfully decomposed, with the first term representing the equilibrium level of output to which the system will tend if $c<v$, the second term generating exponential growth when $c>v$ and asymptotic growth to a new equilibrium when $c<v$, and the third term generating cycles which remain in step with the level of output (see Figure 6). The rate of growth and the size of the cycle both accord with observed data, in contrast to the nonsensical cycles generated when realistic parameter values are used for the standard Hansen-Samuelson model. The long run equilibrium rate of growth generated by this simulation of 4.9% per period compares very

favourably to the nonsensical rate of growth required by Minsky's adaptation of Hicks's second order model of 73% per period (see Figure 3). Figure 6 also indicates that the relative magnitude of the cycle remains in step with the level of output.

Figure 6 Decomposition of solution into Growth & Cycle Components



The proportionality between the level of output and the size of the cyclical fluctuations in this model is no accident. The ratio of the constant in the growth expression to the constant in the cycle expression is

$$(26)\quad \frac{\sqrt{v\times c}\times(y_1-y_0)+v\times(y_2-y_1)}{\sqrt{v\times c}\times(y_1-y_0)-v\times(y_2-y_1)}\times\frac{c+\sqrt{v\times c}}{c-\sqrt{v\times c}}$$

which guarantees that the cycles will always be in proportion to and significantly smaller than the level of output. If $c<v$, then the growth and cycles engendered by an initial positive disturbance in output gradually dissipate, resulting in a new, higher equilibrium level. If $c>v$, then the growth and cycles perpetuate, resulting in regular fluctuations in output around an exponential trend. This clarity and intelligible decomposability of this expression should be contrasted with the reduced form for Hicks's second order difference equation:

$$(27)\ Y_t = \left(\begin{array}{c} \frac{1}{2} \times \frac{\sqrt{(1-s+c)^2-4\times c}\times Y_0-(1-s+c)\times Y_0+2\times Y_1}{\sqrt{(1-s+c)^2-4\times c}} \times \\ \left(\frac{1}{2}\times((1-s+c)+\sqrt{(1-s+c)^2-4\times c})\right)^t + \\ \frac{1}{2} \times \frac{\sqrt{(1-s+c)^2-4\times c}\times Y_0+(1-s+c)\times Y_0-2\times Y_1}{\sqrt{(1-s+c)^2-4\times c}} \times \\ \left(\frac{1}{2}\times((1-s+c)-\sqrt{(1-s+c)^2-4\times c})\right)^t \end{array} \right)$$

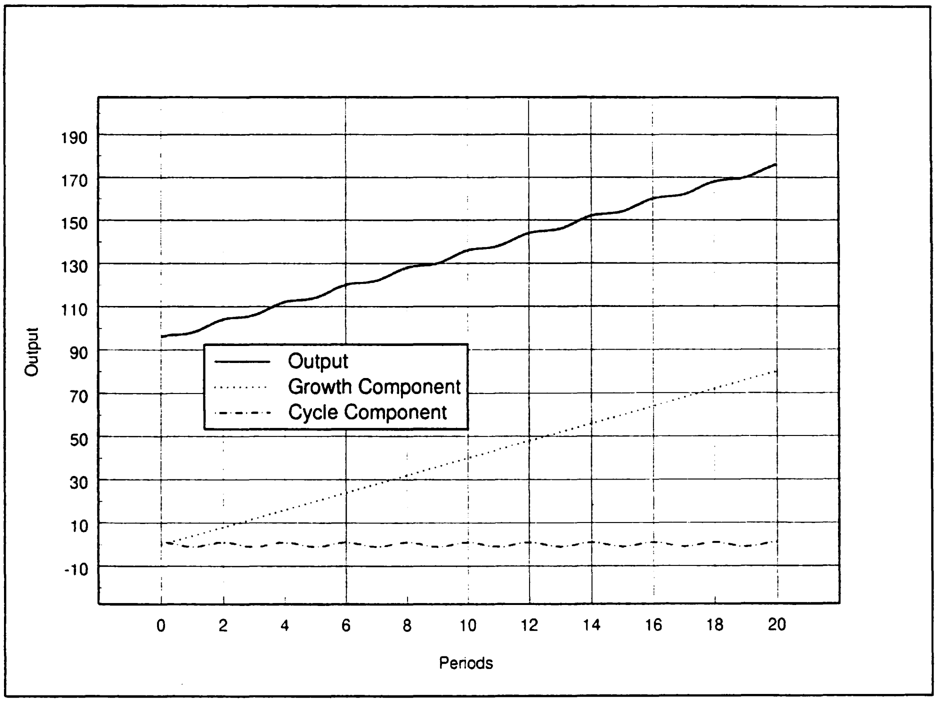
It is quite impossible to give any economic interpretation to this, since as outlined above, it is based on an economic mistake.

The third order relation has a different solution in the special case in which $c=v$. The system's eigenvalues are now *(1,1,-1)* and the closed form solution is:

$$(28)\ Y_t = \frac{Y_0-Y_1}{2}\times(-1)^t + \frac{3\times Y_0+Y_1-2\times Y_2}{2} + (Y_2-Y_0)\times t$$

As with the general case in which $c \neq v$, this system can be decomposed into a growth trend and a cycle (see Figure 10). The first term generates cycles of constant amplitude if Y_0 and Y_1 differ in magnitude, the third generates linear growth if Y_0 and Y_2 differ in magnitude, and the second term gives the result that $Y_t = Y_0$ if the initial levels of output are identical.

Figure 7 Growth Cycle for Special Case where c=v



3.4.2 The Propensity to Save

The propensity to save can be introduced into the model by assuming, as did Samuelson (1939), that investment is motivated by changes in consumption, rather than changes in total output. Its introduction provides a dynamic equivalent to Keynes' static concept of the "paradox of thrift", since increasing the propensity to save will reduce the rate of growth. The initial equations in the model are replaced by

$$(29)\quad \begin{aligned} C_t &= (1-s)\times Y_{t-1} \\ I_t &= c\times((1-s)\times Y_{t-1}-(1-s)\times Y_{t-2}) \end{aligned}$$

This simply modifies the constant in the investment equation, and can readily be incorporated into the general solution for the case where $c \neq v$, to yield

$$(30)\ Y_t = \left(\begin{array}{c} \frac{v\times Y_2 - c\times(1-s)\times Y_0}{v-c\times(1-s)} + \\ \frac{1}{2}\times\left(\frac{\sqrt{v\times c\times(1-s)}\times(Y_1-Y_0)+v\times(Y_2-Y_1)}{c\times(1-s)-\sqrt{v\times c\times(1-s)}}\right)\times\sqrt{\frac{c\times(1-s)}{v}}^{\,t} \\ -\frac{1}{2}\times\left(\frac{\sqrt{v\times c\times(1-s)}\times(Y_1-Y_0)-v\times(Y_2-Y_1)}{c\times(1-s)+\sqrt{v\times c\times(1-s)}}\right)\times\left(-\sqrt{\frac{c\times(1-s)}{v}}\right)^t \end{array} \right)$$

In this model, an increase in the propensity to save will result in a fall in $\sqrt{\frac{c\times(1-s)}{v}}$, which is the long run rate of growth This result conflicts with Harrod's original formula $G = \frac{s}{c}$, which asserted a positive relationship between savings and growth. The error lies with Harrod, since an increase in the rate of savings will only lead to greater growth if it is matched by an increase in investment. However, since an increased propensity to save means a reduced level of consumption, and since, in this revised model, investment is motivated by changes in consumption, reducing the level of consumption reduces the level of investment, and hence reduces the growth.

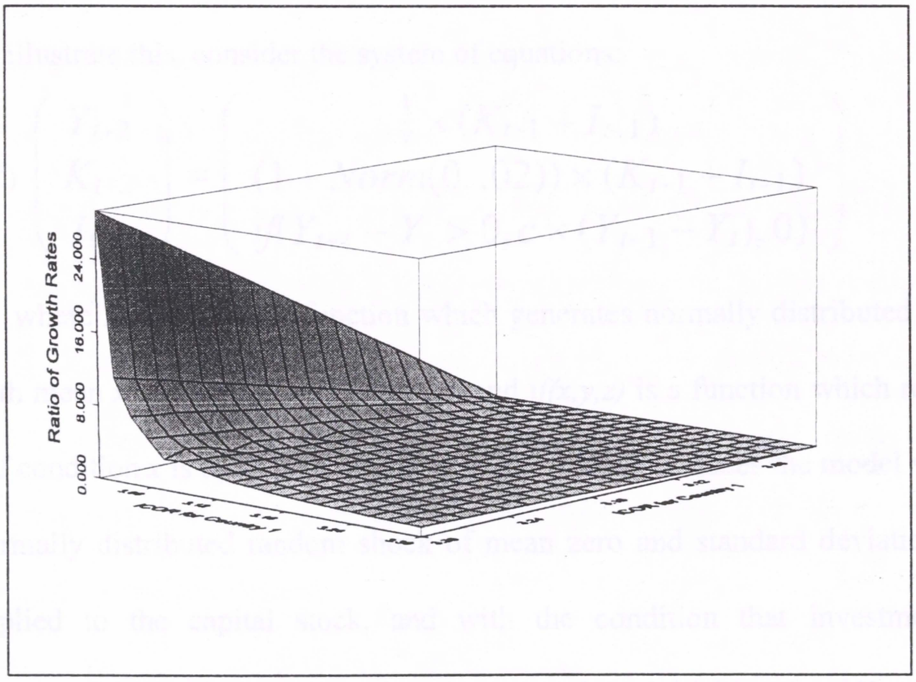
3.5 Divergent Growth

This model provides a very Keynesian explanation for the occurrence of sustained differences in the rates of economic growth of nations—something which is difficult to explain from a Solow/Swan perspective. Economic growth in this model depends upon the willingness of a nation's capitalists to invest, so that a higher propensity to invest is associated with a higher rate of growth. The percentage rate of growth is extremely sensitive to small changes in the value of c. For two economies which differ only in their preferred *ICOR*s, and where $c>v$ for both economies, the limit as $t \rightarrow \infty$ of the ratio of their percentage rates of growth is

$$(31)\ \frac{g_1}{g_2} = \frac{\sqrt{c_1\times c_2}+\sqrt{c_1\times v}-\sqrt{c_2\times v}-v}{c_2-v}$$

which is a quasi-linear but very steep function of the ratio of the preferred ICORs. Figure 8 plots this function with $v=3$ and c values for the two countries ranging between 3 and 3.3. At one extreme of the function, a one per cent difference in *ICOR*s results in a *10* per cent difference in rates of growth for values of the base c and v of 3.3 and 3 respectively. As Figure 8 indicates, the ratio of relative growth rates is more extreme the closer c is to v in the "denominator" country. For $v=3$, $c_2=3.1$, a *1* per cent difference between c values results in a *30%* difference in growth rates. At the other extreme, when c_2 is 3.01 and c_1 is 3.3, the growth rate of country *1* is *32* times that of country *2*.

Figure 8 Ratio of Growth Rates for Countries with Differing ICORs



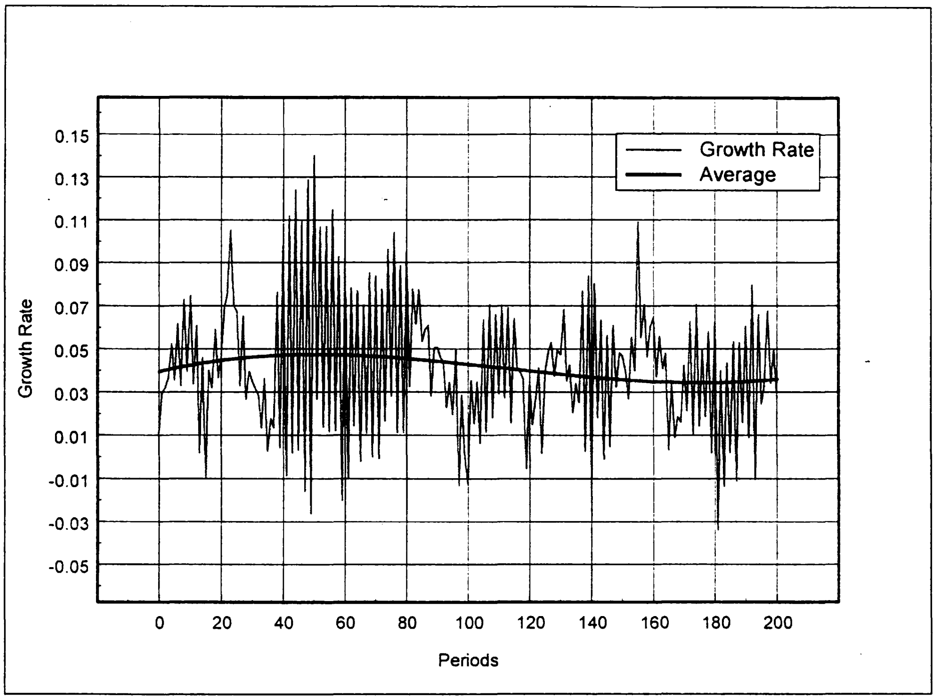
3.6 Random fluctuations and growth

The introduction of random shocks to this model (along with the assumption of non-negative investment)[25] adds further weight to the importance of the investment preferences of capitalists in explaining divergent national growth rates. Any real economy will be subject to random disturbances in output whose causes are unrelated to the economic cycle—-with causes ranging from weather fluctuations to wars. Such disturbances, when fed into Frisch and Hansen-Samuelson type trade cycle models, generate irregular cycles, but—-prior to the imposition of a trend in autonomous investment—-no growth. In contrast, the impact of fluctuations on this model is to engender both irregular cycles and growth. As could be expected, the degree of growth generated depends on the ratio of the desired *ICOR* to the accelerator. If a country has a ratio that exceeds one, the impetus from each fluctuation will persist through time; if the ratio is less than one, then the influence of each disturbance will diminish towards zero over time. To illustrate this, consider the system of equations:

$$(32)\quad \begin{pmatrix} Y_{t+2} \\ K_{t+2} \\ I_{t+2} \end{pmatrix} = \begin{pmatrix} \frac{1}{v} \times (K_{t+1} + I_{t+1}) \\ (1 + Norm(0,.02)) \times (K_{t+1} + I_{t+1}) \\ if(Y_{t+1} - Y_t > 0, c \times (Y_{t+1} - Y_t), 0) \end{pmatrix}$$

where *Norm(x,y)* is a function which generates normally distributed noise with mean x and standard deviation y, and *if(x,y,z)* is a function which returns y if condition x is true, otherwise it returns z. This reproduces the model with a normally distributed random shock of mean zero and standard deviation .02 applied to the capital stock, and with the condition that investment is non-negative. With the same values for c, v and the initial levels of Y as in the preceding simulations, equation block 32 generates the pattern for the rate of growth shown in Figure 9.

Figure 9 Irregular Cycles from Random Shocks to the Third Order Model



The average rate of growth shown in Figure 9 is generated by a cubic spline interpolated moving average. As can be seen, this average varies between 3 and 5 per cent for this simulation (given the random nature of the shock applied to this model, a different pattern of growth rates occurs with every simulation), while the spot rate of growth varies from a low of -3 per cent to a high of 14 per cent. These results are within realistic limits, which further heightens the contrast between this model and those of the Hansen-Samuelson variety.

3.7 The exception to the linear rule

Linear models of the trade cycle have had a poor history. Though Hansen-Samuelson models should have been rejected as erroneous from the outset, they dominated mainstream trade cycle analysis, while at the same time their limitations in practice contributed to the waning of interest in dynamics in general—and the trade cycle in particular—by the mainstream of the economics profession. On the other hand, their weaknesses led to an interest in nonlinear theories of the trade cycle by non-mainstream economists such as Kaldor and Goodwin.

Blatt (1983) argues that though linear models are inadequate, the mainstream reacted incorrectly to their limitations by abandoning dynamics entirely, while Kaldor, Goodwin et al were correct to focus attention upon nonlinear dynamical models. Blatt defines a linear model in linear algebra terms (149-150), and argues that if such a system is locally unstable, any initial deviation will in time become infinitely large. He asserts that this feature of locally unstable linear models makes them untenable as models of the trade cycle, and he concludes that "*If a system is truly linear, then instability of the equilibrium is a fatal flaw of the theory*" (1983: 150. Italics in original).

Though this statement is in general true, Blatt's criticism of linear models does not apply to the model presented in this Chapter. Since the magnitude of the eigenvalue which generates cycles is identical to that of the eigenvalue which generates growth, the cyclical deviation remains in step with the level of output. Thus while its equilibrium is unstable, a chance deviation from equilibrium will result in growth as well as cycles, with the magnitude of the former perfectly in step with the latter. The cycles do tend towards infinity as time goes to infinity, as Blatt argues, but this model also generates growth so that the level of output likewise tends to infinity (where $c>v$). This model is thus the exception to the linear rule outlined by Blatt.

The economic interpretation of this dynamic and generally stable Harrod-Hicks-Keynes system is relatively straightforward. Capitalists react to fluctuations in the level of output in making their investment decisions; thus if fluctuations do not occur, neither does investment, and hence the level of income remains constant. If fluctuations occur, then (given anything other than perverse initial values) the system will expand without limit if capitalists are "aggressive"—that is, they desire a higher capital to output ratio than the productive system in fact possesses. If they are "passive"—wanting a lower ratio—then investment will gradually diminish (compared with the level of output), leading to a stationary level of capital and a stationary state. Continuous random shocks will maintain both cycles and growth, in contrast to Hicks and Frisch-type models where only cycles are generated.

The effective stability of the system (the disappearance of Harrod's knife-edge, despite this system's mathematical instability) emanates from the reactive, stock-adjustment perspective postulated for the investment behaviour of capitalists. Since they react to changes in output, if that change was great two periods ago, then investment will spurt in the current time period, leading to a consequent spurt in output via the accelerator. The subsequent investment will necessarily be less, however, since the next time period in capitalists' backward-looking investment horizon will necessarily involve a smaller change in output, leading to a lower rate of output growth.

A more sophisticated version of this model would introduce a forward-looking perspective to investment, nonlinear investment and accelerator functions, and would generate asymmetric cycles which would go some way towards meeting the objections set out in Blatt 1980. However, as it stands, this constant coefficients linear model is a powerful demonstration of the efficacy of Keynesian concepts for modelling cyclical growth in a dynamic economy. It establishes that it is possible to realise the apparently conflicting visions of Harrod and Hicks, using mathematical techniques which were well known when they made their seminal contributions. One can only speculate as to how growth and trade cycle theory may have evolved had this simple resolution been achieved in their day.

3.8 Minsky and nonlinearity

Minsky's abandonment of the quest to put his model into mathematical form can thus be traced to the flawed nature of the model of cycles onto which he attempted to graft the Financial Instability Hypothesis. While the model developed in this chapter is free of that flaw, it remains inadequate for Minsky's purposes because of its linearity, since it is easily shown that Minsky's Hypothesis is fundamentally based on a nonlinearity—the reaction of capitalists to conditions of constant economic growth.

Minsky argues that capitalist expectations will rise during a period of sustained economic growth, since such a period will always have been preceded by a period of crisis which depressed expectations. Using E for expectations and π for the rate of profit, this implies a relation of the form

$$(33)\ \frac{1}{E} \cdot \frac{dE}{dt} = \frac{dY}{dt} \text{ or } \frac{1}{E} \cdot \frac{dE}{dt} = \pi$$

As a nonlinear insight, its proper foundation is a truly nonlinear model of the trade cycle. The model I have chosen to use is Goodwin's 1967 predator-prey model of cyclical growth. In the next chapter, I outline this model and extend it in preparation for the development of models of the Financial Instability Hypothesis in Chapters Five and Six.

4 Goodwin's Growth Cycle

Economics has a long tradition of borrowing concepts from the physical sciences, with the dominant source being mechanics (Mirowski, 1984). Goodwin (1967) instead turned to biology when he drew an analogy between predator-prey cycles in nature and the social phenomenon of the trade cycle. Goodwin was attempting to put into mathematical form the analysis given by Marx in Chapter 25 of Capital Volume I:[26]

> a rise in the price of labor resulting from accumulation of capital implies ... accumulation slackens in consequence of the rise in the price of labour, because the stimulus of gain is blunted. The rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e. the disproportion between capital and exploitable labour power. The mechanism of the process of capitalist production removes the very obstacles that it temporarily creates. The price of labor falls again to a level corresponding with the needs of the self-expansion of capital, whether the level be below, the same as, or above the one which was normal before the rise of wages took place... To put it mathematically, the rate of accumulation is the independent, not the dependent variable; the rate of wages the dependent, not the independent variable. (Marx 1867, 1954: 580-581)

Goodwin saw this argument as being strongly akin to that underlying analysis of predator-prey interaction in biology.

4.1 Predator-Prey Models

These were a relatively recent innovation intended to explain the commercial fishing statistics of World War I, when reduced fishing levels during the War led to the apparently inexplicable result of a relative increase in the number of predators. Since no explanation for this phenomenon was extant in biology, the biologist D'Ancona asked the mathematician Volterra whether one could be found in dynamics (Braun 1993: 444).

Volterra's model assumed that the feed available to the prey was effectively limitless, so that in the absence of predators their population would grow at the constant rate of *a*% p.a.. Using *p* for the number of prey, this can be modelled as:

(34) $$\frac{1}{p}\frac{dp}{dt} = a$$

The impact of predators depends on the number of interactions between themselves and prey, which depends on how many predators there are. Using *P* for the number of predators, this gives:

(35) $$\frac{1}{p}\frac{dp}{dt} = a - b \cdot P$$

where the constant *b* indicates the number of fatal contacts between predators and prey per unit of time. Predators, on the other hand, would starve to death at the constant rate *c*% p.a. in the absence of prey:

(36) $$\frac{1}{P}\frac{dP}{dt} = -c$$

The growth of predator numbers depended on their catching prey, which Volterra again modelled as a simple product:

(37) $$\frac{1}{P}\frac{dP}{dt} = -c + d \cdot p$$

where the constant d indicates the impact of each fatal predator-prey interaction on the reproduction of predators. The complete predator-prey interaction is thus described by

$$(38)\quad \begin{aligned} \frac{dp}{dt} &= a \times p - b \times p \times P \\ \frac{dP}{dt} &= -c \times P + d \times p \times P \end{aligned}$$

The solution to this pair of coupled differential equations describes an interdependent cycle in predator and prey numbers which can easily be described verbally:

- An initially high number of prey and low number of predators leads to a rapid growth in the number of prey;
- The large number of prey enables the predator population to expand;
- The growing predator population reduces the prey population;
- The reduced prey population leads to the predators dying off, thus restoring the cycle.

Nonlinearity, a key concept in Goodwin's analysis of capitalism,[27] plays a crucial role here: the reversal of the initial growth in prey numbers is guaranteed, because the growth of prey numbers is a linear function of the number of prey, and a nonlinear function of the *product* of the number of prey and the number of predators. The inverse nonlinearity likewise applies to predator numbers.

Goodwin's genius was to see that Marx's argument above can be put into a more complex but nonetheless similar cycle of causation to that captured by Volterra's equations:

- The level of output determines the rate of employment, so that a high initial level of output requires a high rate of employment;
- The rate of employment determines the rate of change of wages, so that a high rate of employment results in a high rate of change of wages;
- The level of wages determines the rate of profit, so that a high rate of change of wages means falling levels of profit;
- The rate of profit determines the level of investment, so that falling profit means low rates of investment;
- The level of investment determines the rate of growth of the capital stock, so that low rates of investment means slow or negative growth in the capital stock;
- The capital stock determines the level of output, so that a slowly growing or declining capital stock means static or falling levels of output, which will eventually lead to falling wages;
- There is thus a cycle between the level of employment and the level of wages.

4.2 The basic model

Goodwin's model begins with the output-employment relation, with the direction of causation being from output Y to employment L (a is the level of labor productivity):

(39) $L = \frac{Y}{a}$

Goodwin assumed constant growth in labor force productivity (at α % p.a.) and population growth (at β % p.a.):

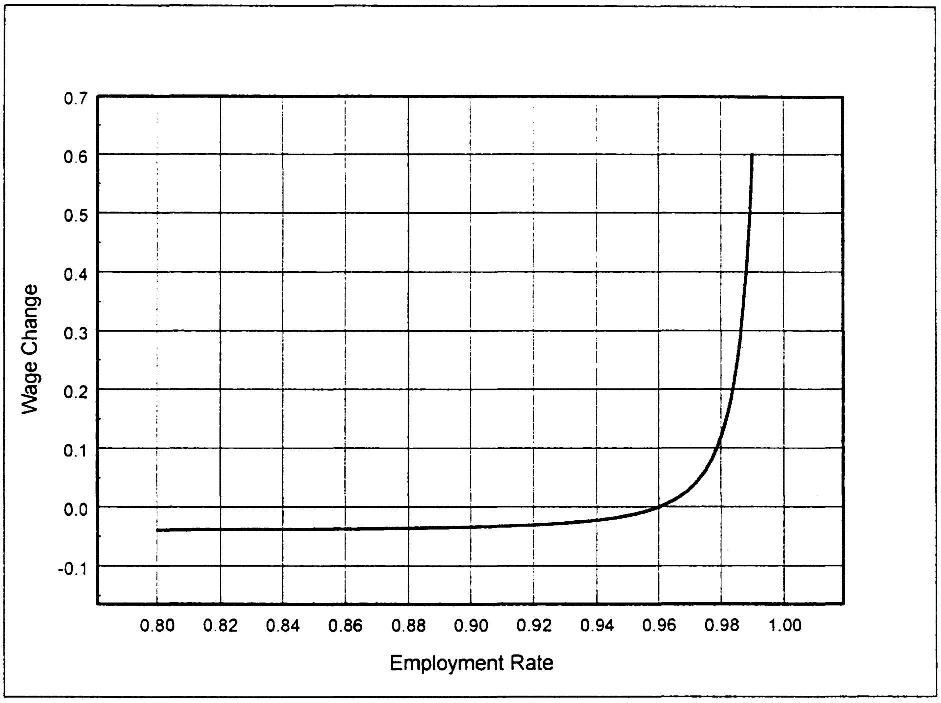
(40) $$a = a_0 \cdot e^{\alpha \cdot t}$$
$$N = N_0 \cdot e^{\beta \cdot t}$$

The sole nonlinearity in Goodwin's model was a "Phillip's curve" relation between the level of employment λ and the rate of increase of wages w:

(41) $$\frac{1}{w}\frac{dw}{dt} = w(\lambda) \text{ where } \lambda = \frac{L}{N}$$

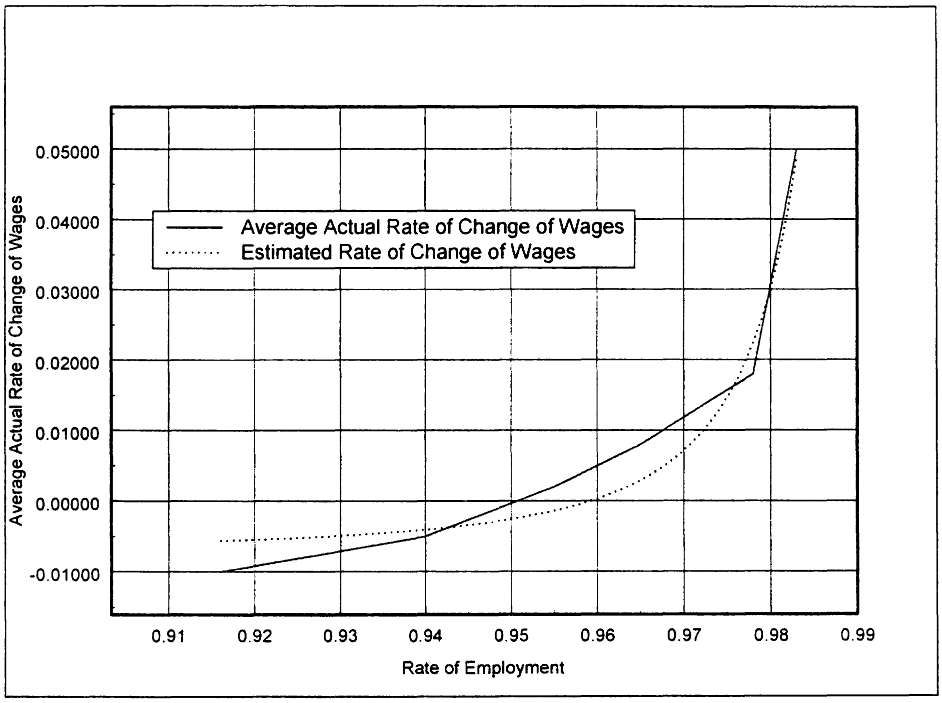
While the symbolic derivation can be done without a functional form, one is needed for the subsequent numerical simulations. The equation used is that first suggested by Blatt of $w(\lambda) = \frac{A}{(B - C \times \lambda)^2} - D$ (1983: 213), while the parameter values used (A=.0000641, B=1, C=1, D=.0400641) reproduce Blatt's simulation results (209).[28] This equation is plotted in Figure 10, which shows that these parameter values result in a constant real wage at an unemployment rate of 3.6%, real wage cuts at higher levels of unemployment (to a maximum of 4% per period), and real wage rises at lower rates (rising asymptotically at full employment):

Figure 10 **"Phillips Curve" Wage Change Function**



The functional form itself can be justified by reference to the nonlinear regression employed by Phillips in his original statistical work (Phillips 1958: 290) on the relationship between the unemployment rate and the rate of change of money wages. Though Phillips used a exponential form, this approximates the asymptotic form employed by Blatt, and as indicated by the regression and average data plots in Figure 11, with appropriate parameter values Blatt's function can fit the data Phillips used to construct his exponential fit:[29]

Figure 11 Regression of Average Phillips Data Against Blatt's Functional Form



The asymptotic form employed by Blatt is preferable to Phillip's exponential form in these simulations because it confines the employment rate to below 100 per cent. The unbounded exponential form employed by Phillips could conceivably return an employment rate of more than 100 per cent.

Goodwin used the Kaleckian assumption that workers spend all their wages and capitalists invest all their profits. Expressing this as a proportion of total output, profit share of output π_S is identical to the investment to output ratio:

(42) $\frac{\Pi}{Y} = \pi_S = \frac{I}{Y}$

Gross investment, also expressed as a proportion of output, is thus:

(43) $\frac{I}{Y} = 1 - \frac{W}{Y} = 1 - \omega$

(where ω is the wages share of output and W is the total wages bill). Investment is also the change in capital stock:[30]

(44) $$I = \frac{dK}{dt}$$

Finally, the capital stock determines output via a linear accelerator:

(45) $$Y = \frac{K}{v}$$

Since output in turn determines employment, the system is closed, and from it can be derived two coupled differential equations for the rate of change of the wages share of output and of the rate of employment:

(46)
$$\begin{aligned}
\frac{d\omega}{dt} &= \frac{d}{dt}\left(\frac{w \cdot L}{Y}\right) \\
&= \frac{d}{dt}\left(\frac{w \cdot L}{a \cdot L}\right) \\
&= \frac{1}{a} \cdot \frac{dw}{dt} - \frac{w}{a^2} \cdot \frac{da}{dt} \\
&= \frac{1}{a} \cdot w \cdot w(\lambda) - \frac{w}{a^2} \cdot \alpha \cdot a \\
\frac{d\omega}{dt} &= \omega \cdot (w(\lambda) - \alpha)
\end{aligned}$$

$$
\begin{aligned}
\frac{d\lambda}{dt} &= \frac{d}{dt}\left(\frac{L}{N}\right) \\
&= \frac{1}{N}\cdot\frac{dL}{dt} - \frac{L}{N^2}\cdot\frac{dN}{dt} \\
&= \frac{1}{N}\cdot\frac{d}{dt}\left(\frac{Y}{a}\right) - \frac{L}{N^2}\cdot\beta\cdot N \\
&= \frac{1}{N}\cdot\left(\frac{1}{a}\frac{dY}{dt} - \frac{Y}{a^2}\frac{da}{dt}\right) - \frac{L}{N}\cdot\beta \\
&= \frac{1}{N}\cdot\left(\frac{1}{a}\frac{d}{dt}\left(\frac{K}{v}\right) - \frac{Y}{a^2}\alpha\cdot a\right) - \lambda\cdot\beta \\
&= \frac{1}{N}\cdot\left(\frac{1}{a}\cdot\frac{1}{v}I - L\cdot\alpha\right) - \lambda\cdot\beta \\
&= \frac{1}{N}\cdot\left(\frac{1}{a}\cdot\frac{1}{v}\frac{Y-W}{v}\right) - \frac{L}{N}\cdot\alpha - \lambda\cdot\beta \\
&= \frac{1}{N}\cdot\left(\frac{1}{a}\cdot\frac{1}{v}(a\cdot L - w\cdot L)\right) - \lambda\cdot\alpha - \lambda\cdot\beta \\
&= \frac{L}{N}\cdot\left(\frac{1-\frac{w}{a}}{v}\right) - \lambda\cdot\alpha - \lambda\cdot\beta \\
\frac{d\lambda}{dt} &= \lambda\cdot\left(\frac{1-\omega}{v} - \alpha - \beta\right)
\end{aligned}
\tag{47}
$$

The reduced form of the Goodwin model is thus:

$$
\begin{aligned}
\frac{d\omega}{dt} &= \omega\cdot(w(\lambda)-\alpha) \\
\frac{d\lambda}{dt} &= \lambda\cdot\left(\frac{1-\omega}{v} - \alpha - \beta\right)
\end{aligned}
\tag{48}
$$

While these equations are more complex than those in Volterra's model, the basic features are the same: the rate of change of employment is a positive linear function of the current level of employment, and a negative nonlinear function of the *product* of employment and wages share; the rate of change of wages share is a positive nonlinear function of the *product* of current wages share and the wage change function, and a negative linear function of the current wages share. It is also possible to interpret the two reduced form equations intuitively: the equation for the percentage rate of change of the wages share of output states that this equals the percentage rate of change of the real wage ($w(\lambda)$) minus the percentage rate of change of labor productivity; the equation for the percentage rate of change of the rate of employment says that this equals the percentage rate of growth of output minus the percentage rates of change of labor productivity and population, since it is easy to show that $\frac{1-\omega}{v}$ equals the percentage rate of growth of output:

$$
\begin{aligned}
\frac{1}{Y}\frac{d}{dt}Y &= \frac{1}{Y}\frac{1}{v}\frac{d}{dt}K \\
&= \frac{1}{Y}\frac{1}{v}I \\
&= \frac{1}{Y}\frac{1}{v}\Pi \qquad (49) \\
&= \frac{1}{Y}\frac{1}{v}(Y-W) \\
&= \frac{1-\omega}{v}
\end{aligned}
$$

This system can be solved analytically by considering the rate of change of wages share with respect to employment:

$$
\begin{aligned}
\frac{\frac{d\omega}{dt}}{\frac{d\lambda}{dt}} &= \frac{\omega\cdot(w(\lambda)-\alpha)}{\lambda\cdot\left(\frac{1-\omega}{v}-\alpha-\beta\right)} \qquad (50) \\
\frac{1}{\omega}\left(\frac{1-\omega}{v}-\alpha-\beta\right)\frac{d\omega}{dt} &= \frac{(w(\lambda)-\alpha)}{\lambda}\cdot\frac{d\lambda}{dt}
\end{aligned}
$$

The equations are now in separable form, and they can both be integrated with respect to time to yield:

$$(51)\ \int\frac{1}{\omega}\left(\frac{1-\omega}{v}-\alpha-\beta\right)d\omega = \int\frac{(w(\lambda)-\alpha)}{\lambda}d\lambda$$

This equality between two integrals with respect to different variables can only apply if they both equal the same constant. Integration yields the interdependent solutions:

$$\text{(52)}\quad \omega(t) = e^{\left(\frac{A}{(1-\lambda(t))^2}-D-\alpha\right)\cdot t}$$
$$\lambda(t) = e^{\left(1-v\cdot\alpha-v\cdot\beta-\frac{\omega(t)}{v}\right)\cdot t}$$

These two equations describe a closed cycle, the mathematical form for the cyclical vision sketched by Marx. The process is distinctly a non-equilibrium one: unless the system begins with the equilibrium values for λ and ω, it will forever gravitate around them. Unlike cycles in a linear system, the nonlinear cycles of this model have a long term impact: while the average value of wages share equals the equilibrium value, the average value of employment (though not the average rate of growth) is lower the further the system is from equilibrium (Blatt 1983: 215-216).

Marx was the inspiration for this model, and Marx is normally associated with an apocalyptic secular vision of capitalism, with the relative or absolute immiserisation of the working class leading to the breakdown of capitalism. However in the passage cited above, such a perspective is absent, and it is likewise absent from Goodwin's model. Instead, he argued that his model supported the empirical outcome that "wages rates went up; profit rates stayed down" (Goodwin 1967: 58). This paints workers as the ultimate long-run beneficiaries of a symbiotic relationship with capitalists (and the means of production).

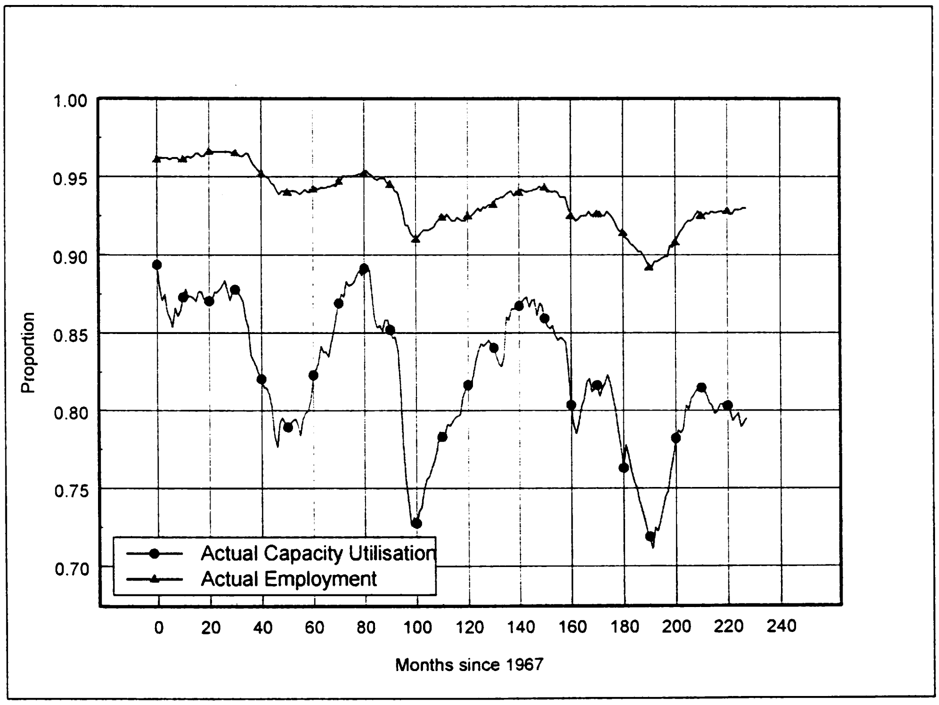
Goodwin's model has been the inspiration for a large range of cyclical models (Desai 1973, 1995, Sasakura 1992, Skott 1989, Sportelli 1995, Yabuta 1993) with flavours ranging from Marxian to neoclassical. While being a strong advocate of Goodwin's model, Blatt regarded the neutral (neither stable nor unstable) equilibrium of the model, and the fact that it could generate arbitrarily large cycles, as flaws (1983: 210). He surmised that these could be overcome by more disaggregation or by the "introduction of a financial sector". In the following sections, I show that the same ends result from two more immediate extensions to the model. The first modification replaces the fixed accelerator coefficient between the capital stock and output by a relation which simulates procyclical variations in capacity utilisation. The second replaces the assumption of constant disembodied technical progress with the partially endogenous determination of the rate of technical change. These alterations, singly or in concert, convert the model from one which has a neutral equilibrium and a closed loop in phase space to one which has an unstable equilibrium and an open loop. Finally I prepare the model for the introduction of finance—and of Minsky's FIH—by replacing the linear investment function with a nonlinear one. Depreciation is also introduced at the constant rate of γ per cent per period.

4.3 Variable capacity utilisation

The base motivation of capitalists in Goodwin's growth cycle is profit, and the model's driving force is investment. The rate of profit determines the level of investment, the level of investment determines the addition (if any) to the capital stock, the capital stock determines output, output determines employment, and employment determines the rate of growth of wages, which in turn determines the rate of profit. The link between capital stock and output is rigid, and based on the statistical regularity that the ratio of the capital stock, broadly measured, to output, broadly measured, is relatively constant.

However this omits the incontrovertible fact that the rate of capacity utilisation—the proportion of the capital stock actually used to produce output—varies pro-cyclically, very much in step with the rate of employment. The fact that both labour and capital become unemployed when a capitalist economy turns down is starkly evident in Figure 12, which plots the monthly employment rate and monthly rate of capacity utilisation for the USA from 1967 till 1985:

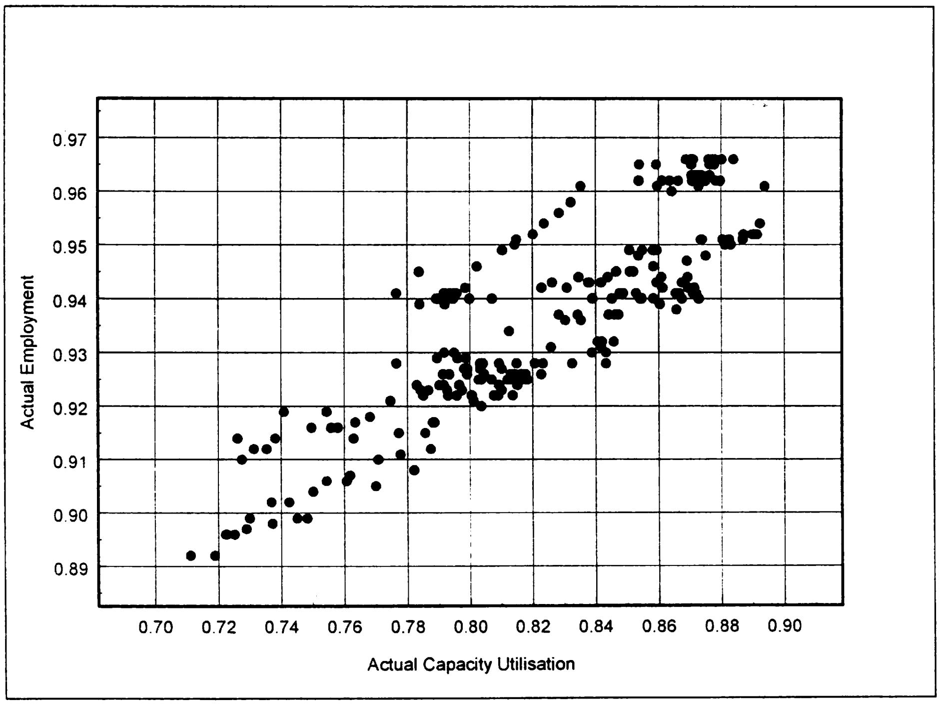
Figure 12 USA Rate of Employment & Rate of Capacity Utilisation, 1967-1985



(The data source for capacity utilisation was the Federal Reserve Statistical Release; the data file can be found at www.bog.frb.fed.us/releases/g17/iphist/utlhist.sa. The data source for the rate of employment was the Bureau of Labor Studies; the data file can be found at gopher://hopi2.bls.gov:70/00hopiftp.dev/special.requests/lf/unemplr.mon.)

Figure 13, which plots the employment rate against the rate of capacity utilisation, makes it obvious that the relationship between the two variables can be reasonably approximated by a linear function. The R^2 of .75 for a linear regression confirms that, to normal statistical standards, a linear function does fit the data.

Figure 13 USA Rate of Employment against USA Rate of Capacity Utilisation, 1967-1985



However a linear relationship implies that either one of the variables can exceed 100 per cent, or fall below zero, when in fact both are constrained in the range of 0 to 100 per cent. A more suitable function to use is a sigmoid:

(53) $$c(\lambda) = \frac{U}{V + e^{-W\cdot(\lambda - X)}}$$ [31]

where U, V W and X are constants and λ is the rate of employment. The sigmoid function has a number of characteristics which make it suitable for modelling capacity utilisation (and economic phenomena in general). Near its minimum, maximum and intermediate values, it approximates a linear function, yet it has maximum and minimum asymptotes, whereas as Figure 15 shows by way of contrast, a linear relation would allow capacity to exceed 100% or fall below 0%. A sigmoid can be given parameter values which result in capacity utilisation rising to 100 per cent for 100 per cent employment, and falling to zero for 100 per cent unemployment:

Figure 14, which plots a regression of this function against the USA employment and capacity utilisation data for the period 1967-1985,[32] indicates that this functional form can fit the observed statistical relationship, and that its fit is very close to that achieved with a linear regression:

Figure 14 Regression of Sigmoid Relationship Between Employment and Capacity Utilisation against USA Data, 1967-1985

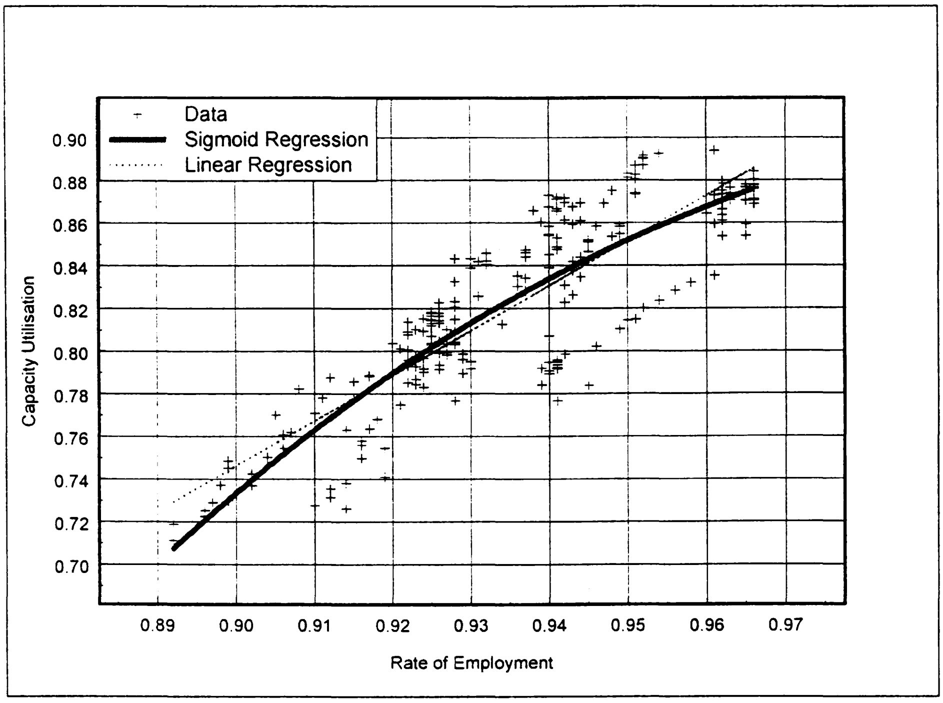
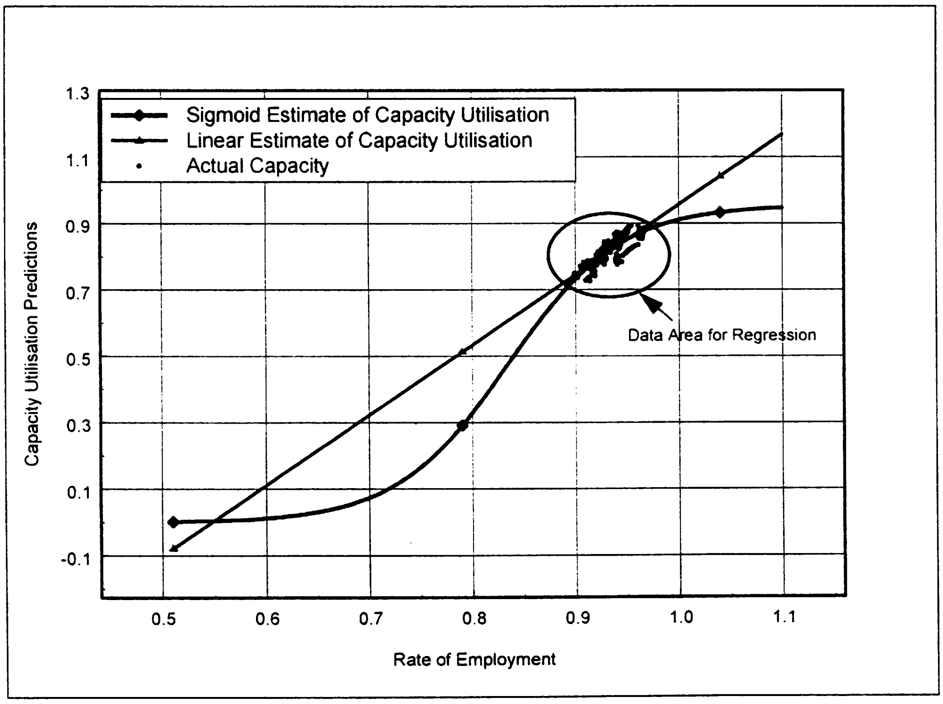


Figure 15 shows that the sigmoid fit is superior to the linear, in that an extrapolation of it yields upper and limits to capacity utilisation, whereas extrapolation of the linear relation predicts no upper or lower limits. For example, the linear extrapolation thus predicts a capacity utilisation level of *minus* 10 per cent when the rate of employment is 50 per cent; the sigmoid extrapolation suggests zero capacity utilisation at 40 per cent unemployment, and never predicts anything less than zero capacity utilisation.

Figure 15 Extrapolation of Sigmoid and Linear Regressions for Employment/Capacity Utilisation Relationship

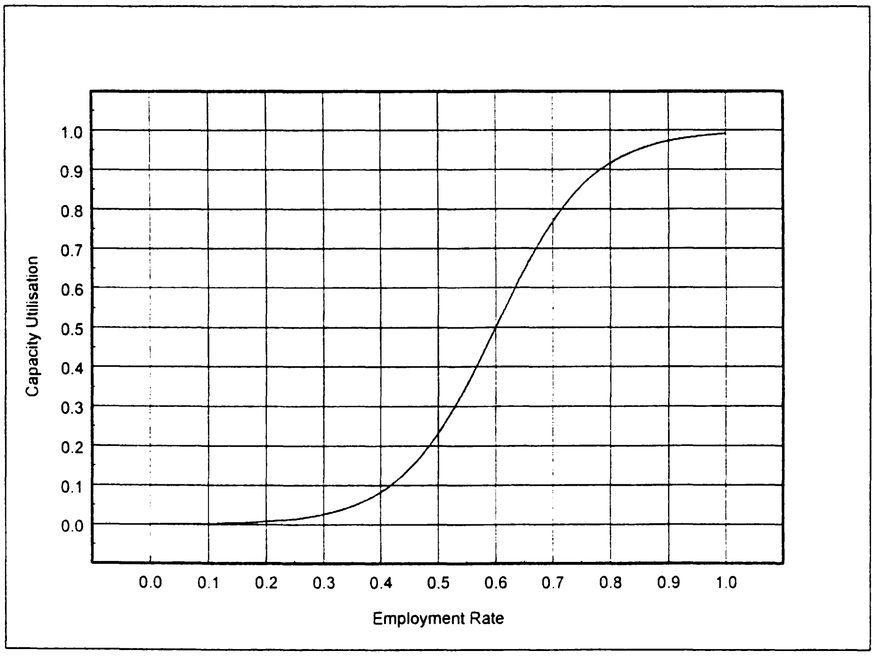


The realism of the basic Goodwin model can thus be improved by replacing the rigid accelerator relation between capital and output with the accelerator times a capacity utilisation function which is a function of the rate of employment:

$$(54)\ \frac{Y}{K} = \frac{1}{v} \cdot c(\lambda)$$

As with the function for the rate of change of real wages, the parameter values used in these simulations (of $U=1$, $V=1$, $W=12$, $X=.6$) are not those derived from this regression (though they are very close; the regression values were *.954*, *.999*, *18.346* and *.835*). The values used were chosen primarily to confine capacity utilisation to the relevant range of zero to *100* per cent, and to achieve zero capacity utilisation at zero employment, and *100* per cent capacity utilisation at *100* per cent employment.[33] This function is plotted in Figure 16, which reveals a relationship between the rate of employment—and hence the level of economic activity—and the rate of capacity utilisation which shows little change for small falls in employment, then progressively larger falls as unemployment becomes significant, tapering to lesser falls as capacity utilisation approaches zero at zero employment:[34]

Figure 16 **Capacity Utilisation Function**



The reduced form of this system is:

$$
\begin{aligned}
&\frac{d\omega}{dt} = \omega \times (w(\lambda) - \alpha) \\
(55)\quad &\frac{d\lambda}{dt} = \frac{\lambda \times \left(\frac{c(\lambda)(1-\omega)}{v \cdot} - \gamma - \alpha - \beta \right)}{1 - \lambda \cdot \frac{U}{V + e^{-W \cdot (\lambda - X)}} \cdot e^{-W \cdot (\lambda - X)}}
\end{aligned}
$$
[35]

As the subsequent simulations show, this model addresses Blatt's first objection to Goodwin's basic model, that it has a neutral equilibrium and generates a closed loop rather than a limit or open cycle.

4.4 Variable Technical Change

In common with many economic models, the basic Goodwin model treats the rate of change of productivity as exogenous.[36] In fact, though the technical innovation-economic growth relation is at one and the same time the most accepted and least understood relation in growth theory, there is little doubt that just as technical change enhances growth, so growth enhances technical change (see Verspagen 1994 and Scheidner & Ziesemer 1994 respectively for recent surveys on the empirical evidence, and the relationships postulated by "new growth theory" between technical change and economic growth). The Goodwin model can therefore be made more realistic by postulating a feedback between the rate of growth, and the rate of technical change.

The most obvious source of this feedback is the relationship between the rate of profit and the rate of private investment in research and development: an increased rate of profit should lead to more funds being available for research and development, and this in turn should produce a higher rate of technical change. There is however an upper limit to how much the rate of technical change can be increased via research and development, and a lower limit set by the impact of public funding, "learning by doing" and so on. The relationship between the rate of profit and the rate of technical change can therefore be characterised as being bounded between exogenously given upper and lower limits. As with the capacity utilisation function, this points to the sigmoid as being the most appropriate functional form for modelling the rate of profit/technical change relationship. The pure sigmoid has to be modified however to account for the existence of an exogenously given lower limit to technical change. The form used is therefore

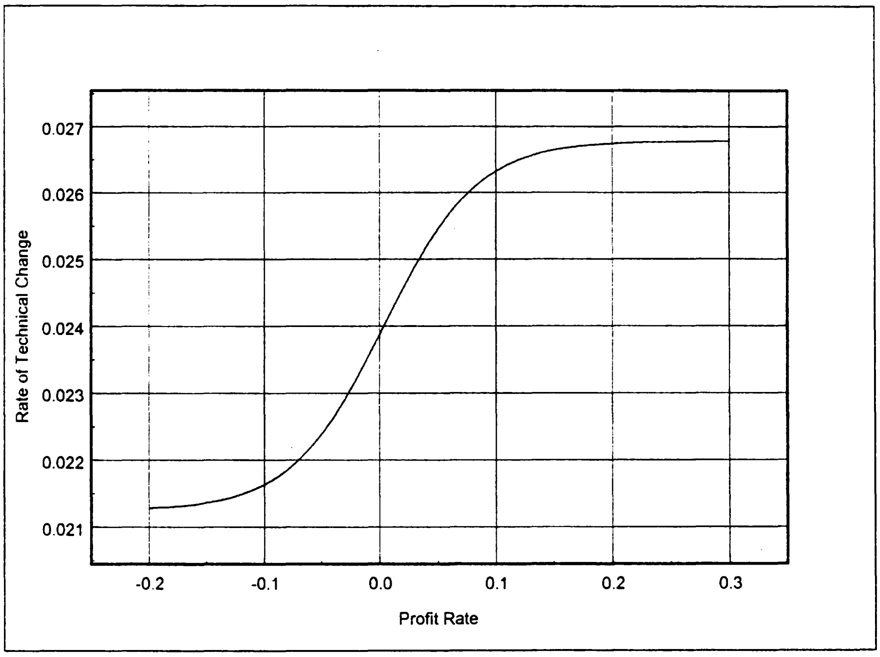
(56) $a(\pi) = \frac{Q}{R+e^{-S \cdot \pi}} - T$

The percentage rate of change of technical change is now given by the equation:

(57) $\frac{1}{a}\frac{da}{dt} = \alpha \cdot a(\pi)$

The parameter values used for this simulation were *Q=.2, R=1, S=25, T=.15*. This generates a relation where the rate of technical change peaks at 2.67 per cent for rates of profit of 20% and above, and tapers to a minimum of 2.1% for rates of profit of minus 30% and below. This range of values for the rate of growth of labor productivity is compatible with the range of rates of growth found empirically (see, for example, Verspagen 1994: 19)

Figure 17 Technical Change Function



The reduced form of this system is:

$$(58)\quad \begin{aligned} \frac{d\omega}{dt} &= \omega \times (w(\lambda) - \alpha \cdot a(\pi)) \\ \frac{d\lambda}{dt} &= \lambda \times \left(\frac{1-\omega}{v} - \gamma - \alpha \cdot a(\pi) - \beta \right) \end{aligned}$$

This model, like the preceding one, converts Goodwin's closed loop into an unstable open cycle.[37]

4.5 The generalised Goodwin growth cycle

The separate extensions made above can be combined to produce a model in which both technical change and capacity utilisation are endogenously determined (within the limits set by exogenously determined parameters). Its reduced form is:

$$(59)\quad \begin{aligned} \frac{d\omega}{dt} &= \omega \times (w(\lambda) - \alpha \cdot a(\pi)) \\ \frac{d\lambda}{dt} &= \frac{\lambda \times \left(\frac{c(\lambda)\pi_S}{v} - \gamma - \alpha \cdot a(\pi) - \beta \right)}{1 - \lambda \cdot \frac{W}{V + e^{-W\cdot(\lambda - X)}} \cdot e^{-W\cdot(\lambda - X)}} \end{aligned}$$

4.6 A nonlinear investment function

The models specified above still contain one glaringly unrealistic element: capitalists are assumed to invest all their profits. While this assumption accords with conventional Kaleckian practice, it is inappropriate in a truly dynamic analysis of capitalism, for two reasons.

Firstly, it takes no account of capitalist behaviour in the determination of investment which, in Kaleckian analysis, is of fundamental importance in determining the performance of the capitalist economy. This assumption stands in marked contrast to Minsky's own analysis of the cyclical nature of capitalist expectations and their role in the determination of investment, and he was aware of this, despite his later reliance upon Kaleckian identities (see for example Minsky 1977, 1982:81-82, where he describes these assumptions as "heroic").

Secondly, the assumption that capitalists invest all their profits makes investment a linear function of the rate of profit. As Figure 18 indicates, such a function of course predicts negative investment when profits are negative. In the context of Goodwin's model, this assumption also has the result that capitalist investment is lowest when the economy is booming, since the profit share of output reaches its perigee when the rate of growth of output has absorbed most of the available labor, thus leading to a blowout in the real wage.

As Keynes emphasises, investment is a function of capitalist expectations of profit, and in a world of fundamental uncertainty these expectations are inordinately influenced by the present performance of the economy (Keynes 1936: 214). A high current level of profit will thus inspire a high rate of investment, and vice versa. As Minsky emphasises, capitalist expectations will become "euphoric" (Minsky 1970, 1982: 120-124) when high profits are sustained, which motivates them to seek out debt to finance those investments which cannot be financed out of retained earnings. Conversely, when profits slump, so too do capitalist expectations, and profits are used to retire debt, rather than to finance new investment. This nonlinear relationship between investment and the rate of profit can be modelled using the asymptotic form used for the wage change function, with parameter values that generate zero investment at zero or lower profits, rising to investment exceeding profits for some level of the rate of profit.[38] This nonlinear investment relation replaces the equation $I=\Pi$ in Goodwin's model with

(60) $I \equiv \frac{dK}{dt} = k(\pi) \times Y - \gamma \times K$

where γ is the rate of depreciation *and* $k()$ is a hyperbolic function of the same form as $w()$:

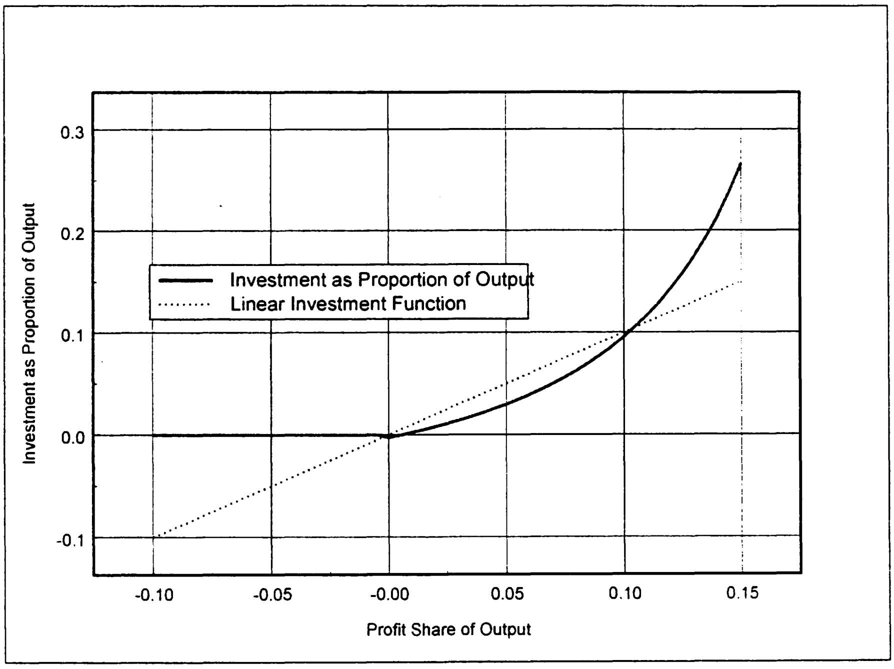
(61) $k(\pi) = \frac{E}{(F-G\times\pi)^2} - H, \pi \geq 0$

The rate of profit is $\pi = \frac{\Pi}{K}$, which can also be expressed in terms of the profit share of output:

(62) $\pi = \frac{\Pi}{K} = \frac{\Pi}{v \times Y} = \frac{\pi_S}{v}$

As Figure 18 indicates, with the parameter values used (E=.0175, F=.53, G=6 and H=.065) the investment function gives zero investment at and below zero profit, rising to equal profits when the profit share of output is 10%, and exceeding profits for higher profit shares.

Figure 18 Investment Function vs Assumption that All Profits are Invested



These modifications to the basic Goodwin model result in the following system:

$$(63)\quad \begin{aligned} \frac{d\omega}{dt} &= \omega \times (w(\lambda) - \alpha) \\ \frac{d\lambda}{dt} &= \lambda \times \left(\frac{k[\pi]}{v} - \gamma - \alpha - \beta\right) \end{aligned}$$

4.7 Nonlinear investment, endogenous technical change and variable capacity utilisation

The combination of all the above extensions resulting in the most generalised version of Goodwin's predator-prey system possible within the confines of a single commodity model.[39] The reduced form is:

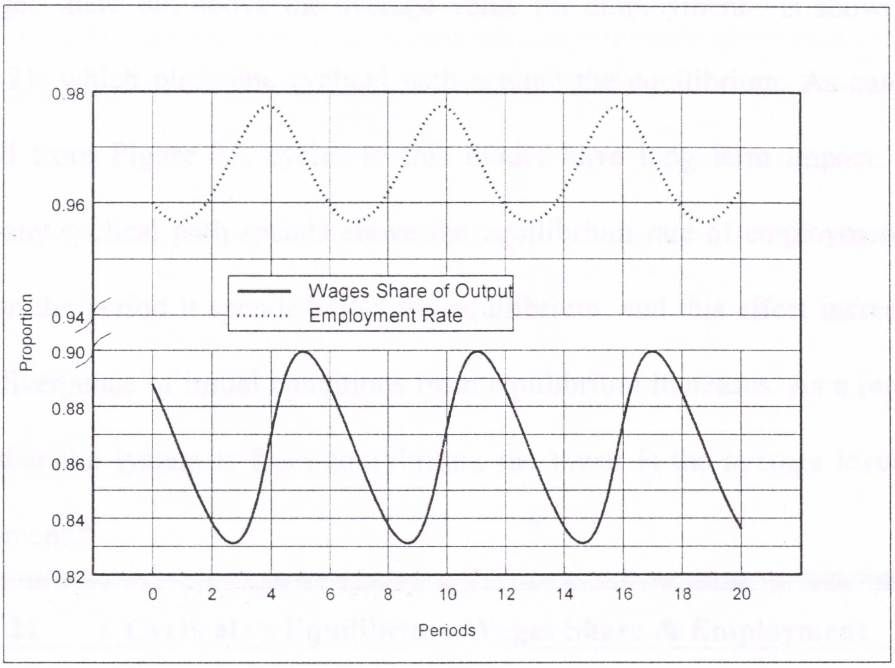
$$(64)\quad \begin{aligned} \frac{d\omega}{dt} &= \omega \cdot (w(\lambda) - \alpha \cdot a(\pi)) \\ \frac{d\lambda}{dt} &= \frac{\lambda \times \left(\frac{c(\lambda)k(\pi)}{v} - \gamma - \alpha \cdot a(\pi) - \beta\right)}{1 - \lambda \cdot \frac{W}{V + e^{-W\cdot(\lambda - X)}} \cdot e^{-W\cdot(\lambda - X)}} \end{aligned}$$

4.8 Simulations

4.8.1 Linear and Nonlinear Investment

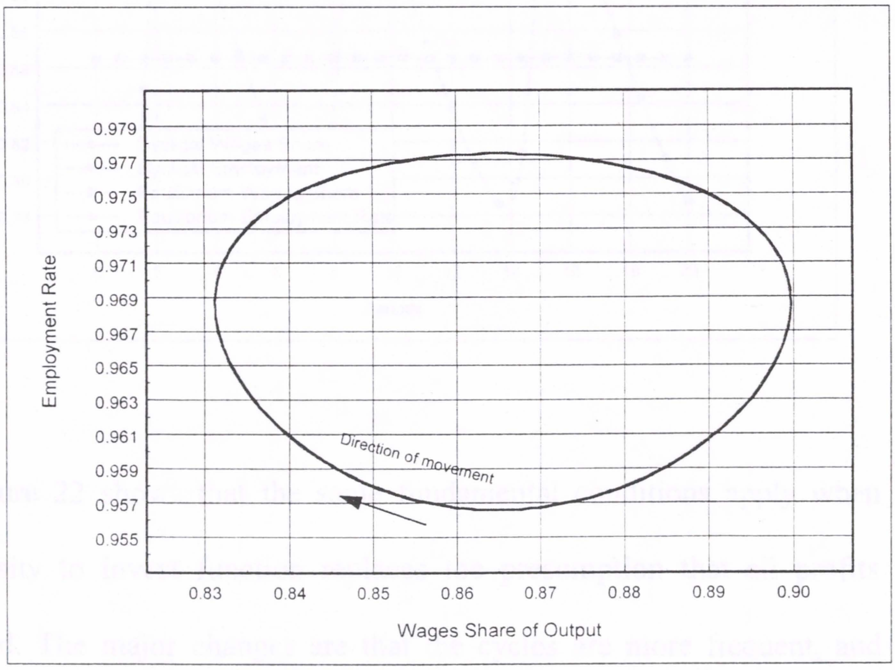
In the basic Goodwin system, where capitalists passively invest all their profits, the driving force in the model is the reaction of workers to the level of employment. An initially above-equilibrium level of employment results in less investment than is needed to sustain the rate of growth of employment above the growth of the workforce, and hence employment falls. Workers accept wage cuts, resulting in a higher profit share, increasing investment and leading to faster output growth, which eventually reverses the decline in employment. However, workers' share of output continues to fall for a while since employment is still below the level at which workers demand a constant real wage, leading to still higher investment, growth, and eventually extreme demands for higher real wages. As Figure 19 indicates, the initial conditions are thus restored and the cycle repeats. The cycles generated by this model are to some extent asymmetric (one of Blatt's tests for the realism of a model of the trade cycle), in that the downward portion of the wage share cycle (of about 4 periods) is significantly longer than the upward phase (of about 2 periods). However the employment rate cycle is almost perfectly symmetric, with both upswing and downswing taking about 3 periods:

Figure 19 Wages Share & Employment Cycles in the Basic Model



These two slightly out of phase cycles are combined in Figure 20 to generate a phase plot which reveals the closed loop generated by the model:

Figure 20 Wage Share/Employment Rate Closed Loop in the Basic Model



The eccentric location of the equilibrium—apparently at the average value for wages share but above the average value for employment—is shown in Figure 21, which plots one cyclical path against the equilibrium. As can be deduced from Figure 21, cycles in this model have long term impact: the period any cyclical path spends above the equilibrium rate of employment is less than the period it spends below the equilibrium, and this effect increases as the divergence of initial conditions from equilibrium increases. As a result, the further the system is from equilibrium, the lower is the average level of employment:[40]

Figure 21 Cyclical vs Equilibrium Wages Share & Employment

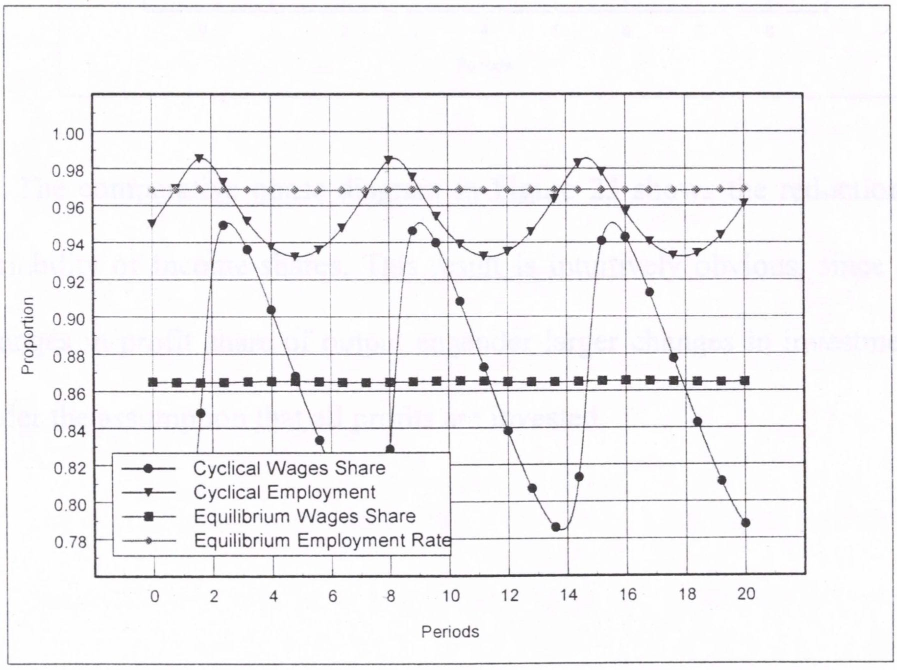
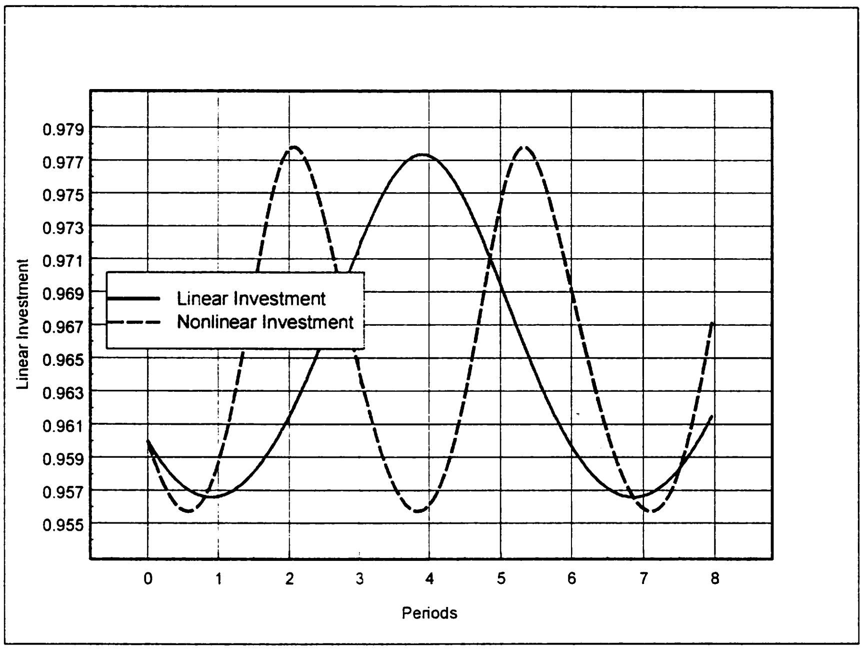


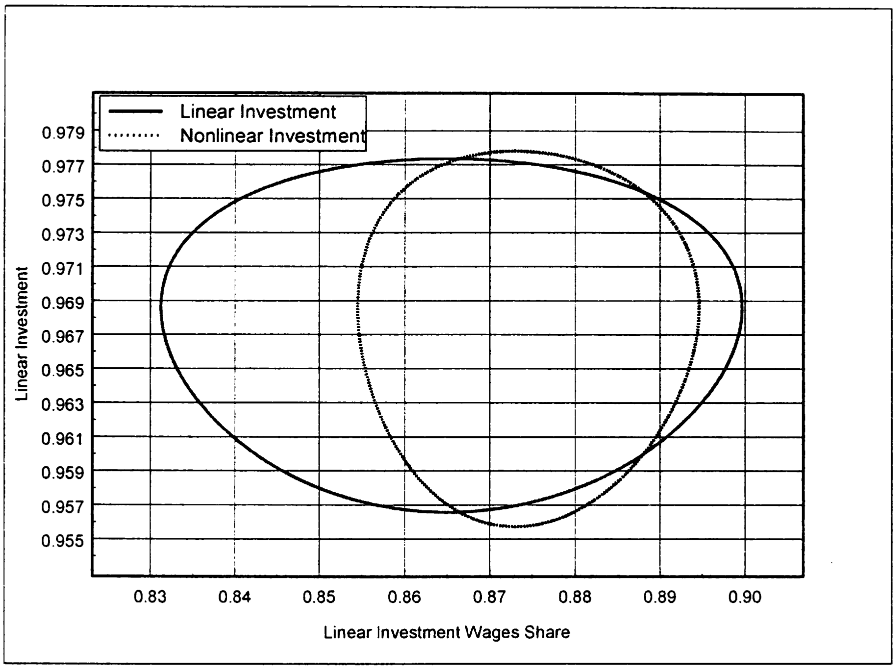
Figure 22 shows that the same fundamental conditions apply when the propensity to invest function replaces the presumption that all profits are invested. The major changes are that the cycles are more frequent, and the variability of income shares reduced. Cycle length decreases from 5.9 periods in the basic Goodwin model to 3.2 periods.

Figure 22 Cycle Duration in the Basic and Nonlinear Investment Models



The comparative phase diagram in Figure 23 shows the reduction in the variability of income shares. This result is intuitively obvious, since smaller changes in profit share of output engender larger changes in investment than under the assumption that all profits are invested.

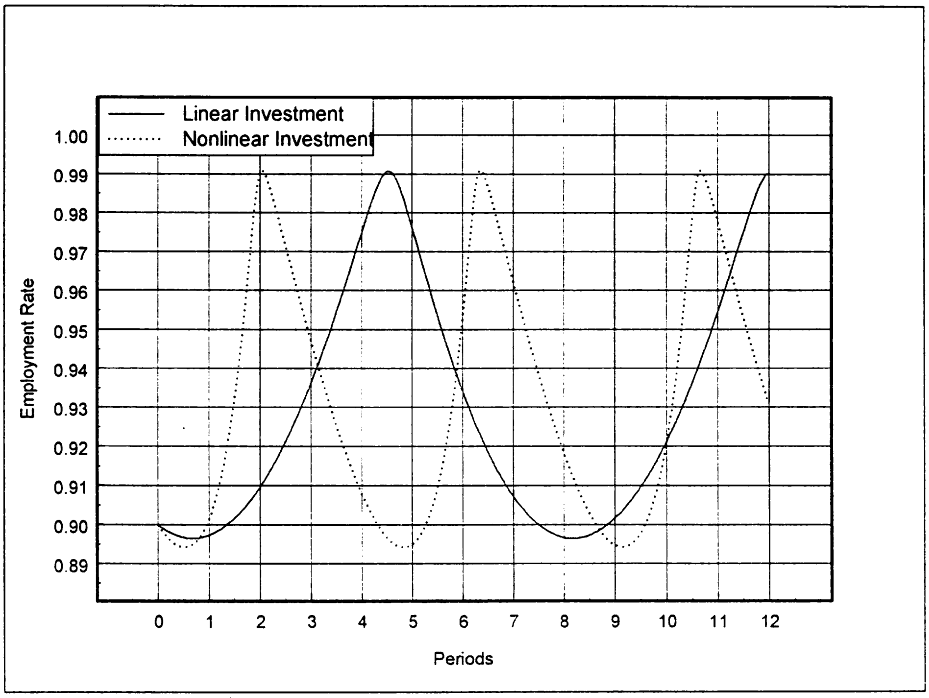
Figure 23 Wages Share & Employment Closed Loops in the Basic and Nonlinear Investment Models



The introduction of a capitalist propensity to invest function thus makes the cycles more frequent (and the time path of wages share and employment more eccentric), but does not fundamentally alter the nature of the system: it still has a stable equilibrium, around which the economy will fluctuate if the initial values are non-equilibrium

The eccentricity of the cycle brings the model closer into conformity with the actual trade cycle, in which downswings are significantly steeper than upswings (Blatt 1983: 227-232). While the basic Goodwin model also generates asymmetric cycles (Blatt 1983: 210), these are more apparent in income shares than in employment. Figure 24 has initial values for λ and ω of .9, and emphasises the extent to which the nonlinear investment function generates more eccentric—and hence more realistic—cycles. Using the employment cycle as the guide to the trade cycle, the nonlinear model spends 64% of its time in the downswing, versus 47% for the linear model (the linear model has a longer downswing in terms of income shares).

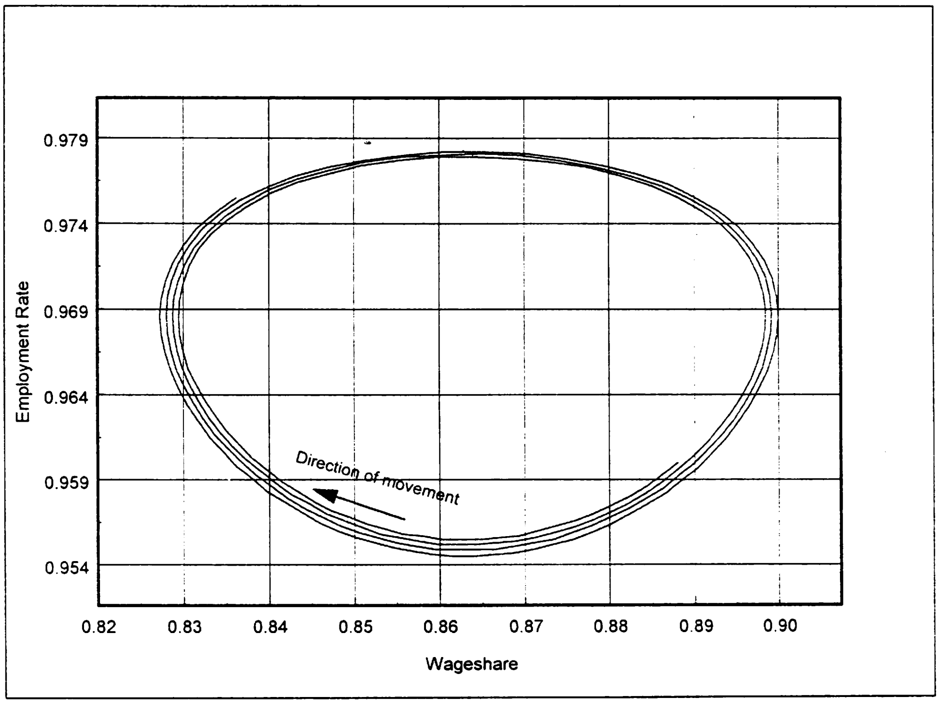
Figure 24 Cycle Shape in Basic and Nonlinear Investment Models



4.8.2 Variable Capacity Utilisation and Partially Endogenous Technological Change

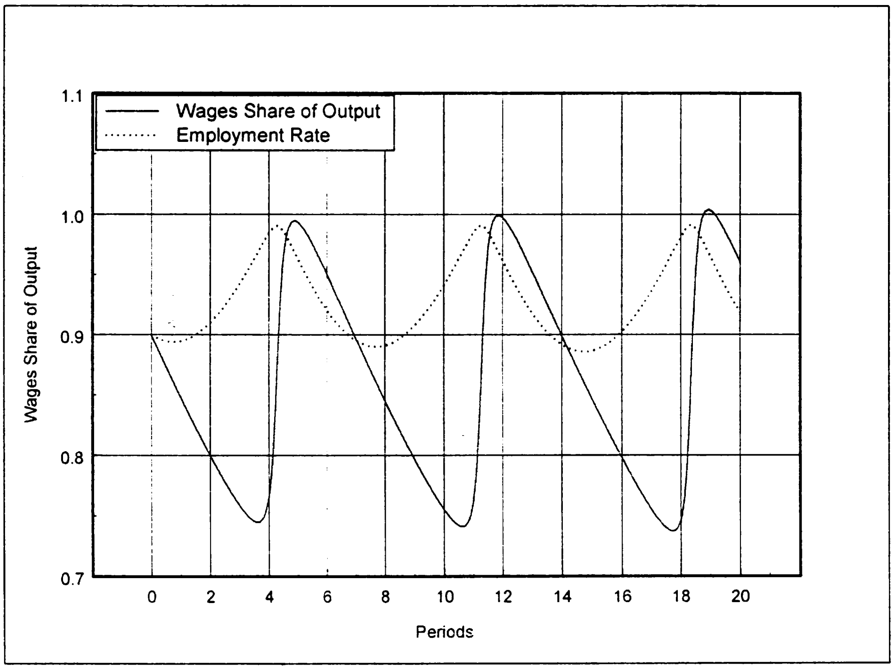
As noted earlier in this Chapter, Blatt (1983: 210) considered that the closed loop generated by the basic Goodwin model was one of its weaknesses, and that further disaggregation, the introduction of a finance sector, or the introduction of a floor to capitalist investment might produce a model whose equilibrium point is "locally unstable, as is desirable" (210). The preceding section indicates that the last of Blatt's nominated alterations, a nonlinear investment function, does not necessarily achieve the end he desired. However the introduction of variable capacity utilisation does result in this fundamental alteration the Goodwin model, replacing a neutral equilibrium with an unstable one, as can be seen from Figure 25. An initial non-equilibrium starting point results in cycles which continue to diverge from the equilibrium, thus resulting in an open loop which indicates increasingly strong cycles as time goes on:

Figure 25 Wages Share/Employment Relation with Variable Capacity Utilisation



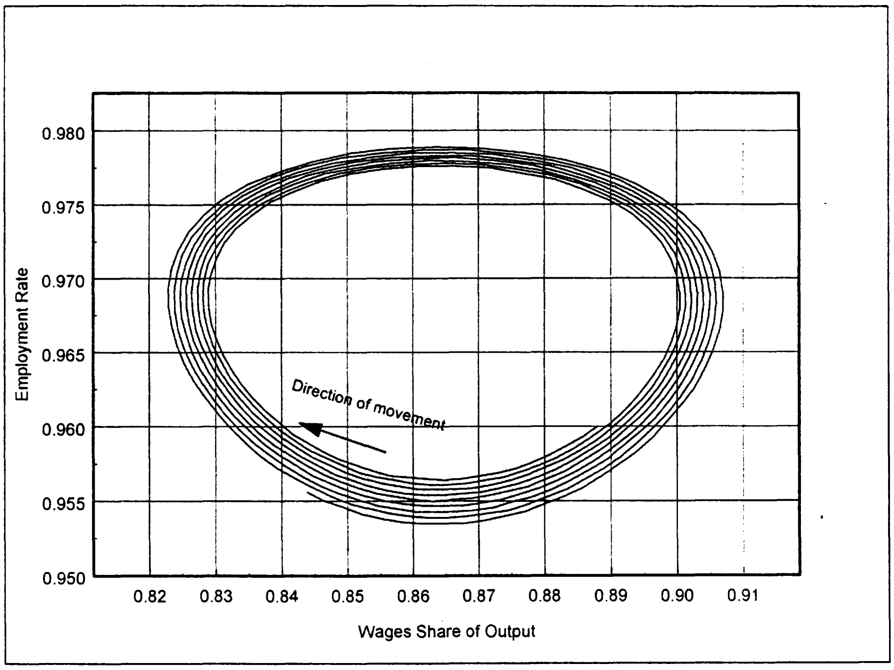
Variable capacity utilisation also increases the eccentricity of the model, and decreases cycle length, again for an intuitively obvious reason. The fall in capacity utilisation when employment is falling increases the rate at which employment falls, thus sharpening the downturn. This leads to a faster restoration of higher levels of profit share, which results in a revival of the rate of growth. The turnaround from slump to boom is therefore faster, while the variability of income shares is increased, and the instability of the system means that these effects are amplified over time. These effects are apparent in Figure 26, which commences with the more extreme initial conditions of (.9,.9).

Figure 26 More Extreme Cycles and Greater Income Variability with Variable Capacity Utilisation



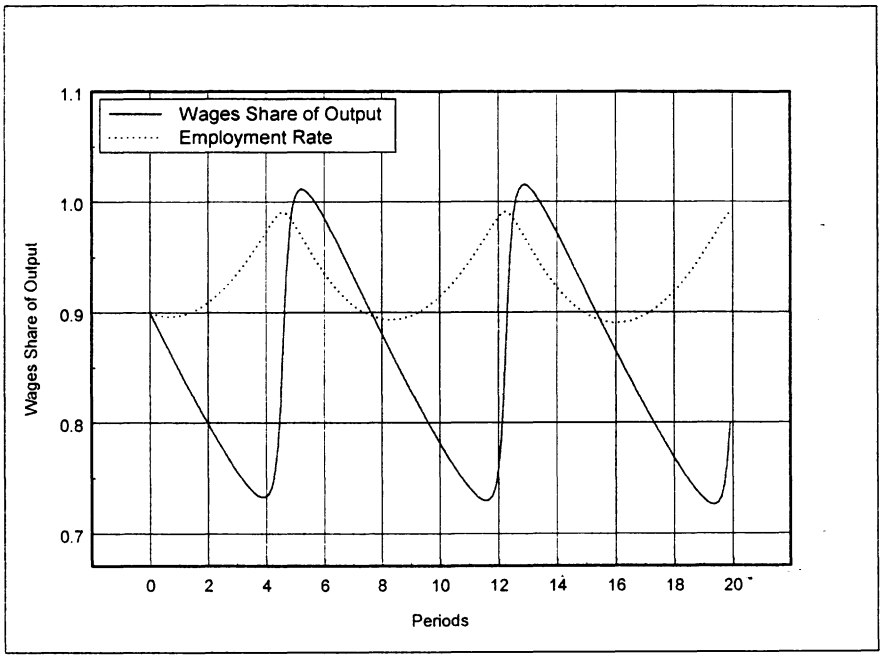
As Figure 27 indicates, the introduction of partially endogenous technical change results in this same fundamental change to the nature of the model. The closed loop of the basic model gives way to cycles of ever increasing intensity.

Figure 27 Wages Share/Employment Relation with Endogenous Technological Change



However the character of the open cycles generated by endogenous technical change differs from that caused by variable capacity utilisation. Whereas variable capacity utilisation acts mainly on the downswing, increasing the fall in employment compared to a model with constant capacity utilisation, endogenous technical change acts on both the upswing and the downswing, increasing the maximum rate of growth achievable as well as decreasing the minimum rate. It thus amplifies the variability of income shares, while extending the duration of the cycle slightly, since the increasing productivity of labour during the boom reduces the rate at which employment rises, thus enabling the boom to last longer. However as Figure 28 indicates using the more extreme initial conditions (.9,.9), this effect is not sufficient to undermine the eccentricity of the cycle.

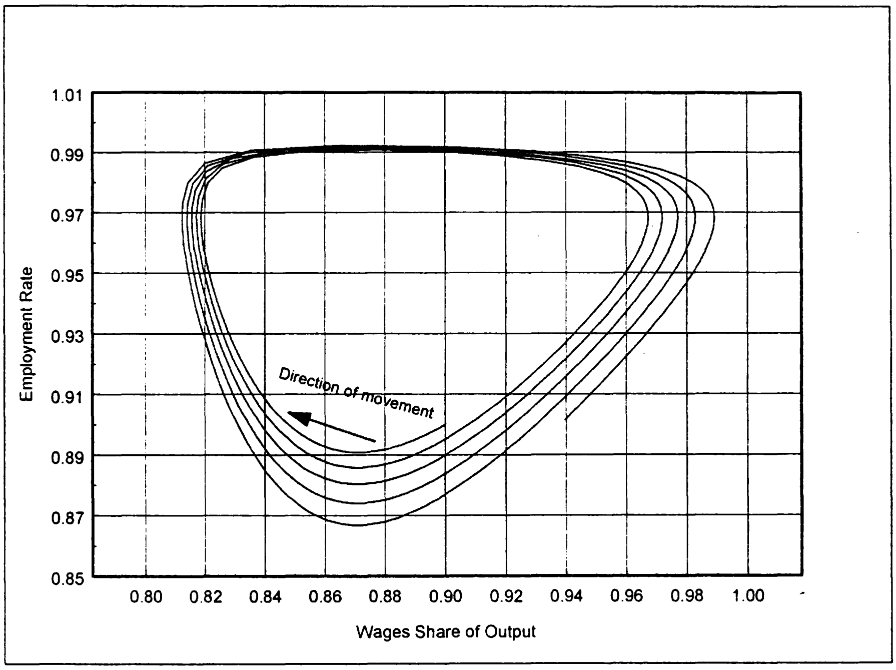
Figure 28 Cycle Shape with Endogenous Technological Change



4.8.3 The combined model

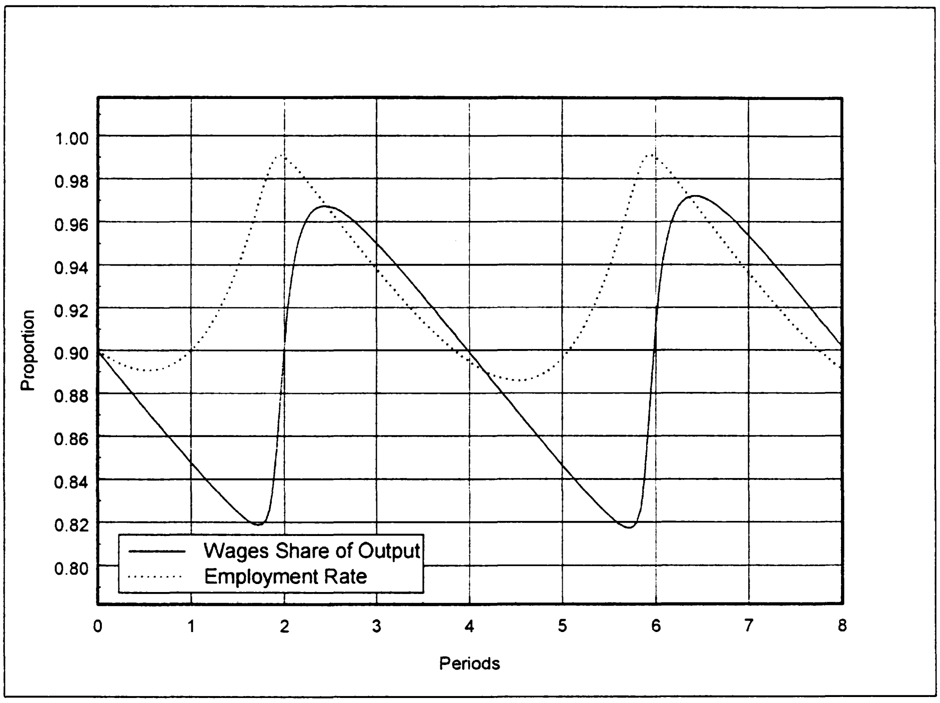
The combined model has a nonlinear investment function, variable capacity utilisation, and endogenous technical change. Figure 29, a phase plot with initial values of (.9,.9), emphasises the instability of the model, with cycles increasing in intensity as time progresses. Upturns are still shorter and sharper than downturns, though the eccentricity is less marked than in the basic model with nonlinear investment.

Figure 29 Wages Share & Employment in the Combined Model



The eccentricity of the cycle generated by the model is apparent in Figure 30, which also starts with the initial conditions of (.9,.9). The model is in an upswing for 1.5 periods and a downswing for the remaining 2.5 periods of a cycle with a duration of 4 periods.

Figure 30 Cycle Eccentricity in the Combined Model



4.9 Conclusion

As Blatt emphasises (Blatt 1983: 211), Goodwin's basic model was an exceptionally good model of the trade cycle in its own right. As many other authors have shown, one of its many virtues is its extensibility, and this Chapter continues this tradition by introducing variable capacity utilisation,[41] with the relatively novel effect of converting the neutral equilibrium of the model into an unstable one.[42]

However the primary objective of this Chapter has been to make the Goodwin model amenable to the introduction of a finance sector, so that Minsky's hypothesis about the potentially destabilising impact of finance upon a capitalist economy can be rendered mathematically. This was achieved by the introduction of a nonlinear investment function, which captures the reality that capitalists invest more than their profits during booms, and less than their profits during slumps. This innovation, which had only a minor impact upon the model in this Chapter, comes into its own in the next two Chapters as we introduce finance and government into the model.

5 Finance and Economic Breakdown

The incorporation of a finance sector into the Goodwin model brings with it the possibility of the accumulation of debt. This enables the simulation of the Fisherian essence of the Financial Instability Hypothesis—that a debt deflation can result from capitalists borrowing more during a boom than they are able to repay during a slump. In the simplest of the simulations, whether a debt deflation actually occurs depends upon the rate of interest charged; in the general model with nonlinear investment, variable capacity utilisation and endogenous technical change, a debt-deflation is inevitable at any rate of interest greater than zero.

Mathematically, the introduction of a finance sector profoundly alters the nature of the model: the two-dimensional Goodwin model is supplanted by a three dimensional model, which brings with it the possibility of chaos.

5.1 A banking sector extension to the Goodwin model

Once the nonlinear nature of investment has been taken into account in the model, it is possible for capitalists to invest more than they earn during a boom—and equally, to invest less than they earn during a slump. The introduction of a nonlinear investment function in the previous chapter was somewhat unrealistic, in that there was no repository for these funds within the model. This chapter closes the model once more by introducing a banking sector which (a) exists to finance capitalist investment when desired investment exceeds profits, and (b) is a repository for capitalist funds when profits exceed desired investment. The banking sector derives its income from interest charged on the balance outstanding. The interest rate is at first treated as exogenous;[43] this extreme "horizontalist" position (see Moore 1988)—in that the finance sector is treated as having an unlimited capacity to finance capitalist investment—is then tempered by a nonlinear interest rate function dependent upon the debt to output ratio, which emulates a lenders' risk premium.[44]

5.2 Variable Interest Rate

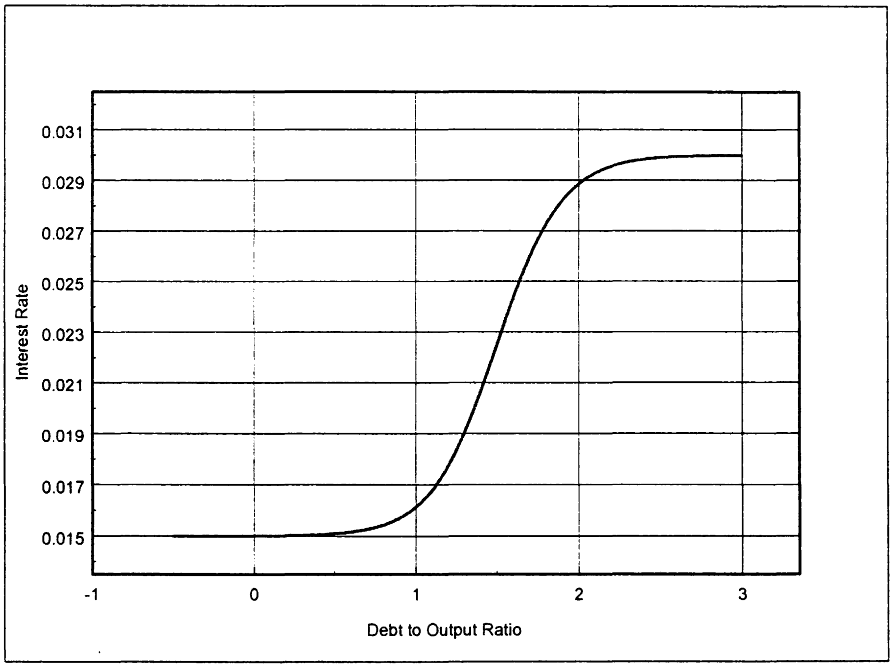
Kalecki's "Principle of Increasing Risk" (Kalecki 1937, 1990: 285-293) and Minsky's concept of marginal lender's risk (Minsky 1977, 1982: 79) both assert that, at the level of the individual firm, an increase in the debt to equity ratio can lead to lenders demanding a higher rate of interest to compensate for the increase in risk. Since Minsky's Financial Instability Hypothesis argues that the accepted corporate debt to equity (and hence debt to output) ratio rises with rising expectations (and falls in a downturn in economic performance), there is no problem with aggregating this individual analysis to the macroeconomic level. It can therefore be argued that the rate of interest should be in part a function of the debt to output ratio.

The question then arises as to the form this functional relationship should take. As with technological change and capacity utilisation, the form employed in this thesis is a generalised sigmoid, $r(d) = \rho \cdot \left(\frac{A}{B+e^{-\Psi\cdot(d-\Delta)}} + E \right)$, where d is the debt to output ratio and ρ, A, B, X, Δ and E are constants. However unlike the capacity utilisation function, the interest rate function is not differentiated in the derivation of the model, and therefore any other functional form could be used without altering the form of the model.

The justification for the choice of the sigmoid is similar to the justification for its choice in the case of technological change. Firstly, as with technological change, the rate of interest can never be negative,[45] and the sigmoid is one functional form which can be so configured as to fulfil this condition. Secondly, again as with technological change, it can be argued that there is an upper limit to interest rates (though this proposition is not as incontrovertible as with technological change): though Third World moneylenders may charge interest rates of 100 per cent per annum, no such rate could be countenanced in a developed capitalist economy.

The parameter values used in these simulations were $\rho = .015$, $A=1$, $B=1$, X=5, $\Delta=1.5$, $E=1$. As Figure 31 indicates, these result in a base rate of interest of 1.5 per cent, with little increase until the debt to output ratio exceeds 50 per cent. From that point it rises, reaching 2.3 per cent at a debt to output ratio of 150%, and tapering to a maximum of 3 per cent at debt ratios of 2.5 and above. These rates are well within historical norms for the real interest rate—while also being significantly lower than the real rates of interest typical of developed countries since 1982[46]—and were chosen to illustrate that the debt-deflationary phenomena was not an artefact of unrealistically high interest rates.

Figure 31 **Interest Rate Function**



5.3 Reduced form equations

The equations needed to introduce a banking sector into the basic Goodwin model are:

(65) The debt ratio: $d = \frac{D}{Y}$

(66) Capitalists use debt solely to finance investment:

$$\frac{dD}{dt} = r \times D + I_g - \Pi$$

(67) Gross investment is a nonlinear function of the rate of profit:

$$I_g = k(\pi) \cdot Y$$

(68) The profit rate is now net of debt repayment commitments:

$$\pi = \frac{Y - W - r \cdot D}{K} = \frac{Y - W - r \cdot D}{v \cdot Y} = \frac{1 - \omega - r \cdot d}{v}$$

(69) Profit share is likewise net of interest on outstanding debt:

$$\pi_s = 1 - \omega - r \cdot d$$

These new definitions result in the following reduced form equations.

5.3.1 Basic model

The reduced form of this system is:

$$(70)\ \frac{d\omega}{dt} = \omega \cdot (w(\lambda) - \alpha)$$

$$(71)\ \frac{d\lambda}{dt} = \lambda \cdot \left(\frac{k(\pi)}{v} - \gamma - \alpha - \beta\right)$$

$$(72)\ \frac{dd}{dt} = \left(r - \left(\frac{k(\pi)}{v} - \gamma\right)\right) \cdot d + k(\pi) - \pi_s$$

The fixed and variable interest versions of both the basic and the general model have identical form, with the difference in behaviour arising from the fact that in the latter the rate of interest is a function, rather than a constant.

5.3.2 General Model

The reduced form for this model is:

$$(73)\ \frac{d\omega}{dt} = \omega \cdot (w(\lambda) - \alpha \cdot a(\pi))$$

$$(74)\ \frac{d\lambda}{dt} = \frac{\lambda \cdot \left(c(\lambda)\left(\frac{k(\pi)}{v} - \gamma\right) - \alpha \cdot a(\pi) - \beta\right)}{1 - \lambda \cdot \left(\frac{W}{V + e^{-W \cdot (\lambda - X)}} \cdot e^{-W \cdot (\lambda - X)}\right)}$$

$$(75)\ \frac{dd}{dt} = \left(r - \left(\frac{k(\pi) \cdot c(\lambda)}{v} - \gamma + \lambda \cdot V \cdot e^{-V \cdot (\lambda - W)} \cdot \frac{\frac{k(\pi) \cdot c(\lambda)}{v} - \gamma - \alpha \cdot a(\pi) - \beta}{U + (1 - \lambda \cdot V) \cdot e^{-V \cdot (\lambda - W)}}\right)\right) \cdot d + k(\pi) - \pi_s$$

5.3.3 Interpreting the equations

As is evident from equations 70-75, the introduction of a banking sector does not alter the equations for the rate of change of the wages share of output and the rate of change of employment. Instead, it acts on these via the altered definition for the rate of profit, and adds a separate equation for the rate of change of debt.

With this equation introduced, it is now possible to consider the Fisher/Minsky argument that a capitalist economy can undergo a debt deflation. As with the Goodwin model itself, the percentage rate of growth is an important aspect of the dynamics of this expanded system. As is easily shown for the basic model, the term $\left(\frac{k(\pi)}{v} - \gamma\right)$ is the percentage rate of growth of the model:

$$
\begin{aligned}
\frac{1}{Y}\frac{d}{dt}Y &= \frac{1}{Y}\frac{d}{dt}\frac{K}{v} \\
&= \frac{1}{Y}\frac{1}{v}I \\
&= \frac{1}{Y}\frac{1}{v}(k(\pi)\cdot Y - \gamma\cdot K) \\
&= \frac{1}{Y}\frac{1}{v}(k(\pi)\cdot Y - \gamma\cdot v\cdot Y) \\
&= \left(\frac{k(\pi)}{v} - \gamma\right)
\end{aligned} \tag{76}
$$

[47]

Making the substitution of g_y for the percentage rate of growth of output, equation 72 (and equation 75) can thus be rewritten as

$$\frac{d}{dt}d = (r - g_y)\cdot d + k(\pi) - \pi_s \tag{77}$$

The net investment term $k(\pi)$ will always be non-negative, so that its contribution to the percentage rate of growth of debt will always be non-negative. The profit share π_s will reduce the debt to output ratio when it is positive, but add to it when it is negative. This implies that the rate of growth of debt depends crucially on the relationship between the rate of interest r and the rate of growth g_y.

If the latter exceeds the former sufficiently often, then the percentage rate of growth of the debt to output ratio will tend towards zero, and thus a stable debt to output ratio.[48] According to equation 77, for the debt to output ratio to stabilise the condition

(78) $(r - g_y) \cdot d = \pi_s - k(\pi)$

must apply. In this situation in which the debt to output ratio stabilises, g_y will exceed r, so that investment must also exceed the profit share of output. This implies a growing level of absolute debt—since capitalists will borrow the difference between the amount they invest and profit—but a stabilised debt to output ratio, since output grows under the influence of positive net investment.

There will also be cyclical fluctuations en route to this position of long term stability, since a rate of growth which substantially exceeds the rate of interest will lead to a fall in the debt to output ratio d, and hence an increase in the profit share of output π_s (which equals $1 - \omega - r \cdot d$). However this will inspire additional investment via the nonlinear investment function $k(\pi)$, which in turn will increase d, and a rising wages share of output ω via the impact of rising employment. This in turn leads to a fall in the rate of growth, and a renewal of the cycle.

In the alternative case where the rate of growth is less than the rate of interest sufficiently often, the level of debt will grow exponentially. As with the previous case, complex relationships between the rate of growth, the rate of profit and the wages share of output come into play which generate cyclical behaviour on the way towards this exponential endgame. If the debt ratio rises, then profit share will tend to fall. The falling profit share will lead to falling investment, since investment is a function of the rate of profit. Reduced investment in turn will reduce the rate of growth, thus compounding the initial problem that growth was lower than the rate of interest. However the collapse in the rate of growth will cause employment to fall, which will lead to workers accepting wage cuts. This reduction in wages share increases profit share, and can thus partially counteract the rate of accumulation of debt.

This analysis indicates that there will be cyclical fluctuations in this model, and that there will be two classes of outcomes: fluctuations towards long run stability, and fluctuations towards a debt-induced breakdown. These phenomena are evident in the following simulations.

5.4 Simulations

The combination of a banking sector with the numerous extensions to the Goodwin model in Chapter Four results in a potential multiplicity of models. In the following sections, four combinations are explored: the basic model (augmented by a nonlinear investment function) with a fixed rate of interest; the basic model with a variable rate of interest; the general model with a fixed rate; and the general model with a variable rate.

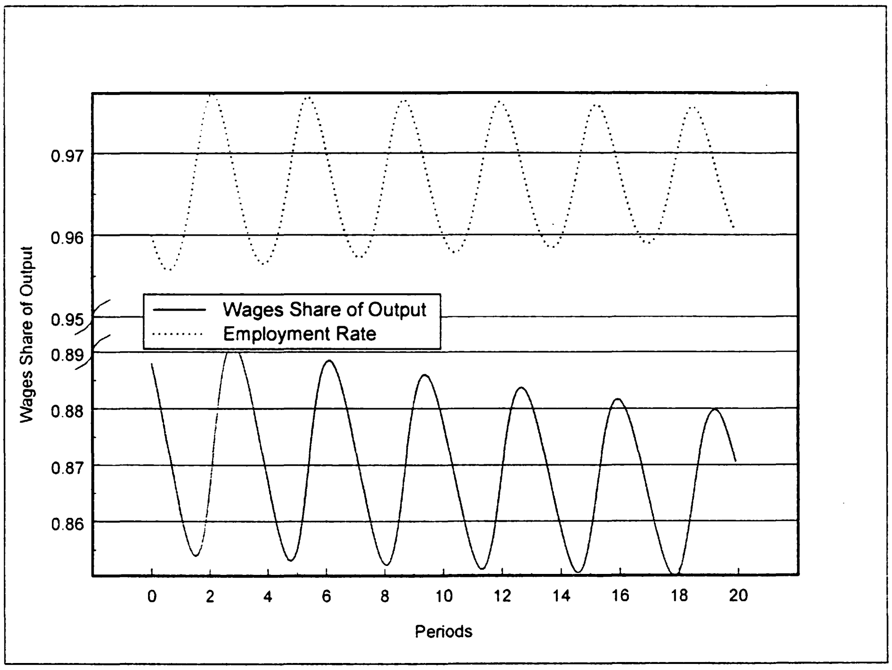
The simulations all begin from a situation in which the economy is extremely robust in Minsky's sense, in that the level of debt is zero, and dynamic in Fisher's sense, in that the initial conditions are not those of long run equilibrium. As the simulations establish, only in the simplest of cases does the model progress from initial robustness to eventual equilibrium, and then only when the rate of interest is low. For all other models, the equilibrium is either delicately poised, or non-existent. In the former case, even an infinitesimal increase in the rate of interest alters the system from one which reaches a stable wages share, employment rate (of 96.9 per cent) and debt to output ratio, to one in which the only stable levels for wages share and employment are zero. In the more general case, no equilibrium exists at all: the dynamic process involves ever-spiralling levels of debt, as wages share and employment collapse to zero.

5.4.1 Basic Model Fixed Interest

5.4.1.1 Finance and Stability at Low Interest

With a low rate of interest, the introduction of a finance sector leads to a model which has a stable equilibrium, as opposed to the neutral equilibrium of the basic Goodwin model. The initial behaviour of the model is the same no matter what rate of interest applies: the closed loop of the previous chapter is replaced by a system which is apparently converging to an equilibrium. This is evident in the time plot in Figure 32, where the cycles in both employment and wages share of output are clearly attenuating, at least in the short term.

Figure 32 Wages Share & Employment Cycles in Basic Fixed Interest Rate Model at Low Interest



Appearances are not deceptive when the rate of interest is low: stability is indeed the final outcome. Figure 33 shows that with the parameter values used in these simulations, a base rate of less than 3 per cent results in a system which, over time, approaches a stable equilibrium.

Figure 33 Employment and Wages Share Converge to Equilibrium at Low Interest

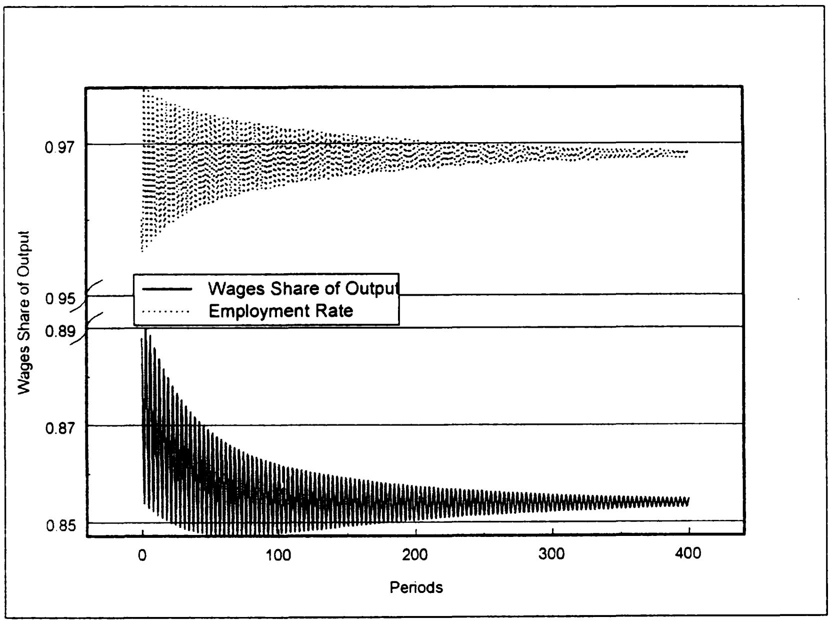
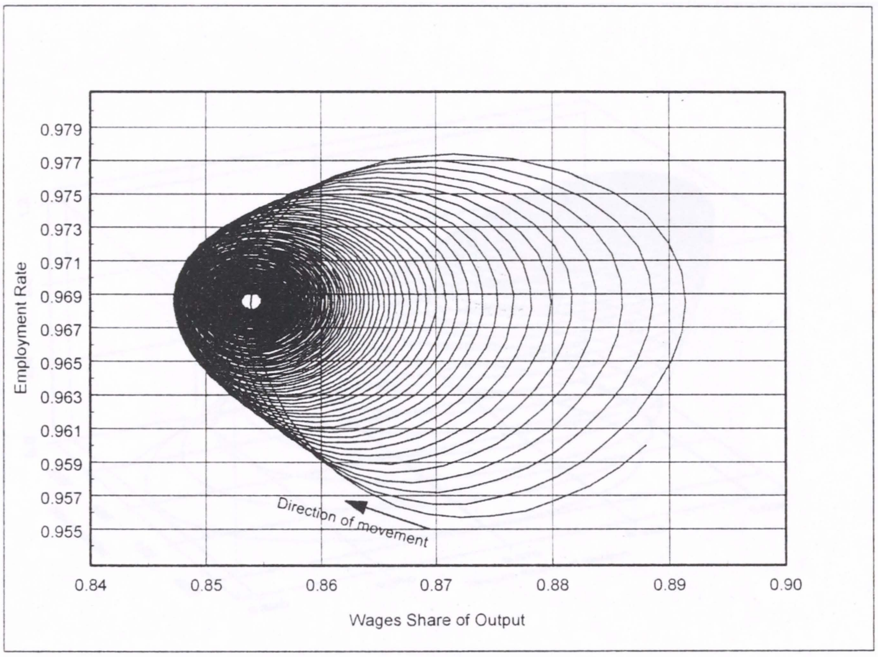


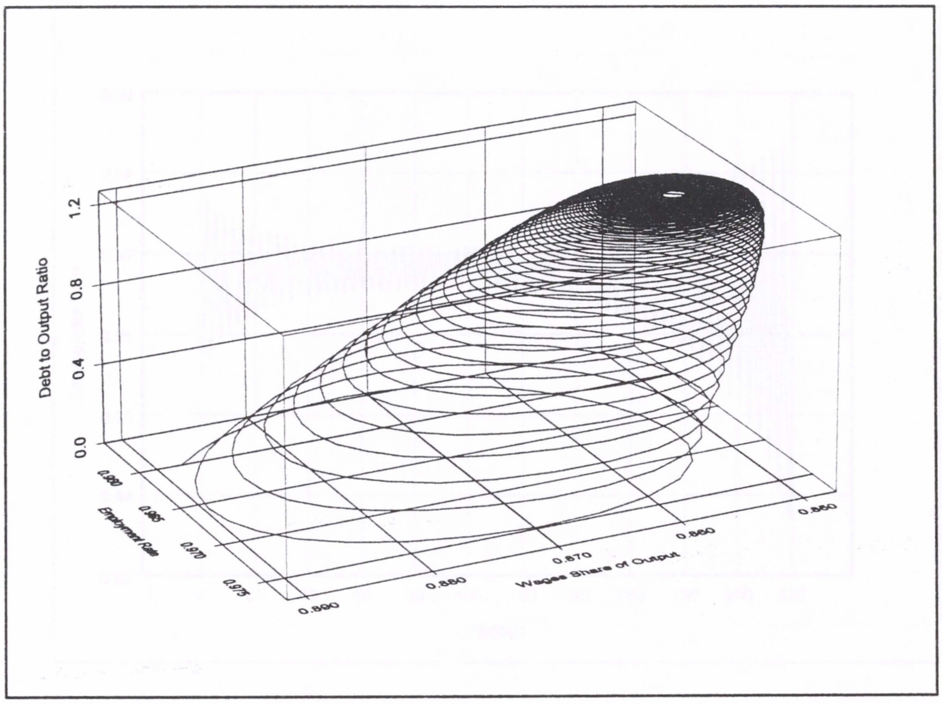
Figure 34 shows this same process more clearly in a two-dimensional phase plot, where the equilibrium point to which the system is converging can be seen clearly.

Figure 34 Convergence to Equilibrium in Basic Fixed Interest Rate Model at Low Interest



However the introduction of finance makes this a three-dimensional system, in which the debt to output ratio is also needed to close the system, so that a three-dimensional phase plot (see Figure 35) is needed to show the real dynamics behind this convergence. At this low level of the rate of interest, the interest payments occasioned by the growth in debt (which results from capitalists borrowing to finance investment during booms) gradually attenuate the level the booms reach. This results in capitalist investment cyclically tapering down to a level at which the ratio of debt to output stabilises. Constant income shares then ensue for the three "classes" in the model—workers, capitalists, and bankers—and the system thereafter grows at a steady pace.

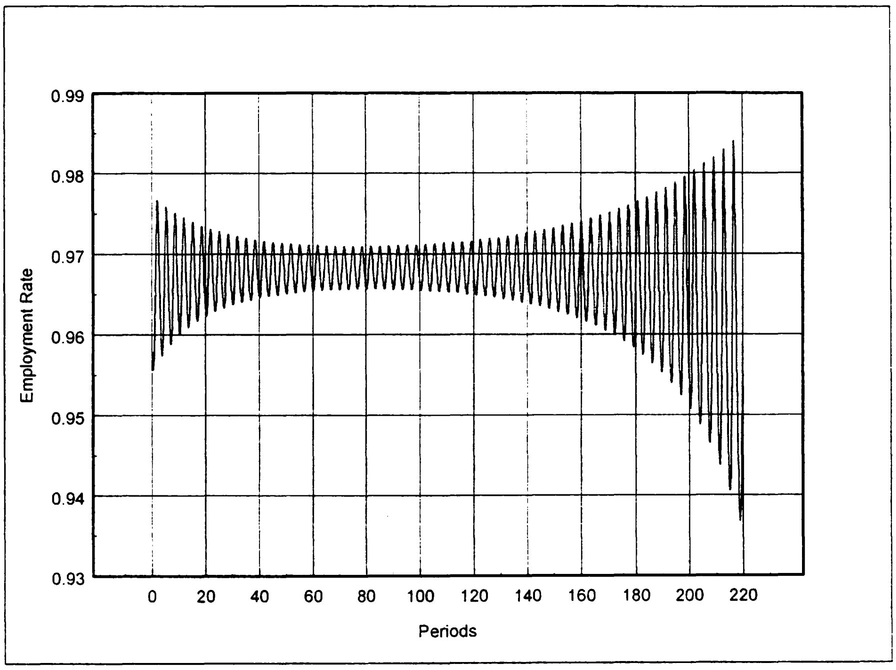
Figure 35 Three-Dimensional Convergence in Basic Fixed Interest Rate Model at Low Interest



5.4.1.2 Instability—Rising Debt with a Wages Blowout

At an interest rate of 3 per cent or above, a different dynamic emerges, which appears quite bizarre when viewed only from the perspective of the rate of employment. As Figure 36 shows (in a simulation with an interest rate of 3.5 per cent), employment appears to be converging to equilibrium for the first 80 periods, but then more extreme cyclical behaviour appears "out of nowhere", leading eventually to extreme cycles and (not shown here) a complete collapse of employment, and with it, of output.

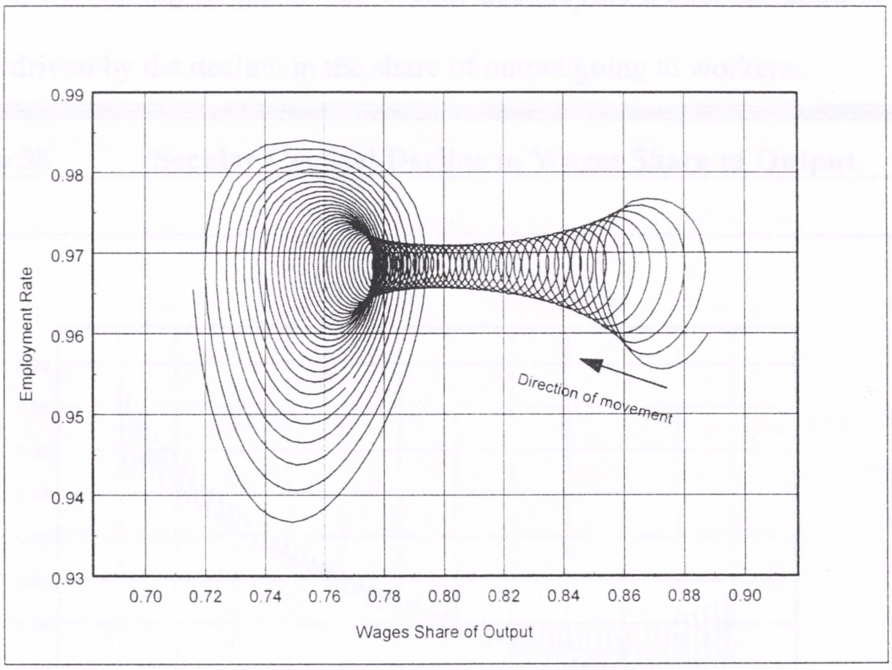
Figure 36 Diminishing and then Increasing Cycles in Basic Fixed Interest Rate Model at Low Interest



This is impossible in a linear model: in the absence of external shocks, cycles either decrease in severity on the road to equilibrium, or rise ever more as the system diverges towards breakdown. A conventional economic interpretation of this data would therefore have to rely on the rather unconvincing explanation of a change in the severity of the external shocks.

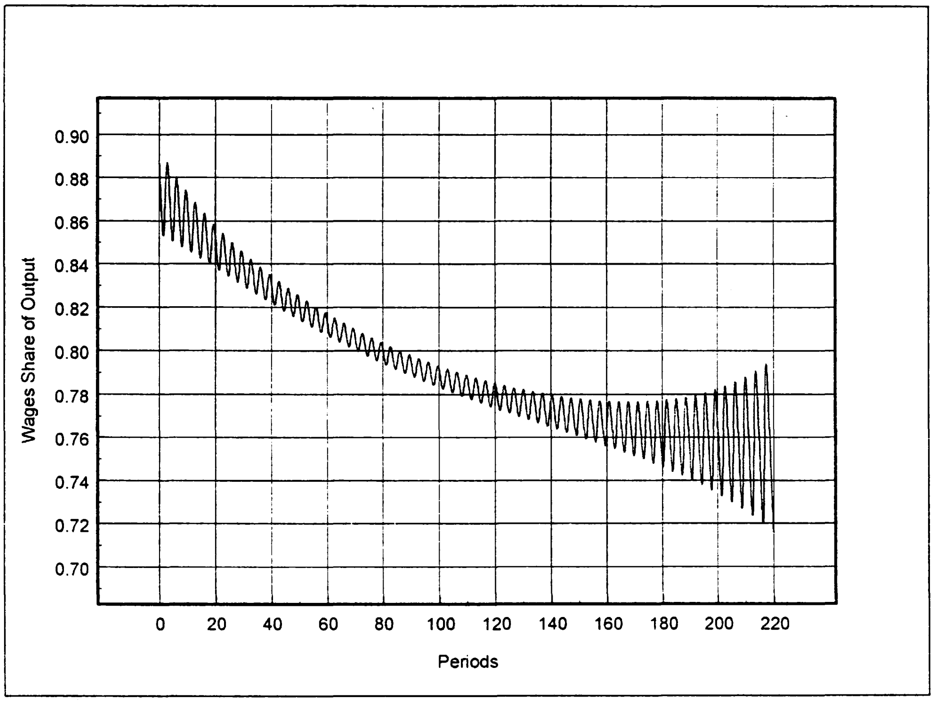
The true underlying dynamics of this nonlinear system are somewhat more apparent in the 2D phase diagram (Figure 37), though the causation is still not obvious. In contrast to the low interest simulation, where the phase portrait showed an inward spiral towards equilibrium (see Figure 34), the later cycles of this simulation intersect with earlier ones as the wages share undergoes a cyclical but secular decline (see Figure 38).

Figure 37 Cyclical Breakdown in Basic Fixed Interest Rate Model



This secular decline of wages share is one significant difference between this debt-deflationary simulation and the previous one where the system converged to equilibrium. Unlike the decline in workers share in the low interest simulation (see Figure 33), this decline is not a gradual progression towards an equilibrium level, but a contributing factor in the debt deflationary process. During a slump, the secular and cyclical decline of wages share interacts with the level of debt (and the nonlinear investment function) to encourage a recovery in investment before all the debt accumulated in the preceding boom has been repaid. The recovery in investment boosts output and employment, thus leading to a growth in wages share as employment passes the point in the "Phillips curve" at which workers accept real wage cuts. At the same time, the increased level of investment increases the debt ratio once more. The rise in wages share then reduces profits, with ever increasing severity, thus leading to a sharper fall in investment and output, and a renewal of the cycle at a higher pitch. The overall decline into a debt deflation is thus in part driven by the decline in the share of output going to workers.

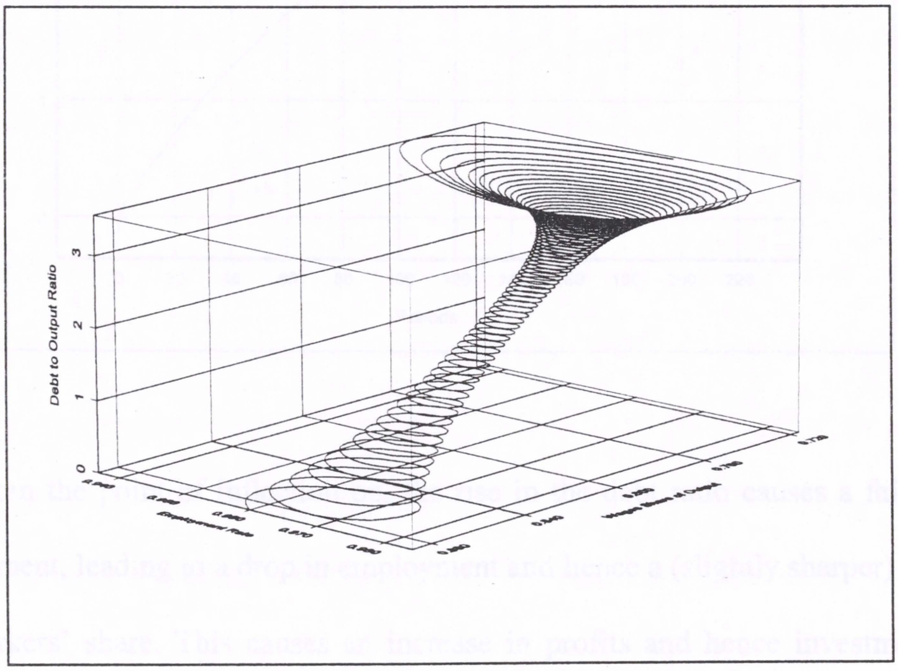
Figure 38 Secular Cyclical Decline in Wages Share of Output



The full dynamics are visible in the 3D phase diagram of Figure 39. An initially low level of employment, near average workers' share and zero debt to output ratio leads to workers accepting wage cuts. As the capitalist share of output rises, rising investment ensues. The investment is financed in part by borrowing, which results in a rise the debt ratio. The increased employment leads to wage rises for workers, resulting eventually in a sharply rising workers' share, which, coming on top of a rise in the debt ratio, reduces profit and results in a fall-off in investment. The reduced investment leads to lower employment, which initially tempers and then reverses the increase in workers' share. In the early stage of the cycle, profits then exceed investment, resulting in some bank debt being repaid. The decline in debt and the wages share of output restores capitalist profits, but since bank debt has not been completely repaid, the cycle is not as extreme when it next repeats.

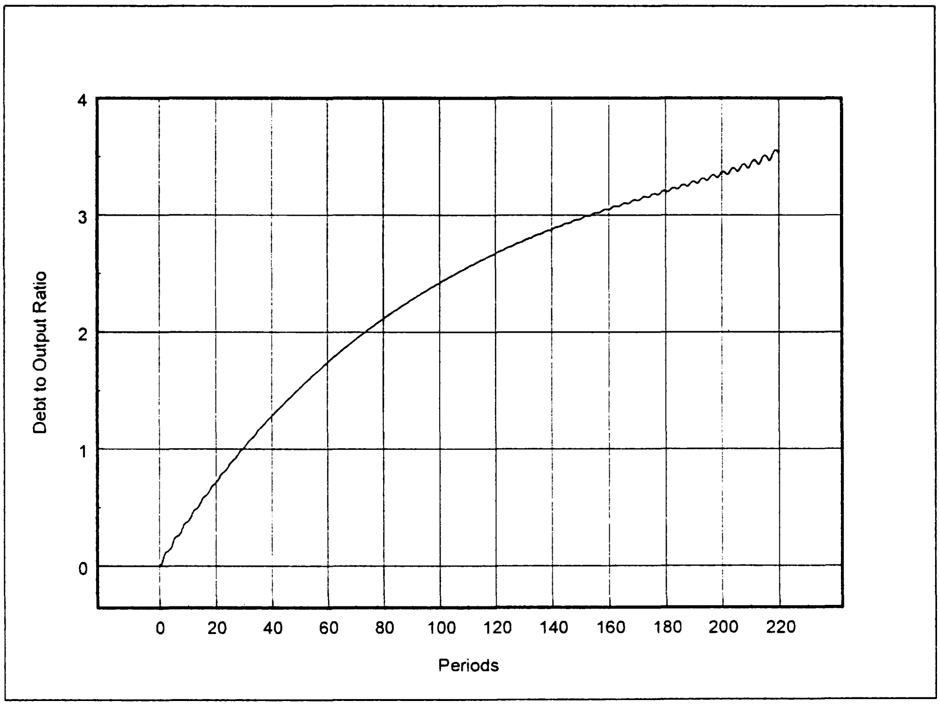
However, rather than the cycle being damped away, the higher rate of interest leads to the formation of a wage-employment vortex, where the debt ratio continues to grow rather than reaching a plateau.

Figure 39 Three-dimensional view of Cyclical Breakdown in Basic Fixed Interest Rate Model



In this region, as Figure 40 indicates, the exponential component $\frac{dD}{dt} = r \cdot D$ of the overall debt relation $\frac{dD}{dt} = r \cdot D + I_g - \Pi$ comes to dominate the system dynamics. There is a point of inflection at approximately period 160 where the rate of change of the debt ratio ceases its monotonic decline and instead begins a cyclical but exponential rise. This is the “real” foundation beneath the monetary phenomenon of Fisher’s paradox that in a debt-deflation, “The more the debtors pay, the more they owe” (Fisher 1933: 344).

Figure 40 The Debt Ratio Rises at a Decreasing and then Increasing Rate

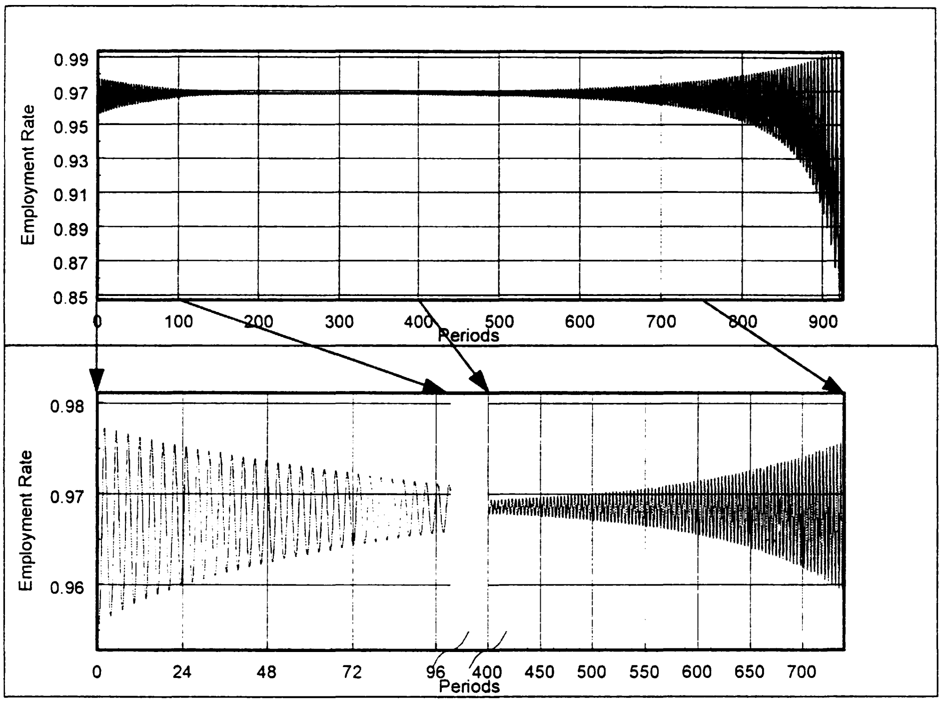


From the point of inflection on, the rise in the debt ratio causes a fall in investment, leading to a drop in employment and hence a (slightly sharper) fall in workers’ share. This causes an increase in profits and hence investment, leading to further debt and a rise in the debt ratio again. Rather than attenuating, the cycles now become more intense, with the strong fall in workers’ share causing a big growth in profits, a commensurately larger surge in investment (given the non-linear investment function), a further increase in debt, then the increase in workers’ share due to increased employment (and the non-linear wage change function), and a renewal of the cycle at a more extreme level. Eventually the boom is so extreme that the extra debt incurred results in profits falling and remaining below zero, given the level of debt which has been accumulated. The system then collapses towards zero employment, wages and profits, with bankers' share spiralling ever upwards. It is now in a debt-induced breakdown, from which—without a change in the rules, such as a debt moratorium—it cannot escape.

5.4.2 Basic model, Variable Interest Rate

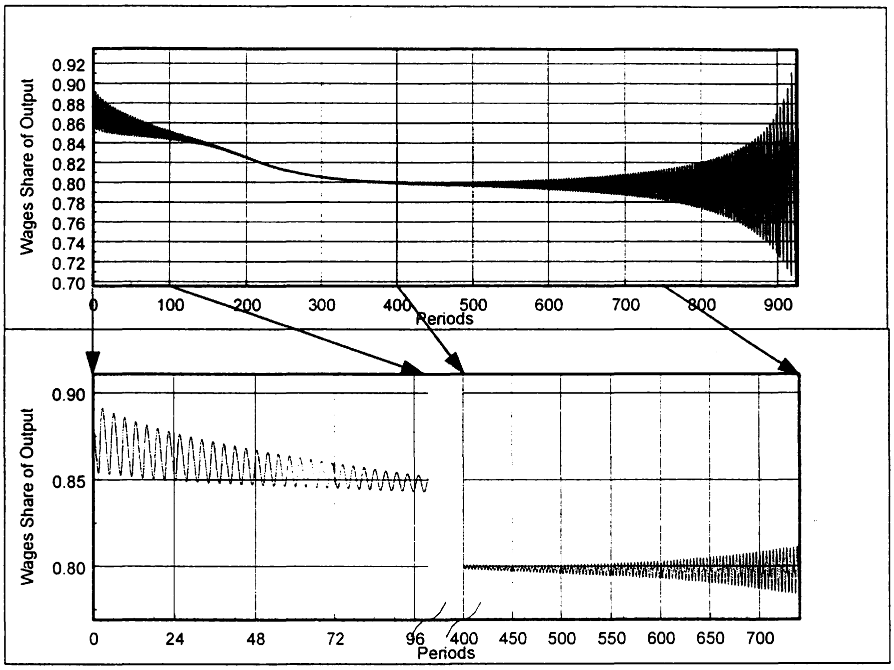
As with the fixed interest case, the occurrence of a breakdown in the variable interest rate case is conditional upon the degree of sensitivity to debt postulated for the interest rate function. At a low level of sensitivity (so that a very high debt to output ratio is required to trigger an increase in the rate of interest), the economy will converge to a stable equilibrium. However at the level of debt sensitivity specified above, a variable interest rate leads to breakdown, but with a substantially different time pattern to the fixed interest case. As the bottom chart in Figure 41 indicates, from the point of view of employment, the system apparently settles down rapidly, while the top chart shows that this apparent stability lasts for a considerable period. In an actual economy, this apparent stability would doubtless encourage great complacency in economic policy makers.

Figure 41 Diminishing and then Rising Cycles with Variable Interest



The wages share of output likewise appears to be approaching stability, though in addition there is an apparent secular trend for the wages share of output to decline (Figure 42). While this might have caused some concern during the relatively egalitarian 1950s, during the more efficiency-oriented 1980s and 1990s such an effect would doubtless be interpreted as the elimination of "real wages overhang", and the restoration of workers share of output to the marginal product of labor.

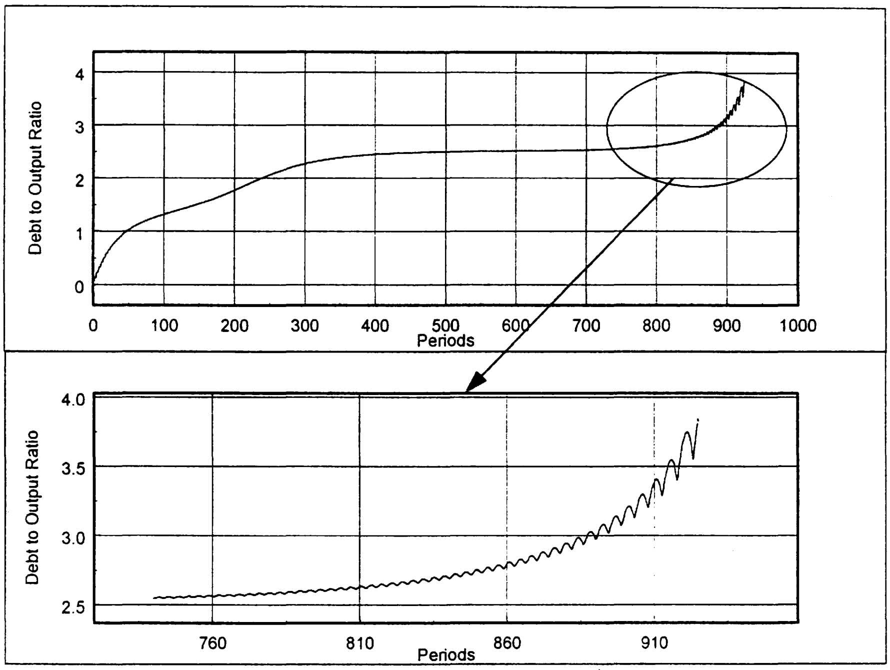
Figure 42 Secular Decline in Wages Share with Diminishing and then Rising Cycles with Variable Interest



However this lengthy period of apparent stability gradually gives way to increasing cyclical behaviour until eventually the cycles overwhelm the system.

These peculiar affects are somewhat more explicable when the debt to output ratio is also considered in Figure 43. The first phase, with stable employment and declining wages share, involves a non-monotonic acceleration in the debt to output ratio. The second, with rising cyclical behaviour, involves an at first almost imperceptible but eventually exponential rate of growth in the ratio, until such time as the interest payments on debt overwhelm capitalist profits.

Figure 43 Non-monotonic Change in the Debt to Output Ratio with Variable Interest

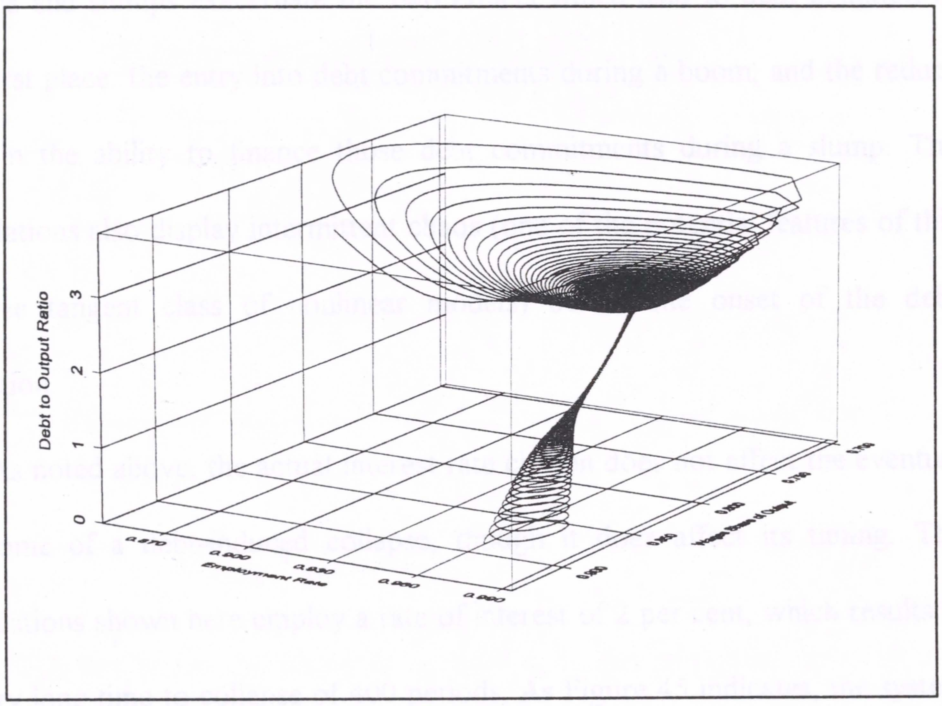


The non-monotonic shape of the debt to output time profile can be traced to the nonlinear shape of the interest rate function. The first segment up till period 150, when the debt to output ratio is rising quickly but at a decelerating rate, corresponds to the region in which a sigmoid function has a low but increasing rate of growth. The second, where the debt ratio is rising slowing and apparently rising, corresponds to the period of a high but decreasing rate of change in the rate of interest, while the third when the ratio appears to have stabilised corresponds to the period in which the rate of change of the rate of interest drops to zero. In this range, the model is akin to the high fixed interest rate case considered above, and the rate of interest now exceeds that which leads to breakdown in that case. The extreme cycles in the debt to output ratio in the final phase of the model, shown in the bottom plot in Figure 43, are a new phenomenon however, which become more prominent and significant in the general models.

As before, the full dynamics are apparent in a three dimensional phase plot, shown in Figure 44. After an initial period of cyclical behaviour, the impact of debt on capitalist investment keeps output growth just below the rate of growth of the labor force, resulting in a level of employment below that which leads to a constant wages share. This continual fall in workers share of output keeps profits to a level just above that at which the investment function generates a demand for borrowing. The debt to equity ratio rises slowly, rather than reaching a plateau as it did with low debt sensitivity. This gradual increase eventually triggers a rise in the rate of interest, which affects not just new investment as in IS-LM analysis, but the accumulated debt from previous investment. Investment slows and eventually halts, leading to a "free fall" collapse in workers share which is overwhelmed by the more rapid rise in the debt to output ratio as the exponential side of the debt relation overwhelms the system (even though the rise in the rate of interest ceases, since the upper limit of the interest rate function has been reached).

Though the initial behaviour is similar to the high fixed rate case, the dual force of the rise in the debt ratio (which itself drives the interest rate higher) and the rising interest rate results in a greater rate of acceleration of the debt to output ratio, with little or no cyclical variation of wages and employment. There is however a secular fall in workers' share—hence the very narrow "vortex" in Figure 44 as compared to Figure 39—so that overall the period would erroneously be interpreted as one of prolonged stability by conventional economic analysis.

Figure 44 Three-dimensional Phase Plot for Basic Model with Variable Interest



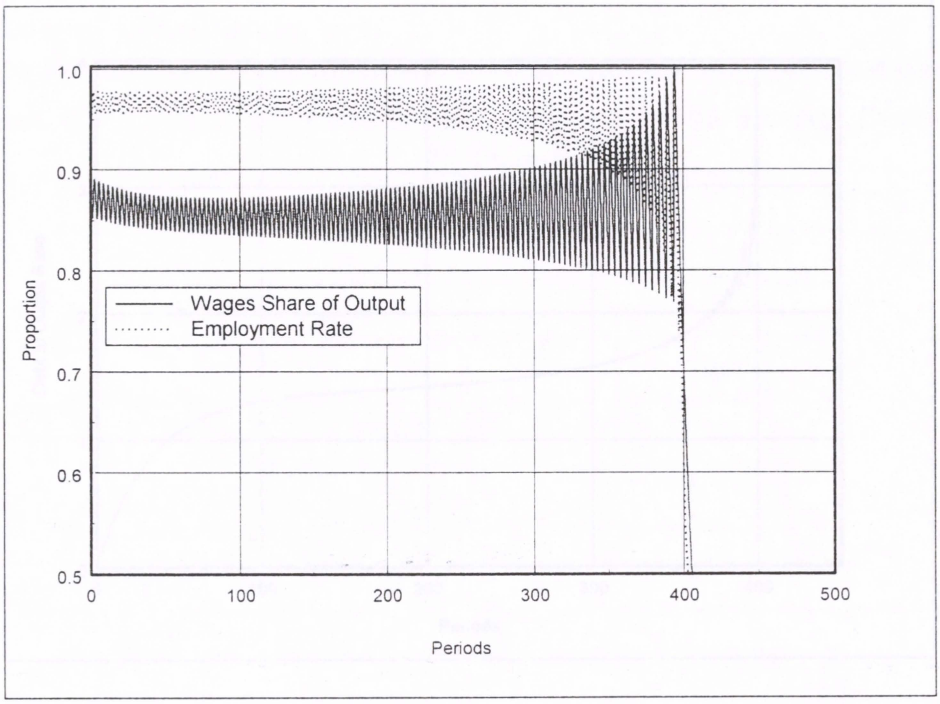
The breakdown in these last two simulations confirms Minsky's arguments about the systemic tendency to increasing fragility. Investment behaviours which were unproblematic when debt levels were low themselves lead to a secular increase in debt levels. This increase in debt increases the fragility of the system, leading to a point at which investment behaviour which was once sustainable leads instead to a debt deflation.

5.4.3 General Model, Fixed Interest

Breakdown in the basic model was conditional, either upon the rate of interest in the exogenous interest case, or the degree of debt sensitivity in the endogenous interest case. The increased degree of nonlinearity in the general model converts this conditional collapse into an absolute one: the system undergoes a debt deflation for any non-zero rate of interest and with any level of debt sensitivity. This is because the general model's more pronounced booms and slumps exacerbate the two effects which lead to debt deflation in the first place: the entry into debt commitments during a boom, and the reduction in the ability to finance those debt commitments during a slump. The simulations also display intermittent chaos (one of the potential features of this inverse tangent class of nonlinear models) before the onset of the debt deflation.

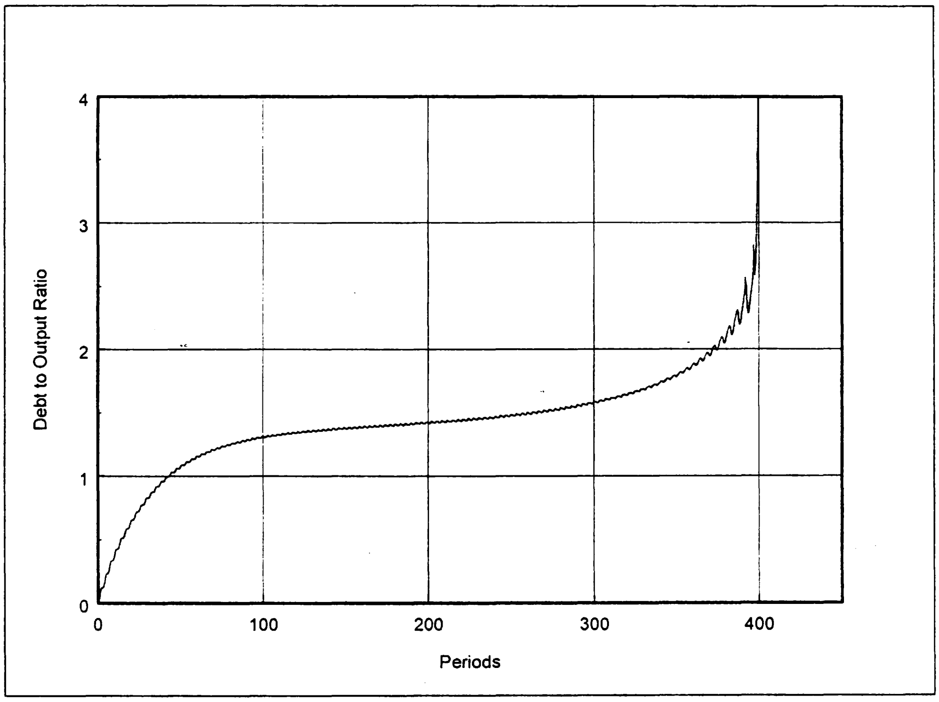
As noted above, the actual interest rate chosen does not affect the eventual outcome of a debt-induced collapse, though it does affect its timing. The simulations shown here employ a rate of interest of 2 per cent, which results in a very long time to collapse of 400 periods. As Figure 45 indicates, the system is considerably more cyclical than the basic model, and there is only a small decline in the intensity of cyclical behaviour as debt levels grow.

Figure 45 Sustained Cycles and Inevitable Breakdown in Fixed Interest General Model



The key to the breakdown is, as always, the debt to output ratio. Figure 46 shows that this has the same basic shape as the fixed interest basic model, though the cyclical behaviour in the exponential phase is much more extreme than that of Figure 40.

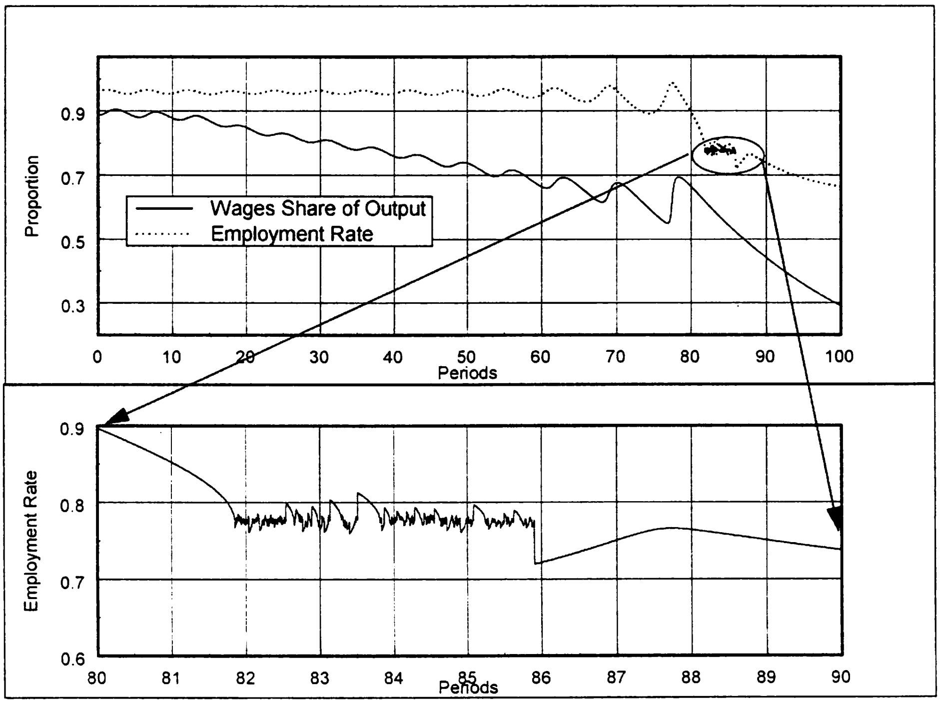
Figure 46 Debt to Output Ratio in Fixed Interest General Model



5.4.4 General Model, Variable Interest

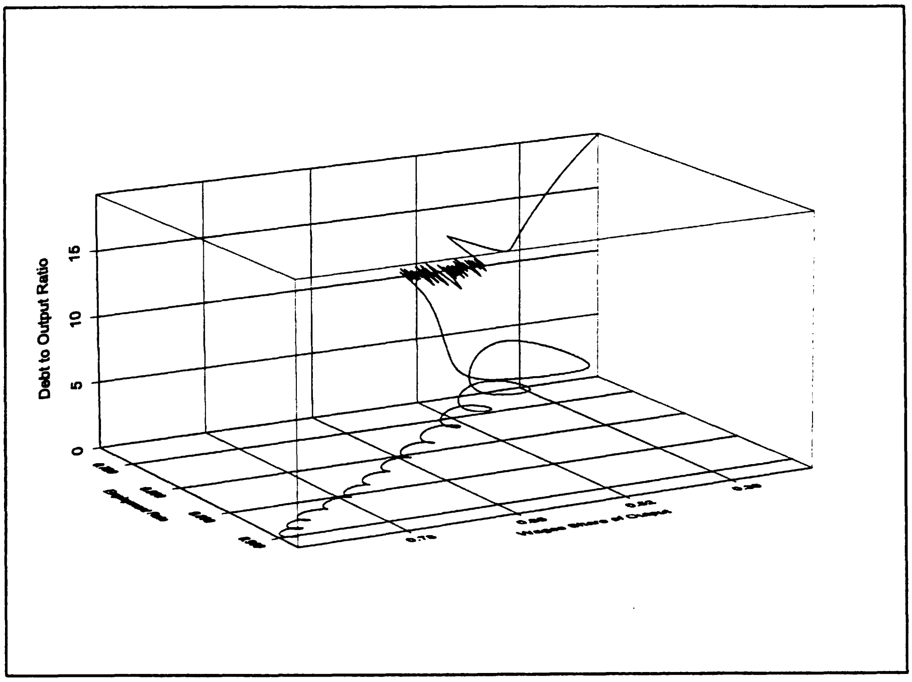
The structure of the collapse in this most general model reinforces Goodwin's most emphatic message, that nonlinearity is essential to an understanding of the dynamics of capitalism. The interplay of nonlinear forces results in a system which, though cyclical, appears stable for a considerable time, yet which is suddenly beset by marked instability and breakdown. In particular, the extreme and apparently random fluctuations in employment shown in the bottom chart in Figure 47 could never be generated by a linear model. The fundamental force underlying the breakdown, the debt to output ratio, now interacts with the additional nonlinearities due to a variable interest rate, endogenous technical change and variable capacity utilisation to generate a cyclical path to breakdown which, in its final stages, is chaotic. The detailed plot of the behaviour of the model at the stage of breakdown gives an indication of just how unstable the system is in this intermittent stage before the onset of debt deflation.

Figure 47 Cyclical and Chaotic Behaviour in Variable Interest General Model



As with the less pronounced chaotic behaviour of the fixed interest model, this instability is mirrored in the behaviour of the rate of employment. The fundamental character of the model remains the same, as indicated by the 3D phase diagram in Figure 48.

Figure 48 Three Dimensional Phase Plot of Variable Interest General Model



The forces behind this collapse reiterate Minsky's message that finance matters in a capitalist economy, and that financial fragility can develop over time as accepted debt ratios rise during a sustained period of tranquil growth. The transition from tranquillity to fragility and crisis is sudden, and the causes and symptoms mysterious to those who deem that economic performance is independent of the debt and equity structure of the economy.

5.4.5 Comparison with related models

There have been several prior attempts either to model Minsky's Hypothesis, or to generalise Goodwin's model to include financial variables, using a range of analytical foundations. Taylor and O'Connell (1989) built an elaborate model which attempted to capture Minsky's commodity and asset price dynamics in a one-commodity model, but which made some questionable assumptions about the pricing of capital goods (1989: 4) and employed some behavioural concepts derived from rational expectations (1989: 7) and IS-LM analysis with exogenous money (1989: 9, 11) which are antithetical to Minsky's method. The model could not be simulated, and Jacobian analysis gave ambiguous results concerning the possibility of a debt-deflation, with the outcome depending upon the relative values of wealth holdings and share purchase to interest rate elasticities.

Asada (1989) constructed a quite elaborate model incorporating an investment function, variable capacity utilisation, money and a government sector. However this model also employed an IS-LM framework in which the goods and money markets were assumed to clear (Asada 1989: 148-149), and in which the money supply was exogenously determined. Unsurprisingly, this model demonstrated a separation of real and monetary variables (1989: 151, 153) prior to the introduction of a variable capitalist propensity to save which depends inversely upon the expected rate of inflation.

Jarsulic (1989) generalised Goodwin's model to include a Post Keynesian treatment of finance, but at the expense of assuming constant income shares (Jarsulic 1989: 39). With the loss of one degree of freedom, Jacobian analysis showed that the model would generate a limit cycle, but it could not of course generate phenomena akin to a debt deflation.

Though his model was not explicitly an attempt to model Minsky's Hypothesis, Shaikh 1989 has many features in common with the above analysis, and with the multi-sectoral model of Chapter Eight. However the possibility of a debt-deflation is missed by the assumption that all borrowing is repaid within one period (1989: 73). Sustained cycles were also dependent upon the presence of exogenous shocks (65,66,76).

Andresen (1996) applied a systems dynamics approach to economic modelling to argue via simulations that a debt deflation was an inevitability in a capitalist economy with finance. His method was to proceed from general principles of the behaviour of financial flows within and between production, consumption and finance entities to a dynamic flow chart simulation without the intermediate step of reduced form equations. Despite this quite different methodology, his analysis and conclusions are the closest available in the literature to those reached in this Chapter.

The models presented in this chapter differ from all of the above in that (a) they are based as closely as possible on Goodwin's predator-prey model on the one hand, and Minsky's FIH on the other, with no incursions from other frameworks such as IS-LM analysis; (b) all degrees of freedom are explored, so that in contrast to the models of Jarsulic and Shaikh where one variable was arbitrarily held constant, in the models above all variables are allowed to change with time.

5.5 Conclusion

Minsky's ambition in constructing the Financial Instability Hypothesis was to build a theory which "makes great depressions one of the possible states in which our type of capitalist economy can find itself" (Minsky 1982, p. xi). His purpose was to find "an apt economic theory for our economy" (Minsky 1977, 1982: 68), since it was a manifest fact that capitalist economies periodically find themselves in such a state.

These models of a capitalist economy with finance, which have been constructed via "stylised fact" extensions to Goodwin's growth cycle model, are able to demonstrate the Fisherian essence of Minsky's Hypothesis, that capitalist expectations of profit during booms can lead them to incur more debt than the system is capable of financing during slumps. The breakdown which occurs is analogous to a debt-induced Depression in an actual economy. When such an event occurs, the model indicates a forever-increasing level of capitalist indebtedness. In the real world, however, the system continues but with some form of breakdown: some capitalists go bankrupt, many lenders write-off bad debts and suffer capital losses, and debt moratoria are often enforced.

Both the model and real world breakdowns follow paths predicted by Minsky, capturing his propositions (Minsky 1995: 198) that

> In a heavily indebted economy
>
> 1. even minor declines in profits and wages can lead to increases in nonperforming assets in the portfolios of financial institutions.
> 2. even minor increases in interest rates can lead to increases in nonperforming assets in the portfolios of financial institutions.
> 3. even minor increases in wages can lead to pressure on profit flows and therefore to an increase in nonperforming assets.

In these simulations, booms, which were unproblematic initially, become destabilising later because of the increased debt to output ratios that develop over time. This corresponds with Minsky's predictions of a secular trend towards rising debt to equity ratios as the memory of the previous major crisis recedes, which makes the system more fragile. Income distribution effects are also important. The fall in workers share interacts with rising bankers share (at a slightly slower rate) to lead to a sequence of minor booms of increasing magnitude, until one occurring at a time of greatly increased debt leads to a runaway blowout in debt. In effect, a rise in income inequality (between workers and capitalists) leads to a period of instability and then collapse, a concept explored in Minsky 1986.

In all these simulations, a long period of apparent stability is in fact illusory, and the crisis, when it hits, is sudden—occurring too quickly to be reversible by changes to discretionary policy at the time. The conventional policy response of governments to an overheated economy—increasing the interest rate with the intention of dampening investment and thus tempering the boom—acts not only upon the incentive to invest, but also upon the level of outstanding debt. If this level is already high, then increasing the interest rate may turn boom into crisis. The subsequent attempt to revive the economy by reducing interest rates—and thus stimulating investment, according to IS-LM analysis—amounts to trying to force the economy back down into the stable section of the vortex, when it has already passed into its catastrophic region. However, the centripetal forces which exist in that region—the direct weight of accumulated debt upon a depressed economy, and the indirect weight of depressed capitalist expectations—are so great that any government action at that time may be too little, too late. This emphasises the essential policy message of the Financial Instability Hypothesis, that we should avoid crises in the first place, by developing and maintaining institutions and policies which enforce "a 'good financial society' in which the tendency by businesses and bankers to engage in speculative finance is constrained" (Minsky 10980b, 1982: 69). These institutional arrangements include close and discretionary supervision of financial institutions and financial arrangements, fiscal policy which restrains the development of euphoric expectations during upswings and supplements capitalist cash flows during slumps, and a general bias towards income equity rather than inequality.

The next chapter indicates that Minsky's policy message—that big government can prevent a debt-deflation—is valid. The introduction of a government sector which taxes and spends countercyclically prevents the development of a debt deflation in all relevant variants of the model.

6 Stabilising an unstable economy

Trade cycle stabilisation, which was (along with the maintenance of full employment) the hallmark of economic policy in the 1950s and 1960s, came into disfavour in the 1970s with the rise of inflation on the one hand and the return to dominance of neoclassical economic theory on the other. Since neoclassical analysis views the economy as a self-stabilising system, the trade cycle itself was seen as simply the product of exogenous shocks to this system, and attempting to attenuate these via active policy was tilting at windmills. Instead, stabilisation policy became recast as the removal of one perceived source of exogenous shocks—variations in the rate of growth of the money supply—by requiring the government to maintain the rate of growth of the (exogenous) money supply at the rate of growth of real output.

From Minsky's perspective, both interpretations of stabilisation policy were misguided. The real function of stabilisation policy is not to reduce the perturbations of the trade cycle, nor the variability of the (endogenous) money supply, but to prevent the occurrence of a debt deflation.

As the experience of the Great Depression established, a debt deflation is a far more serious phenomenon than the trade cycle itself. Output, employment and profits can recover from a trade cycle downturn, and in the long term such fluctuations below the average are inconsequential. But output, employment and profits cannot recover from a debt deflation without a severe external stimulus, or a fundamental change in the rules (such as a debt moratorium). Even if recovery eventually occurs, the growth foregone during the deflation is foregone forever, so that long term impact of such an event persists through time. Beyond the economic statistics, the personal and social dislocation a debt deflation generates—the prolonged unemployment, the bankruptcies, the widespread poverty and the suicides—are so abhorrent that a humane society should attempt to make the phenomenon which gave rise to them an impossibility.

Minsky's proposition that Big Government has made modern capitalism immune to debt deflations is thus a much stronger argument for government intervention in the economy than the traditional argument over attenuating the trade cycle. It can also be argued that the development of Big Government—which began in the depths of the Great Depression—was a social evolutionary response to the experience of debt deflations, and that this evolutionary adaptation is in danger of conscious reversal on the basis of misguided economic theories, and a political fetish for balanced budgets.

The goal of a balanced budget at all times is antithetic to Minsky's vision of modern capitalism, in which the essential role of government is to prevent a debt deflation by containing the growth of private sector debt. Government does this by preventing capitalist expectations from becoming euphoric during booms, and boosting cash flows to enable capitalists to repay debts during slumps. Both these actions entail cyclical variation in the government deficit, which is largely automatic once a government has been committed to progressive taxation regimes on the one hand, and social welfare on the other. Progressive taxation scales cut into operation as capitalist income spirals during a boom, thus reducing the flow of funds which would otherwise find its way into speculative ventures. This limits the blowout in asset prices, reduces the amount of debt accumulated during the boom phase of the trade cycle, and generates a public sector surplus. Similarly, spending on social welfare rises automatically (and revenues from progressive taxation also fall) during the slump, the public sector goes into deficit, and the money flows to businesses who in the absence of social welfare would have found their cash flows were insufficient to repay their debts.[49]

This vision clearly suits the practice of government during the 1950s and the 1960s much more so than it does today's market-oriented version. The simulations in this chapter test Minsky's argument that "something is right about the institutional structure and the policy interventions that were largely created by the reforms of the 1930s" (Minsky 1982: xiii), without necessarily confirming that the successes will continue in the more market dominated institutional structure which has been contrived during the 1980s and 1990s.

6.1 Nonlinear Government Taxation and Spending Functions

Minsky's interpretation of government stabilisation policy concentrates on the impact of taxation and spending on corporate cash flows, with taxation constraining these flows during a boom, and social welfare adding to them during a slump. The government sector thus plays the role of a counter-cyclical stabiliser to the profit-driven investment behaviour of capitalists. When falling wages share of output and/or debt cause profit to approach levels which previously induced "euphoric" levels of investment, rapidly rising taxation and rapidly falling government expenditure reduce net profit. When rising wages share and/or rising debt induce an investment slump, diminished taxation and increased government expenditure boost capitalist net income, enabling debts to be serviced.

This notion of government can be introduced with two further nonlinear functions, one relating the rate of change of government spending to unemployment, the other relating the rate of change of taxation (of capitalists only) to the level of profits.

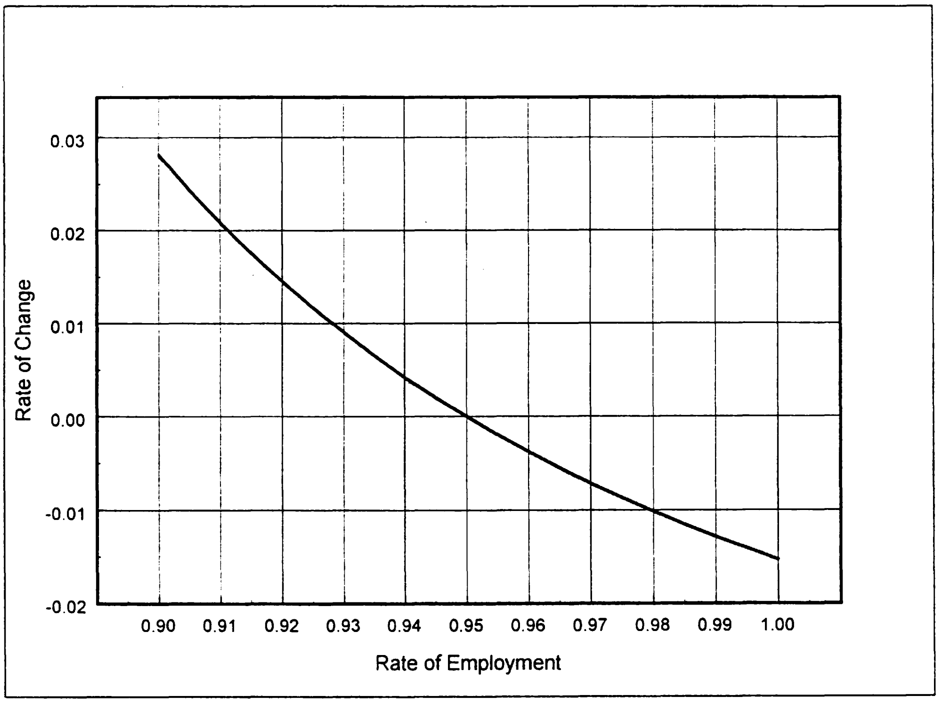
A more conventional approach would have been to describe the ratio of government spending to output, and of taxes to output, as nonlinear functions respectively of the rate of employment and the level of income. This approach was eschewed for two reasons. Firstly, the government being modelled here is a "Minskian" one in which its overriding goal is the avoidance of a debt deflation. The rate of employment is thus used as an indicator of when the economy is in recession, in which case the function of government spending is to sustain capitalist cash flows to enable them to repay accumulated debt, while the level of profits is taken as an indicator of when the economy is booming, and hence when capitalists can be expected to accumulate excessive debt commitments. Government spending will thus accelerate as the rate of unemployment rises, in order to contain the movement into recession, while the rate of taxation will rise as the rate of profit rises. Secondly, the use of the more conventional relation would have necessitated differentiating the functional forms used, which I prefer to avoid except where, as in the case of capacity utilisation, a strong case can be made for the functional form chosen.

The relationship used is the same asymptotic form as for wage change and investment (though since neither function had to be differentiated to derive the closed form of the model, any functions which generate a rising nonlinear response to an input could have been used). The government spending function is:

(79) $$g(\lambda) = \frac{I}{(J - K \cdot (1-\lambda))^2} - L$$

where *I,J,K* and *L* are parameters, with the values in these simulations of *I=.05, J=1.2, K=4*, and *L=.05*. These settings result in the functional relationship between the level of employment and the rate of change of government spending shown in Figure 49. Government spending remains constant at an unemployment rate of 5 per cent, falls at 1 per cent per period for an unemployment rate of 2 per cent, and rises at 1 per cent per period for an unemployment rate of 7 per cent.

Figure 49 The Rate of Change of Government Spending Function

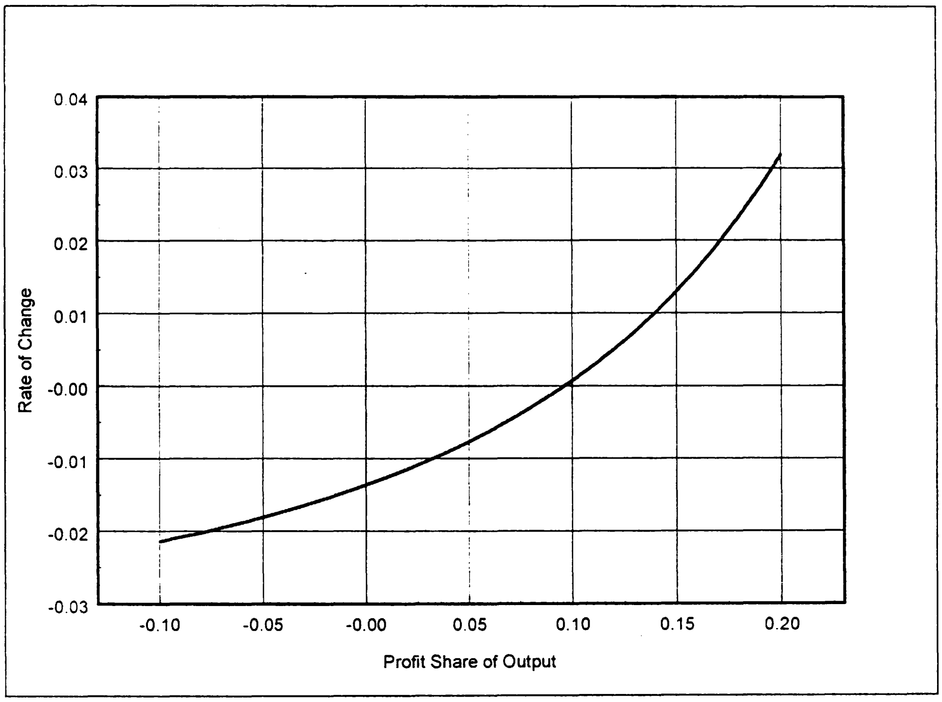


The taxation function is

(80) $$t(\pi) = \frac{M}{(N-O\cdot\pi)^2} - P$$

where *M,N,O,P* are parameters with the values in these simulations of *M=.0175, N=.83, O=5,* and *P=0.039.* These settings result in the functional relationship between the profit share of output and the rate of change of government taxation shown in Figure 50. Government taxation remains constant at profit share of approximately 10 per cent, falls at 1 per cent per period for a profit share of 2.5 per cent, and rises at 3 per cent per period for a profit share of 20 per cent.

Figure 50 The Rate of Change of Government Taxation Function



While these parameter values were not derived from any regression analysis, they provide a reasonable (and perhaps even subdued) picture of the workings of the "automatic stabiliser" functions of government behaviour. It should be emphasised however that the actual role of government goes well beyond management of the economy, let alone the "automatic stabiliser" function modelled in this chapter. These functions often result in the government sector being in practice a net source of funds for the private sector, in that over time the government accumulates a debt. In this "minimalist" model, the government in fact is a net sink of funds from the private sector, so that over time it accumulates a "negative debt". While these differences are significant, the general commonality remains that in both model and reality the government's deductions from the private sector rise during a boom, and fall during a slump.[50] It is the homeostatic impact of this counter-cyclical public behaviour which is explored in the following simulations.

6.2 New Definitions

This government extension to the model requires some redefinitions, as well as several new equations:

(81) Gross profit share reverts to its pre-banking form: $\pi_s = 1 - \varpi$

(82) Net Profit share now includes the impact of government and interest payments: $\pi_{ns} = 1 - \varpi - t + g - r \times d_k$

(83) The rate of change of government spending is a function of unemployment: $\frac{dG}{dt} = g(1 - \lambda) \times Y$

(84) The rate of change of taxation is a function of the rate of net profit: $\frac{dT}{dt} = t(\pi_n) \times Y$

As in the basic model, a simple relationship exists between the rate of net profit and the net profit share of output. The net rate of profit determines the level of investment via the investment function

(85) $\pi_n = \frac{\Pi_n}{K} = \frac{Y-W-T+G-r\times D_k}{v\times Y} = \frac{1-\omega-t+g-r\times d_k}{v} = \frac{\pi_{ns}}{v}$

These equations are common initial propositions to all the following models.

6.3 The impact of government debt

In this model, government subsidies to firms are debt financed, and where taxation exceeds subsidies, the surplus is banked and earns interest. The government sector can therefore accumulate debt. The impact of government debt on the rate of growth of overall debt is instructive however: as equation block 86 indicates, it does not alter the expression for total debt, but instead "redistributes" debt between the private and public sectors:

$$\begin{aligned} \frac{dD}{dt} &= \frac{dD_k}{dt} + \frac{dD_g}{dt} \\ &= (r \times D_k + I_g - \Pi + T - G) + (r \times D_g + G - T) \\ &= r \times D + I_g - \Pi \end{aligned} \tag{86}$$

Though the equation for total debt is unaltered from the simulations in Chapter Five, the actual debt accumulated is significantly different because the behavioural decisions leading to debt are substantially altered. In particular, private investment decisions are now based on significantly attenuated profits and losses, so that the time path of the simulation is altered completely.

6.4 The Models

6.4.1 Basic model

The final system consists of six differential equations:

$$\frac{d\omega}{dt} = \omega \cdot (w(\lambda) - \alpha) \tag{87}$$

$$\frac{d\lambda}{dt} = \lambda \cdot \left(\left(\frac{k(\pi_n)}{v} - \gamma \right) - \alpha - \beta \right) \tag{88}$$

$$\frac{dd_k}{dt} = \left(r - \left(\frac{k(\pi_n)}{v} - \gamma \right) \right) \cdot d_k + k(\pi_n) - \pi_s + t - g \tag{89}$$

$$\frac{dd_g}{dt} = \left(r - \left(\frac{k(\pi_n)}{v} - \gamma \right) \right) \cdot d_g - t + g \tag{90}$$

$$\frac{dt}{dt} = t(\pi_n) - t \cdot \left(\frac{k(\pi_n)}{v} - \gamma \right) \tag{91}$$

$$\frac{dg}{dt} = g(\lambda) - g \cdot \left(\frac{k(\pi_n)}{v} - \gamma \right) \tag{92}$$

The fixed and variable interest rate versions have the same form, with the difference coming from the fact that r is a constant in the former, and a function in the latter.

6.4.2 General model

The reduced form for the general model is significantly more complex, and as with the pure market model, much of this complexity comes from a common term which is the percentage rate of growth of output g_y:

$$(93)\ g_y = \lambda \cdot W \cdot e^{-W\cdot(\lambda - X)} \cdot \frac{\frac{k(\pi_n)\cdot c(\lambda)}{v} - \alpha\cdot a(\pi_n) - \beta}{V + (1 - \lambda\cdot W)\cdot e^{-W\cdot(\lambda - X)}} + \frac{k(\pi_n)\cdot c(\lambda)}{v} - \gamma$$

$$(94)\ \frac{d\omega}{dt} = \omega \cdot (w(\lambda) - \alpha \cdot a(\pi_n))$$

$$(95)\ \frac{d\lambda}{dt} = \frac{\lambda\cdot\left(\frac{k(\pi_n)\cdot c(\lambda)}{v} - \gamma - \alpha\cdot a(\pi_n) - \beta\right)}{1 - \lambda\cdot\left(\frac{W}{V + e^{-W\cdot(\lambda - X)}}\cdot e^{-W\cdot(\lambda - X)}\right)}$$

$$(96)\ \frac{dd_k}{dt} = (r - g_y)\cdot d_k + k(\pi_n) - \pi_s + t - g$$

$$(97)\ \frac{dd_g}{dt} = (r - g_y)\cdot d_g + g - t$$

$$(98)\ \frac{dt}{dt} = t(\pi_n) - t\cdot g_y$$

$$(99)\ \frac{dg}{dt} = g(\lambda) - g\cdot g_y$$

6.5 Simulations

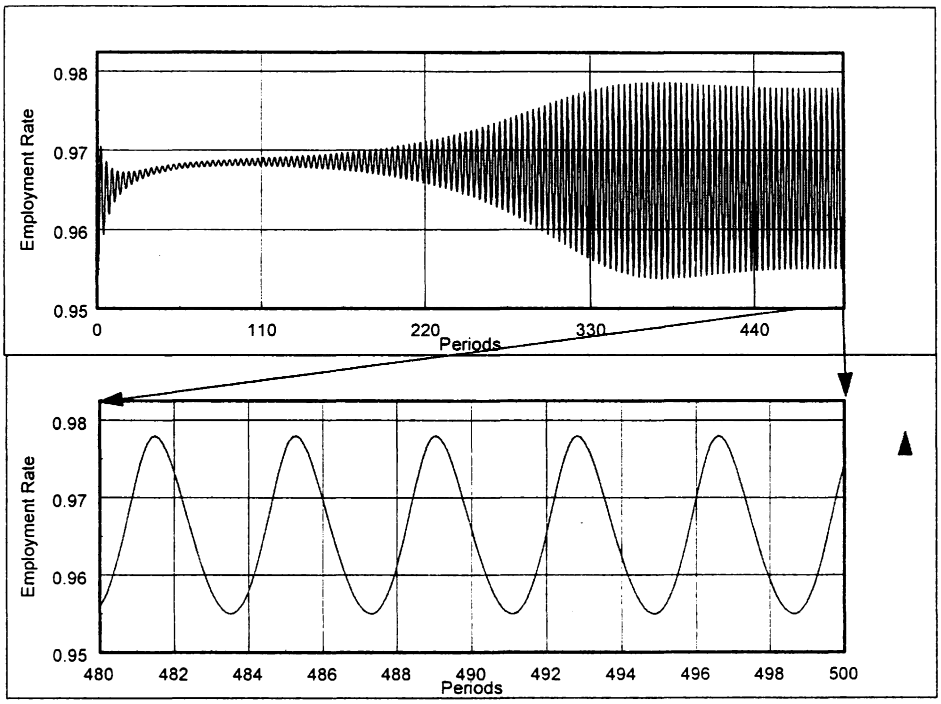
As with the finance extension, a multiplicity of models are now possible. However, unlike the finance extension, the behaviour of the model economy is qualitatively the same in all of these. The following simulations focus upon just two of the possible combinations: the basic model with a fixed rate of interest, and the general model with a variable rate of interest. In both cases, the initial level of taxation and government subsidies was zero, while the other initial conditions were as used in the previous chapters.

6.5.1 Basic Model, Fixed Interest

6.5.1.1 A Stabilised Economy

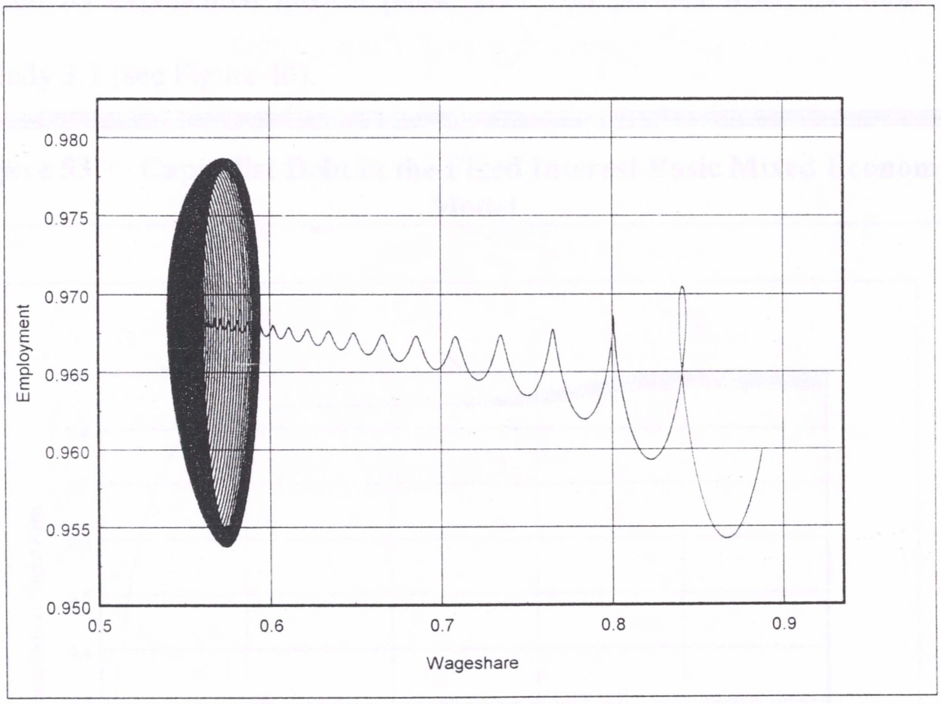
The fixed interest simulation in Chapter Five (Figures 36-40) showed that in the absence of a government sector, a 3.5 per cent rate of interest would lead to a debt deflation. Figure 51 shows the time path of employment in this mixed economy simulation with a rate of interest of 3.5 per cent—half a per cent above the level which led to a debt deflation without a government sector. In the no-government case, the rate of employment ceased its apparent convergence to equilibrium after 80 periods, and then began a cyclical course towards complete collapse (Figure 36). In contrast, the input of the government sector in this simulation results in what can best be characterised as cyclical stability. As the bottom chart in Figure 51 indicates, after a considerable period of time, the apparently stable cycles of the basic Goodwin model return.

Figure 51 Cyclical Stability of Employment in the Fixed Interest Basic Mixed Economy Model



Starting from an initial non-equilibrium set of initial conditions, wages share and employment oscillate towards the system equilibrium. However as the wages share/employment phase diagram of this system illustrates (Figure 52), this equilibrium is unstable: unless the model starts from precisely those conditions, it will diverge from them. This is because the irregular cycles of this model result from the system gravitating towards a chaotic limit cycle—an attractor which is not a single equilibrium point, but a curve in multidimensional space (Lorenz 1993: 35). From the perspective of chaotic dynamics, the introduction of a government sector converts the system from a three-dimensional system—which had a stable fixed point attractor only in the basic model when the interest rate was below a critical level, and which above this level, had no equilibrium and necessarily progressed towards breakdown—to a six dimensional system. This introduces the likelihood of complex attractor behaviour, with interactions which we can no longer visualise (since we can only "see" three dimensions) causing a phase plot to move from one orbit around an apparent equilibrium point to another distinct orbit, and then back again. With an interest rate of 3.5 per cent, unemployment cycles between 2 and 4.5 per cent, while wages share fluctuates between 55 and 58 per cent of output.

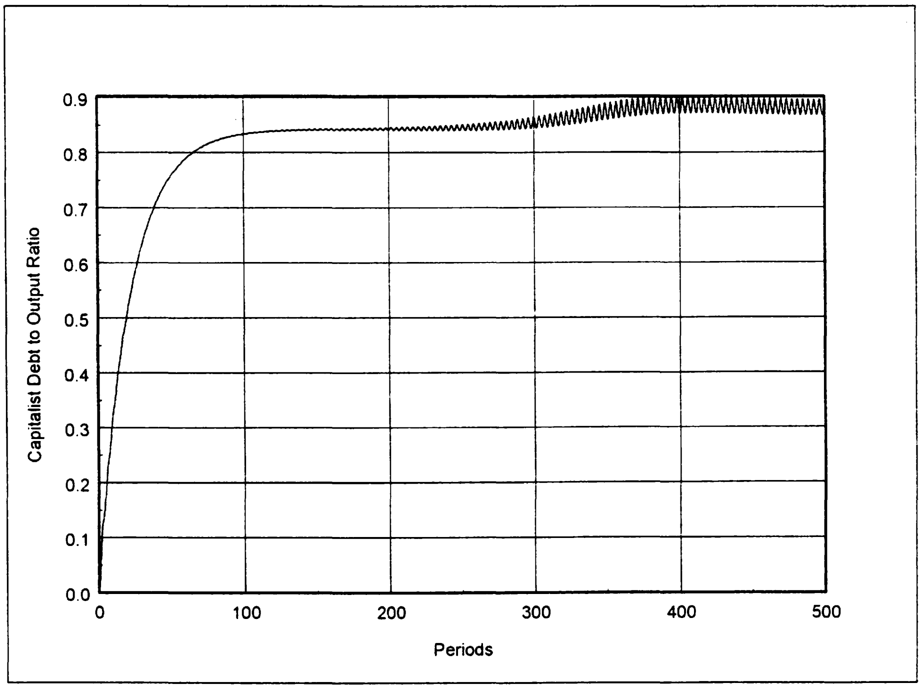
Figure 52 Two-dimensional Phase Plot of Fixed Interest Basic Model



Sustained irregular cyclical behaviour is therefore, somewhat paradoxically, a probable consequence of successful stabilisation policy. However, as is crucial from the point of view of the Financial Instability Hypothesis, this prevents a debt-induced breakdown at levels of the rate of interest which previously led to a debt deflation. The catastrophic simulations of the previous chapter can be seen, in contrast, as indicating the behaviour of an economy lacking the crucial homeostatic input of government intervention.

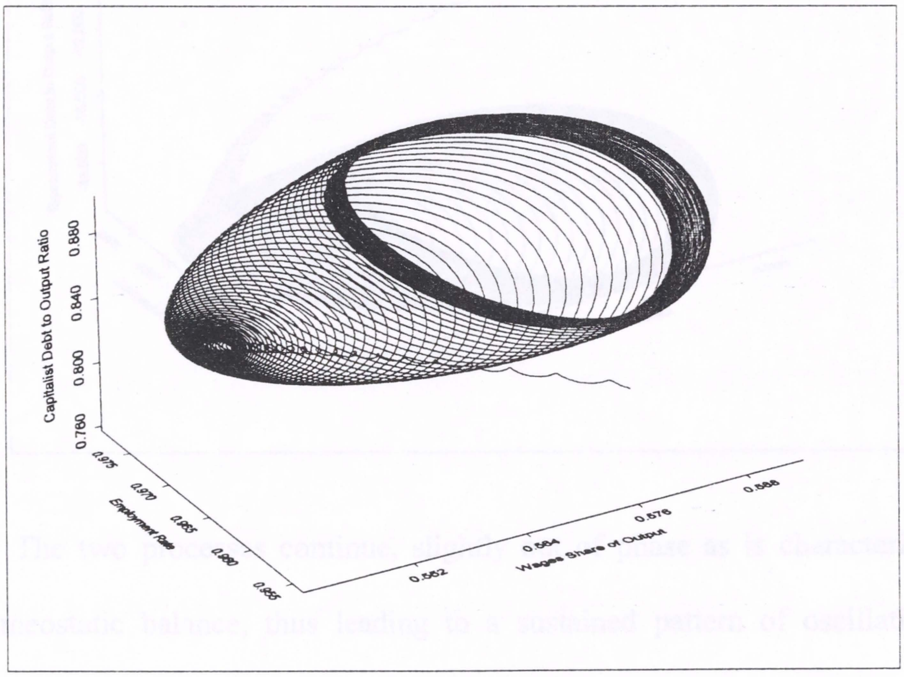
A government sector thus stabilises this model in Minsky's sense of the term: stability exists because a debt deflation becomes an impossibility. The key to this fundamental stability is evident in Figure 53: private debt grows rapidly to about 85 per cent of output after 100 periods, but then tapers chaotically towards a rate of between 85 and 90 per cent of output. By way of contrast, the capitalist debt to output ratio in the non-government simulation started its exponential growth phase after 160 periods when the ratio was already 3:1 (see Figure 40).

Figure 53 Capitalist Debt in the Fixed Interest Basic Mixed Economy Model



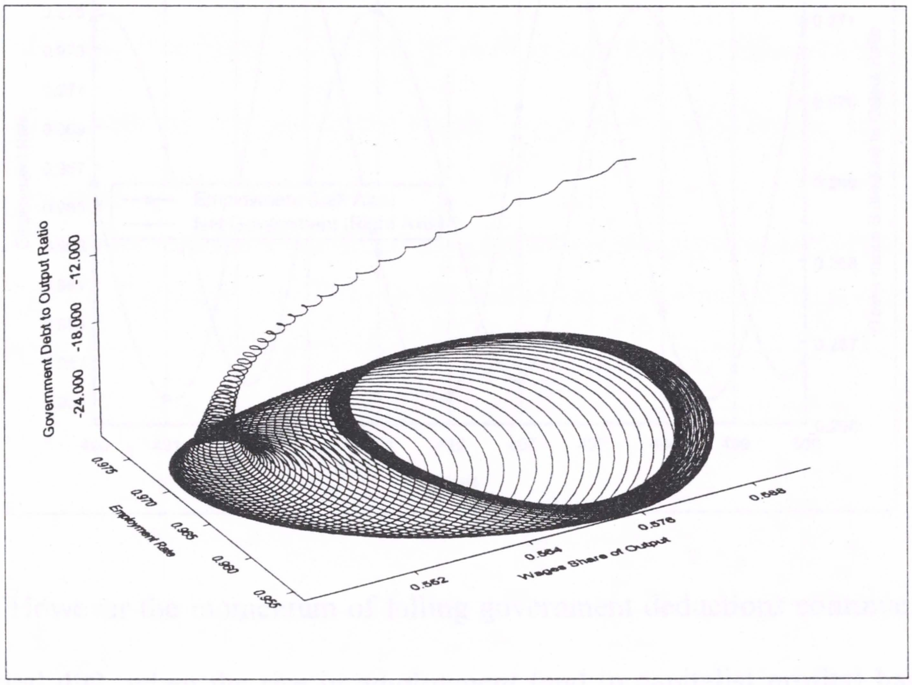
The three dimensional phase plots in Figures 54 and 55 (which begin from period 50, when most of the initial gravitation towards the equilibrium point has occurred) can be read in conjunction with Figures 52, 53 and 56 to develop a picture of the complex interactions taking place in this mixed economy model. Whereas the non-government plot revealed an ever-rising level of debt which eventually caused output and employment to collapse (see Figure 39), in the mixed economy the level of debt reaches an apparent peak at about 85 per cent of output, and then behaves in an oscillatory fashion, cycling between 85 and 90 per cent of output as unemployment cycles between 2.5 and 3 per cent, and wages share of output cycles between 55 and 59 per cent.

Figure 54 Three-Dimensional Phase Plot of "Private Sector" in Fixed Interest Basic Mixed Economy Model



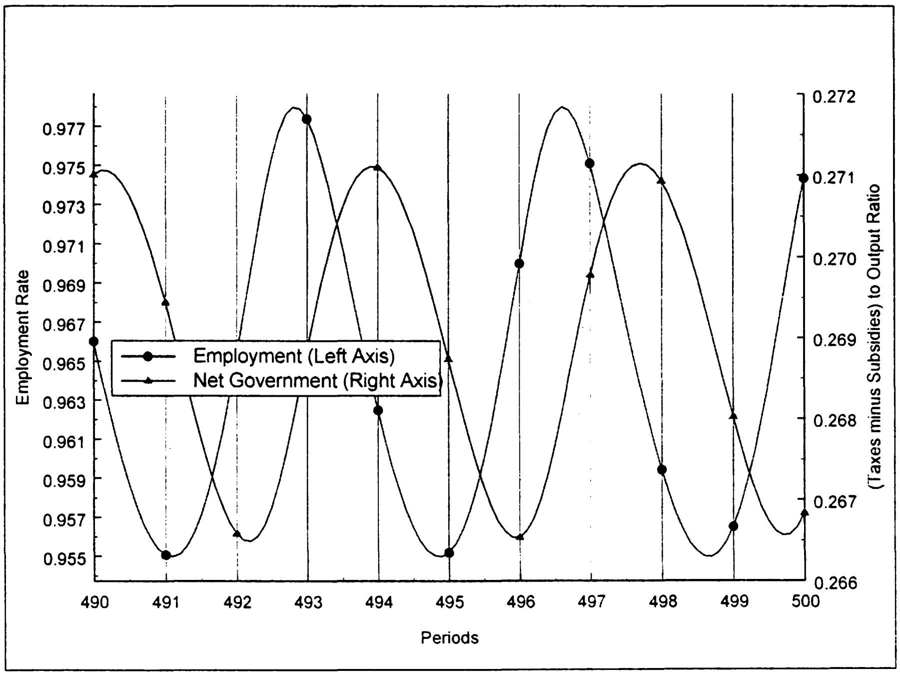
The force which drives this pattern is the countervailing behaviour of government debt, which is driven by changes in the rate of taxation and the level of subsidies (and also by interest on outstanding government debt), which themselves depend primarily upon the rate of employment. When private debt starts to rise—because of an increased level of investment—government debt starts to fall because of an increased level of taxation and a reduced level of subsidies. These in turn are financed by government deductions from capitalist net profit, which thus restrains and eventually reverses the growth in capitalist investment.

Figure 55 Three-Dimensional Phase Plot of "Public Sector" in Fixed Interest Basic Mixed Economy Model



The two processes continue, slightly out of phase as is characteristic of homeostatic balance, thus leading to a sustained pattern of oscillations in private and public debt. This phase relationship is most clearly seen in Figure 56 where in period 490, net government intervention is at its apogee, and employment is falling (as a consequence of depressed levels of investment). Government deductions from capitalist cash flows then decline, but this is not immediately sufficient to reverse the trend in investment, so that the rate of employment continues to fall until period 491. At this point, the level of government deductions has fallen sufficiently to lead to a level of investment which reverses the trend to falling employment.

Figure 56 Cycle Duration and Phase relationship between Private and Public Sectors



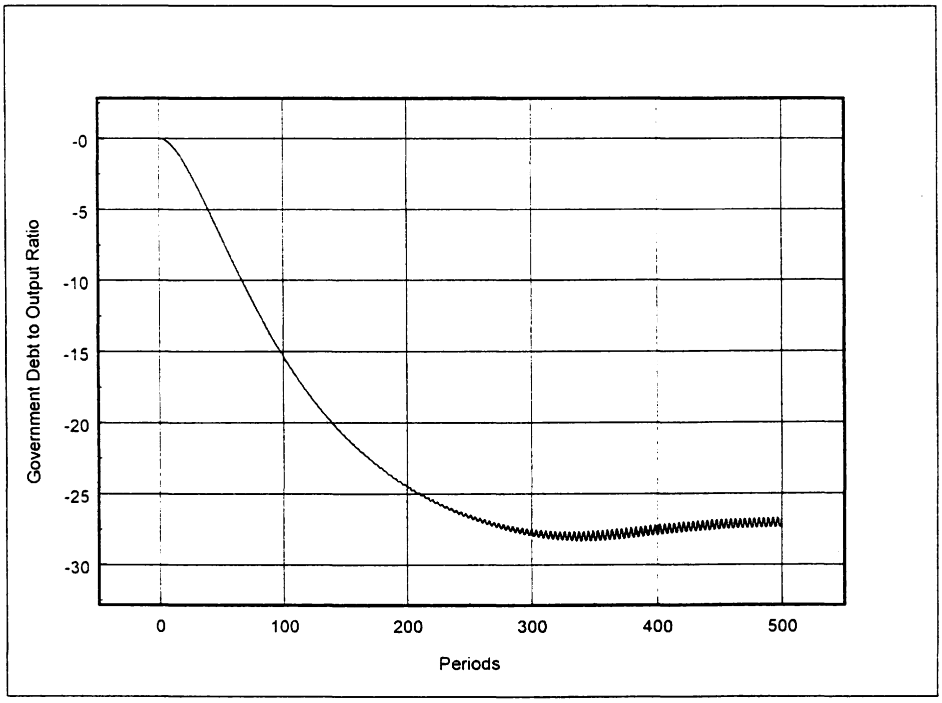
However the momentum of falling government deductions continues until period 492, when the rise in employment (and in capitalist profits) has been sufficient to trigger an increase in the level of net government deductions. This begins to rise from its perigee, but not sufficiently to restrain the momentum of the rate of employment, which continues to rise until half way through period 492. The decline in employment then continues while net government rises to its next apogee in period 494, and the entire process begins again.[51]

Figure 56 also indicates that the trade cycles in this six-dimensional model meet Blatt's criterion of asymmetry, with upturns taking slightly less than 2 periods, and downturns almost exactly 2 periods, to give an overall cycle duration of just under 4 periods.

6.5.1.2 A Bifurcation at Higher Rates of Interest

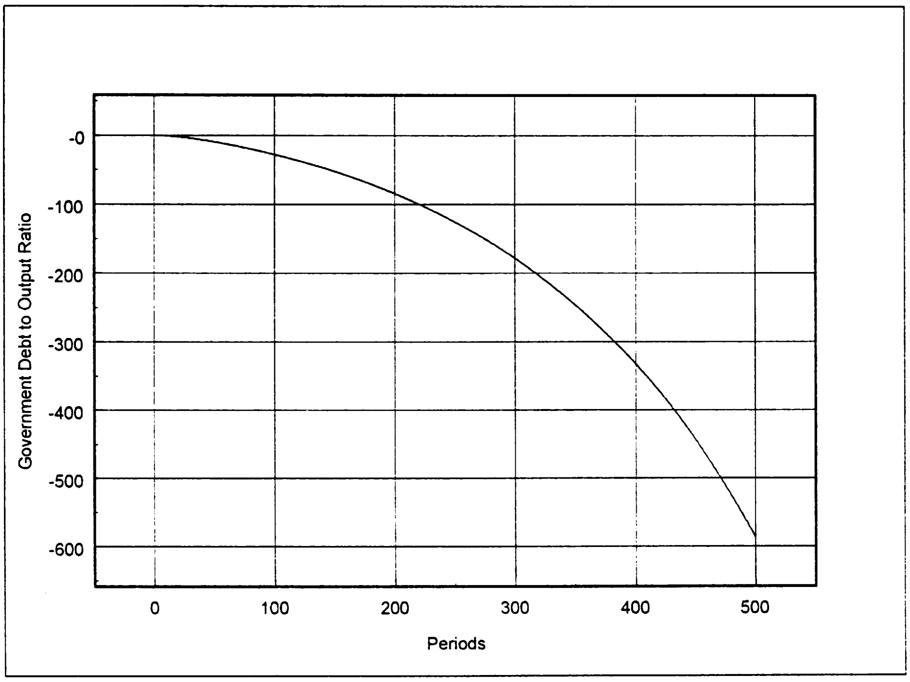
The simulations in the preceding sector, with an interest rate of 3.5 per cent, result in the capitalist debt to output ratio stabilising at just under 100 per cent (see Figure 53), while over the 600 periods of the simulation, the public sector actually accumulates a surplus (a "negative debt") equivalent to 27 times output (see Figure 57). This level of accumulated public sector debt is a function both of the parameter values used in the simulation, and of the interest rate.

Figure 57 Public Sector Debt in the Fixed Interest Basic Mixed Economy Model



As the interest rate rises, so too does the size of the surplus accumulated by the public sector—until such time as the rate of interest exceeds the long run average of the rate of growth. Once this critical point is passed, government intervention can still stabilise the economy, but only at the "price"—in these simulations—of an ever-expanding public sector surplus. Figure 58 shows this effect for an interest rate of 5 per cent.

Figure 58 Public Sector Debt at High Interest in the Fixed Interest Basic Mixed Economy Model



The reason for this effect is apparent when the formula for the relationship between the rate of interest and the rate of growth is considered, as discussed earlier in this chapter. If $(r - g_y)$ is positive sufficiently often and of sufficient magnitude, then the exponential side of the relationship can dominate the function. Whether this occurs depends on the magnitude of the other components of the equation.

In the case of private debt, the other terms include the difference between investment and profits $(k(\pi_n) - \pi)$ and net government $(t - g)$. Given that net government starts from zero in these simulations, the former will initially be positive and larger than the latter, so that a positive debt is accumulated. However as this debt rises, so does the restraining impact of net government. The overall impact of the non-exponential terms is thus negative, which restrains the exponential side of the relationship even when the rate of interest exceeds the rate of growth.

In the case of government debt, since the role of the government sector in this model is to restrain capitalist investment so that a debt deflation is avoided, the government's deductions from the system exceed its additions to it. As a result it accumulates negative debt, and since the other terms in its function are both negative, its "negative debt" will grow exponentially unless $r \leq g_y$ on average.

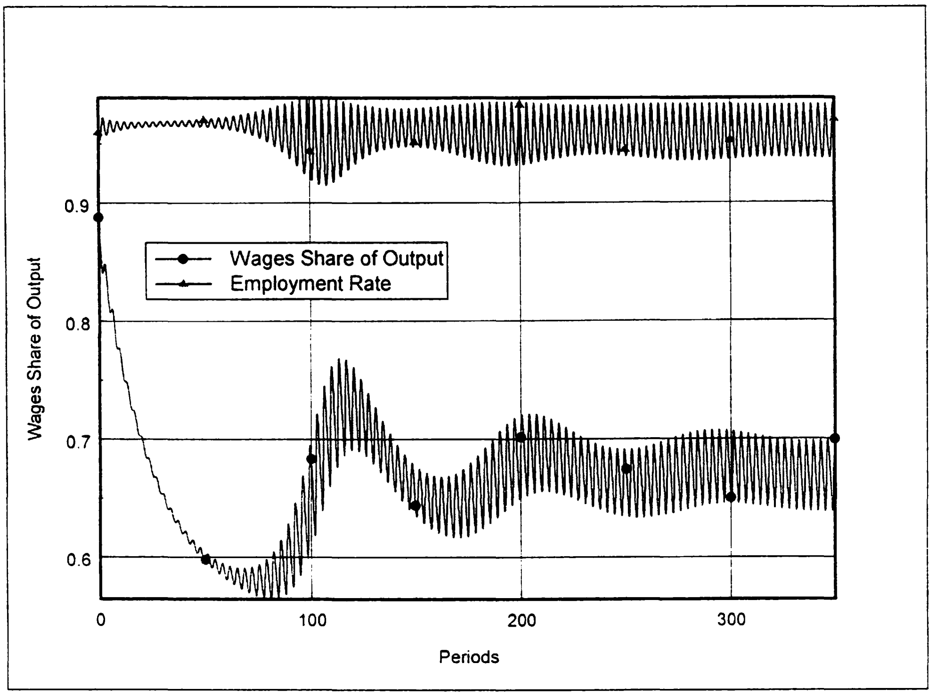
This in effect proposes that the banking sector has an unlimited capacity to pay interest on accumulated government reserves, which is of course unrealistic. However the level of accumulated government reserves is not an input to any other part of the model (whereas the level of private debt is, via its effect on the rate of net profit). This model quirk thus has no causal impact on the validity of the model, and could in any case be attenuated by assuming a zero rate of interest for government debt, or by introducing a "social wage" function of government, so that the net gap between taxation revenue and subsidies became a payment to workers, rather than accumulating in the government's coffers.

A situation in which rentiers expect a higher rate of return than the rate of economic growth is of course one which is replete with potential for economic instability. The simplified nature of this model gives the result that the only symptom of this instability is a rising public sector accumulated surplus. However in the real world, such an excessive rate of interest would doubtless manifest itself in numerous other forms of instability.

6.6 General Model Variable Interest

The patterns found in the simplest mixed economy model are also apparent in the general model, which is the most complete rendition of Minsky's hypothesis possible within the confines of a single commodity model. It results in a very rich pattern of endogenous fluctuations (within the overall context of a growing economy), with no possibility of a debt-deflation. Figure 59, which plots the time path of employment (and the wages share of output), should be compared to Figure 51 to illustrate how much more complex the cycles are in this general model.

Figure 59 Persistent and Pronounced Cycles in Variable Interest General Model



In particular, there appear to be "long waves" in the data, such as between periods 80 and 120, when the rising level of wages share of output indicates a general tendency for the rate of economic growth to exceed the rate of growth of the labor force, and between 120 and 170, when the opposite is true. These effects arise from the endogeneity of capacity utilisation and technical change, and their relationships with the rate of employment and the profit share of output, rather than from any exogenous influences. The two and three dimensional phase plots of Figures 60 and 61 indicate that these complex patterns are simply a generalisation of the behaviour of the basic model, whose phase plots display a more restricted range of movement, but the same broad characteristics (see Figures 52 and 54).

Figure 60 Two-dimensional Phase Plot of Chaotic Limit Cycle in Variable Interest General Model

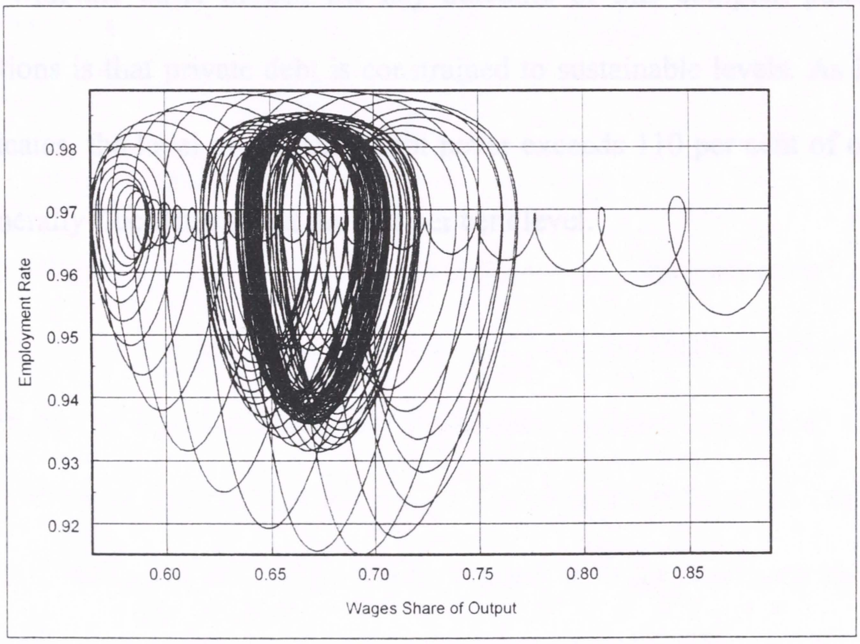
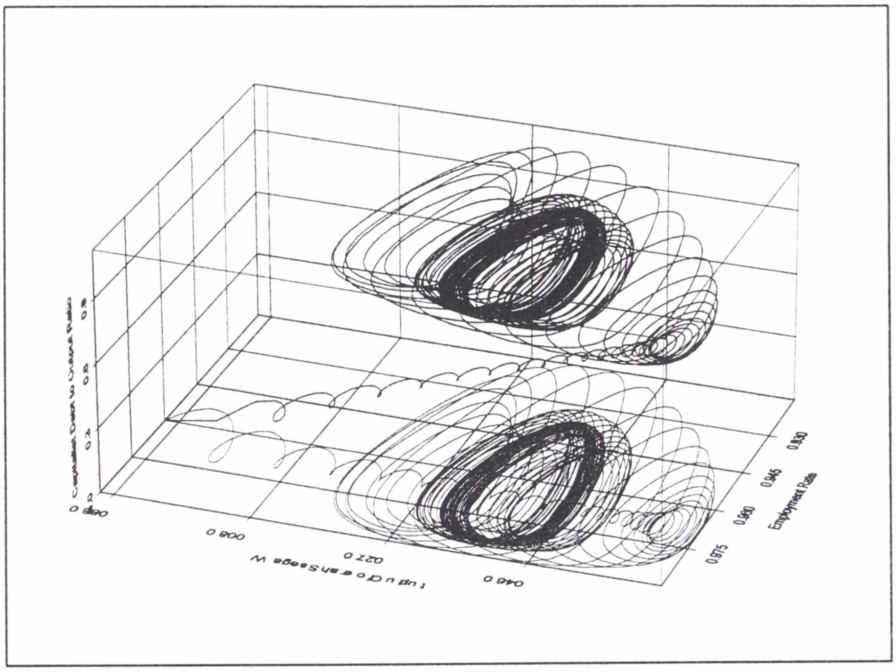
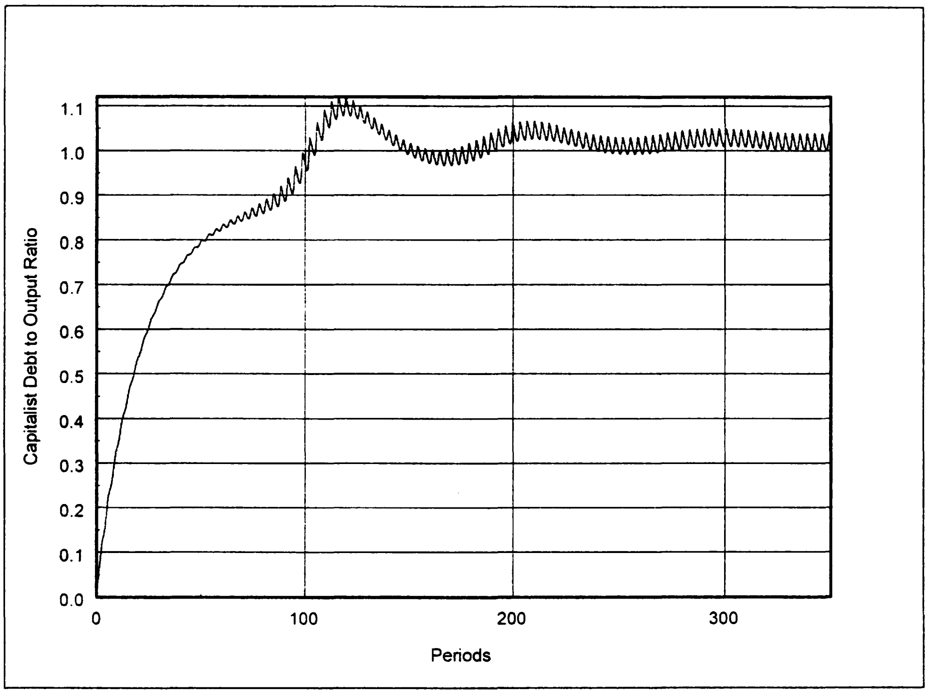


Figure 61 Three-dimensional Phase Plot of Chaotic Limit Cycle in Variable Interest Basic Model (with 2D projection onto Wages Share/Employment Plane)



As with the basic model, the key outcome of this complex pattern of interactions is that private debt is constrained to sustainable levels. As Figure 62 indicates, the level of capitalist debt never exceeds 110 per cent of output, and generally fluctuates about the 100 per cent level.

Figure 62 Capitalist Debt to Output Ratio in Variable Interest General Model



The presence of a government sector thus converts a model in which, in the absence of a government, a debt deflation was inevitable, into one in which a debt deflation is an impossibility.

6.7 Conclusion

The degree of government intervention in the economy which characterised the 50s and 60s has been wound back drastically in the 70s, 80s and 90s, on the practical basis that "Keynesian" policies had failed, and the theoretical basis that such policies were bound to destabilise an otherwise inherently stable system. Minsky has argued that these arguments were misguided, the former by failing to see that these policies in fact worked by preventing a debt deflation, and latter by incorrectly interpreting Keynes through the neoclassical prism of Hicks's IS-LM analysis. The models detailed above provide a modern advocacy for the concept of the mixed economy, at a time when Keynes's message has been completely eradicated from popular and policy consciousness, and almost as completely expunged from economic analysis.

The importance of government is emphasised by the results of incorporating a stylised "Minskian" government into the model: its interventions convert situations which previously led to breakdown into ones which generate irregular cycles, of a kind reminiscent of those experienced during the long post-War boom. These simulations provide strong support for Minsky's proposition that the institutional arrangements instituted in the aftermath of the Great Depression "worked", since though cycles occurred, breakdown did not. The objective of stabilisation policy was not to avoid cycles—which are endemic to any complex system—but to prevent the possibility of complete economic collapse. As emphasised at the beginning of this Chapter, this is a far stronger case for the government sector than the conventional "Keynesian" view of stabilisation policy, in which government automatic stabilisers simply attenuate the trade cycle.

While the above "Minskian" government simulations are easily able to restrain the systemic tendency towards a debt deflation, there must be severe doubts as to whether the kind of government which has been deconstructed over the last thirty years is a sufficiently powerful or balanced stabiliser to capitalist investment behaviour. The most important changes have come on the taxation side, as the near abolition of progressive tax regimes has meant that speculative investment is much less effectively restrained now than it was during the Golden Age of the 1950s and 1960s. The dismantling of the welfare state has reduced the ability of government to underwrite capitalist cash flows during a downturn, though the existence of unemployment benefits and income tax means that this effect still exists in a more muted form. However the fetish for zero budget deficits, regardless of the level of unemployment, means that once the economy has settled into a depressed state after a crisis, it is less likely to emerge from it. We therefore can expect more frequent speculative binges, and more prolonged slumps as we recover from them.

6.8 Limitations of the analysis

While these models do illustrate the impact of long term debt on a capitalist system and the potentially stabilising impact of government, they need significant extension to fully reflect Minsky's hypothesis. In particular, the models above do not consider Minsky's analysis of the role of prices in a capitalist economy.

A verbal outline of the cycles that a model with prices would generate starts with the economy on a path of relatively stable growth, after recovering from a preceding crisis. A period of time on this path leads to rising expectations, and hence a rising valuation of expected future income streams. Hoarding diminishes (liquidity preference falls) and investment increases, as does the level of debt incurred to finance investment. The increased investment leads to a rising rate of economic growth, and sets off both Goodwin's correction mechanism (rising money and real wages) and Minsky's—rising equipment prices, rising debt, rising interest costs and a diminishing net rate of return. (A more complete elaboration of Minsky's model would include a market for assets, and the flooding of this as rising debt made more conservative investments speculative. However this elaboration would involve an enormous increase in mathematical complexity.) The rate of investment thus falters, and with it the rate of economic growth. But unlike the non-monetary models above, this does not necessarily result in a downturn followed by the reproduction of the cycle at a different pitch, since the interaction between the rate of change of the commodity price level and the rate of change of the asset price level can now affect the outcome.

If the initial rate of commodity price inflation is high, then the debts accumulated during the boom can easily be repaid during the slump, because the rising price level in effect reduces the real magnitude of debt. The slump can however alter the rate of commodity price inflation, via its impact on the rate of growth of money wages.

If the initial rate of commodity price inflation is low, then capitalist cash flows during the slump can be below the level needed to meet interest payment commitments, so that the level of debt continues to rise and investment drops to zero. The slump in output causes a rise in unemployment and reduces the rate of growth of money wages even further, and the impact of this on the already low rate of inflation can be compounded by reductions in markups as firms undertake "beggar my competitor" tactics in an attempt to increase sales in a declining market. The commodity price level can thus go into decline, leading to Fisher's Paradox of a rising real level of debt.

A key feature of this model in contrast to neoclassical ones, is that prices have a destabilising rather than stabilising role. With prices set for wage goods by a markup on (constant or falling) average cost and prices for investment goods set by expectations of capital gain, the divergence of the two price levels actually pushes the economy off the equilibrium path. Equally, change in the price level, which is normally irrelevant in neoclassical models, has a potentially stabilising role during times of crisis.

To properly consider these dynamics requires the construction of a multi-sectoral model of the economy, in which both price dynamics and input-output considerations can be explored. It is to these issues of multi-sectoral analysis that I now turn.

7 Markup Pricing Dynamics and Input-Output Analysis

As noted in Chapter Three, Minsky did not attempt to produce a complete mathematical rendition of the Financial Instability Hypothesis, after having made a doomed foray into modelling a finance-driven trade cycle using the Hansen-Samuelson multiplier-accelerator framework. Instead, as he developed the FIH verbally, he increasingly relied upon Kalecki's equations for the micro and macrofoundations of his own theory (Minsky 1978, 1982: 103-104; 1980a, 1982: 36-44; 1980b, 1982: note 19, 88-89).

An important component of Kalecki's own microfoundations was the proposition that the prices of most produced commodities were set by markups on prime costs, where those markups reflected the degree of monopoly in each sector (Kalecki 1942, 1954). Minsky accepted this explanation of price formation, attenuating it by the claim that equipment prices were also in part demand-determined. Recently, Steedman (1992) has challenged the soundness of the Kaleckian price theory, on the basis that the focus upon an intra-sectorally determined markup ignores the avowed multi-sectoral, input-output nature of production. Input-output relations are, as Steedman properly asserts, a "brute fact about modern industrial economies" and he contends that it is "at best, one-sided to say that 'prices are largely cost-determined' without saying also that 'costs are to a significant degree price determined' (1992: 126). He argues that when input-output analysis is incorporated into a short run Kaleckian markup pricing framework, many of the price setting and income distribution propositions made by Kaleckians can be shown to be invalid.

In addition, Steedman challenges the need for dynamic analysis at all, asserting in effect that dynamics merely describes the traverse between one position of long run equilibrium and another.

Were Steedman correct on the first count, it could be argued that at least some of the foundations to Minsky's Hypothesis are unsound. Were he correct on the second, then there would be little point in establishing models of multi-sectoral dynamics, and such models are necessary to explore Minsky's claims about the price dynamics of capitalism. This chapter considers Steedman's critique of Kaleckian mark-up pricing and rejection of the need for dynamic analysis, as a prelude to the development of a multi-sectoral model of an economy with finance in Chapter Eight.

7.1 Steedman's critique

The crucible in which Steedman tests the Kaleckian approach to pricing is a simple Sraffian model of price in an economy with circulating capital only, producing n commodities, with constant returns to scale, unchanging technology and exogenously given labor and imported inputs. In such a model, the equilibrium price vector is given by the equation

$$(100) p = (u + pA)(I + \hat{m})$$

where p and u are row vectors of prices and exogenous inputs respectively, A is the production matrix, I the identity matrix, and $\hat{m}$ a diagonal matrix of industry markups.

After using this system to criticise Kaleckian claims about the impact of a change in markups on prices, the concept of an average markup, the wages share and markups, and the notion of vertical integration, Steedman anticipates potential dismissals of his critique, saying that he would not be surprised "to find the foregoing analysis giving rise to such responses/rejections as: Kalecki was never interested in long-period equilibrium theory!..." (1992: 145). He then turns to dynamics, by considering first of all an economy where "production methods, prices and markups have been unchanged for many periods" (1992:145) so that the above equation, which is an equilibrium condition, is the initial condition for his dynamic model. Next he considers a once-only change in exogenous inputs from *u* to *u* + *du*, so that the system is disturbed from its equilibrium. He then argues that after this disturbance,

> The commodity price vector at the end of period 1 will be given by $p_1 = (u + du + pA)(I + \hat{m})$... At the end of period 2, it will be $p_2 = (u + du + p_1 A)(I + \hat{m})$" (1992: 145-46)

Steedman shows that this process leads in the limit to the same situation as shown in his preceding static analysis, and asserts (1992: 146) that

> The general point which is illustrated by the above examples is, of course, that our previous 'static' analysis does not 'ignore' time. To the contrary, that analysis allows enough time for changes in prime costs, markups, etc., to have their *full* effects.

It is this assertion with which I take issue, for three reasons:

(a) The static equilibrium position is the endpoint of a dynamic process only if that process is stable. It is easily shown that the stability of Steedman's equation is conditional, not absolute.

(b) Steedman's equation assumes that markups would not alter in response to a change in relative profit rates. This can be shown to be an inaccurate reading of Kalecki's writings. When variable markups are introduced, the resulting system is necessarily nonlinear, and generally marginally unstable.[52] When a nonlinear markup adjustment process is used, the equilibrium ceases to be a fixed point of the system.

(c) Steedman's equation considers only price dynamics, when in a general dynamic model of a capitalist economy, several other dynamics—in particular, those of quantity and finance—would also be present. These additional dynamics all but guarantee that the fixed point(s) of such a system will be unstable, with the consequence that conditions which apply at those points are irrelevant to the short and long term behaviour of the model.

7.2 The conditions for price instability

The general form of Steedman's dynamical system is

(101) $$p_{t+1} = (u + p_t A)(I + \hat{m})$$

Recasting this in the form of an autonomous dynamical system, we have:

(102) $$p_{t+1} = (u)(I + \hat{m}) + p_t A(I + \hat{m})$$

The general solution is

(103) $$p_t = c(A(I + \hat{m}))^t + u(I + \hat{m})(I - A - A\hat{m})^{-1}$$

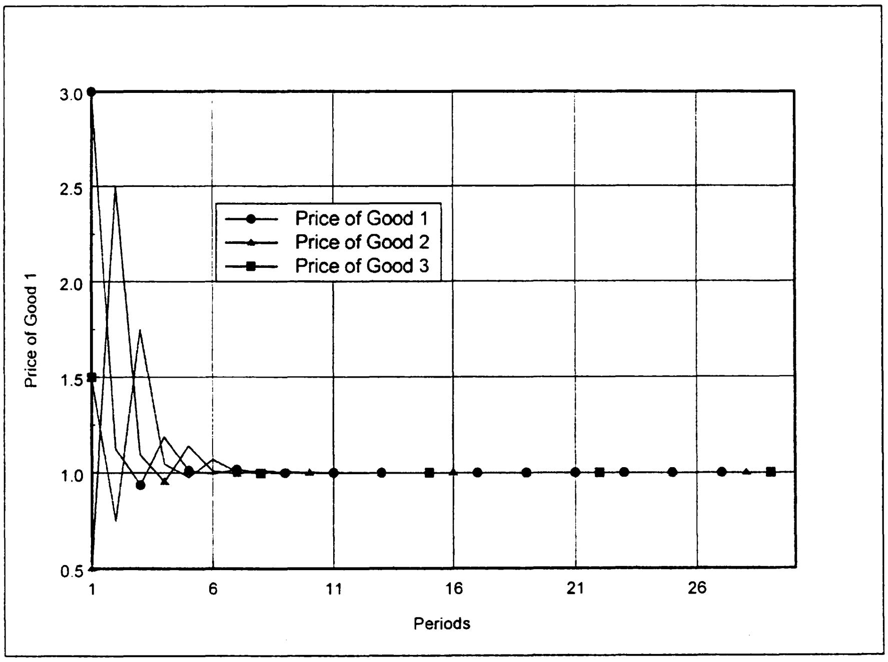
where c is a constant vector (whose elements are determined by initial conditions). The equilibrium value for p of $u(I+\hat{m})(I-A-A\hat{m})^{-1}$ is clearly stable only if the modulus of the dominant eigenvalue of $(A(I+\hat{m}))^{t}$ is less than one. This is true for the example input-output matrix and markups Steedman uses to illustrate his argument, but it is not true in general for any valid input-output matrix A and arbitrary vector of markups m.

This point can be illustrated by two numerical examples, one identical to Steedman's, the other differing only in having a different input-output matrix. The first model has the following characteristics:

$$(104) A = \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 \end{pmatrix}, m = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}, u = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

As Figure 63 indicates, if this system starts with a nonequilibrium price vector—say, *(3,1,.5)*— then it rapidly converges to equilibrium. To all intents and purposes, equilibrium is restored after just nine iterations.

Figure 63 Convergence to Equilibrium in Steedman's Model with his Sample Input-Output Matrix

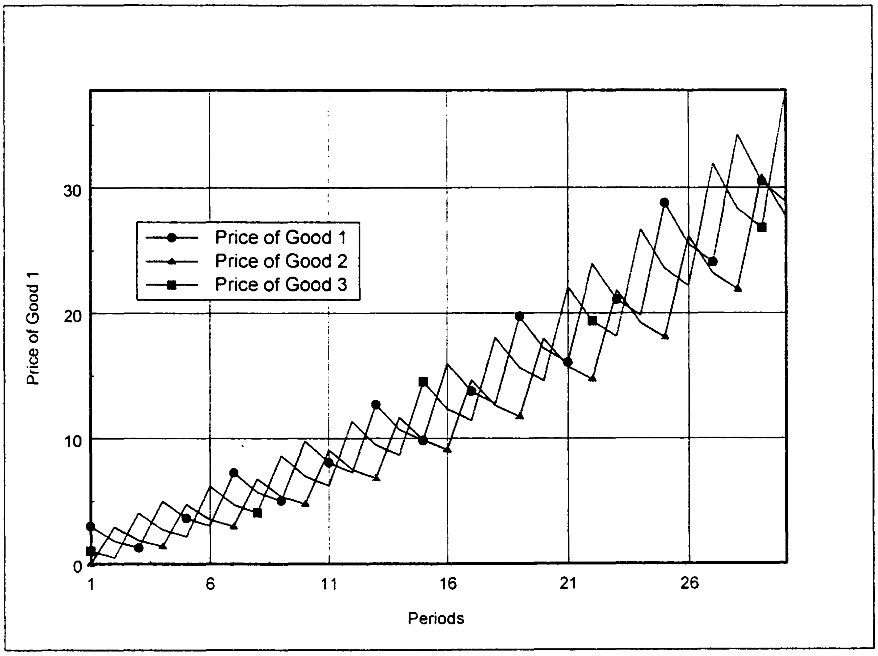


However if we consider a second model economy with a different a different input-output matrix:

$$(105) A = \begin{pmatrix} 0 & \frac{3}{5} & 0 \\ 0 & 0 & \frac{4}{5} \\ \frac{7}{10} & 0 & 0 \end{pmatrix}$$

but exactly the same exogenous inputs and markups vectors, Figure 64 indicates that this economy is quite clearly unstable. The price in each sector fluctuates about an exponential trend as it diverges from the equilibrium vector given by equation 100.

Figure 64 Divergence from Equilibrium In Steedman's Model with a Different Input-Output Matrix



There is an unremarkable explanation for this phenomenon, from a static equilibrium point of view: the arbitrary markups chosen are incompatible with positive equilibrium prices for this input-output matrix. From a static point of view, this would automatically rule out such an input-output/markup combination in the long run. However, the combination of a negative equilibrium price vector and a linearly unstable price system is quite viable in a dynamic, nonequilibrium context, since it ensures that prices will never, in fact, become negative in the long run—because they will forever diverge from the equilibrium point.

This shows that in a dynamic setting, contrary to Steedman's argument, the markups of price over direct unit cost can be set on the basis of intra-sectoral competitive pressures alone and without regard to (but not independently of) input prices—though this will result in sustained price inflation. However, this interpretation will not be continued with, since it is easily shown that the assumption of constant markups over time is not an accurate rendition of Kalecki.

7.3 Markup dynamics

There is ample evidence in Kalecki's writings that he would expect industry markups to alter in response to changes in relative rates of profit, as a consequence of competitive pressures in a general setting of monopolistic or oligopolistic competition. These competitive pressures would lead to lower markups in industries experiencing higher rates of profit, and vice-versa. In Kalecki 1942, he considers a supposed counter-example to his markup pricing model, with two industries, in one of which the capital is twice, and the other half, annual sales. Kalecki argues that "If the degree of monopoly is the same in both industries, the industry B will certainly earn a much higher rate of profit. This will attract new entries into industry B, and its capital will rise in relation to sales..." (Kalecki 1942: 122)

Later in the same paper, Kalecki argues that gross profit margins and prime costs "need not change in exactly the same proportion, because 'my' degree of monopoly cannot be assumed to remain constant." (126-7)

Kalecki's analysis is in general cast in terms of monopolistic or differentiated oligopolistic competition (Kalecki 1971: 160), where the degree of competitiveness in each industry reflects partly its own characteristics and partly its relative profitability. Kalecki (1971: 49-52) and Kriesler (1987: 76) list the factors which might change the degree of monopoly, and these include "increase of concentration" which would increase the "degree of monopoly", leading to higher markups. Conversely, a decrease in concentration should decrease the degree of monopoly and lead to a fall in margins.

This indicates that to properly consider the partial dynamics of price formation, the markup must be regarded as a variable determined by the degree of competitiveness in each sector, where the degree of competitiveness is directly related to the relative profitability of each sector. Thus, in general terms, Steedman's partial dynamic system must be modified to something of the following form in order to properly capture the impact of input-output relations on Kaleckian markup and price dynamics. Equation block 106 shows such a system. Whatever form is chosen for μ, this will be a nonlinear system, because of the existence of product terms of the general form $p_{t-1} \cdot A \cdot m_t$:

$$
\begin{aligned}
p_t &= (u + p_{t-1}A)(I + \hat{m}_t) \\
m_t &= \mu((u + p_{t-2}A)\hat{m}_{t-1})
\end{aligned}
\tag{106}
$$

Unlike linear systems—such as that explored in the previous section—the equilibria of a nonlinear system can be locally unstable but globally stable. Consequently, the conditions which apply at equilibrium can be irrelevant to both the short and long term state of the system, without necessarily leading to runaway price inflation.

To illustrate this, I will develop three "Kaleckian" models of price setting in which the adjustment process (a) is partial dynamic, ignoring quantity, technology and finance dynamics; and (b) presumes markups are inversely dependent upon sectoral profits, due to competitive forces. The first two models use a linear form for *m*, and demonstrate equilibria which are marginally unstable. The third uses a more realistic nonlinear form for the markup adjustment process, and illustrates a novel form of instability.

The markup adjustment process in first model is based simply on changes in gross profits. This apparently contradicts Kalecki's view (Kalecki 1942: 124-26) that it is not the absolute profit margin which reflects the degree of monopoly, but the rate of profit. However as equation block 107 indicates, the gross profit is used here because the partial nature of this model means that the rate of profit collapses simply to the markup:

$$\pi_t = \left(\Pi_t^T \cdot \left(\hat{C}_{t-1}\right)^{-1}\right)^T$$

$$(107) \quad = \left(u^T + p_{t-1}^T \cdot A\right) \cdot \hat{m}_t \cdot \left(\left(\left(\left(u^T + p_{t-1}^T \cdot A\right)^T\right)^{-1}\right)^T\right)$$

$$= m_t$$

While in a general model, the rate of profit would depend upon quantity and other dynamics, in this partial model the rate of profit depends only upon the markup, which implies a markup adjustment mechanism whose dynamics are completely independent of prices:

$$(108) \quad m_t = m_{t-1} - a \cdot \left(\pi_{t-2} - \pi_{t-3}\right) = m_{t-1} - a \cdot \left(m_{t-2} - m_{t-3}\right)$$

A gross profit adjustment mechanism, on the other hand, incorporates a feedback from prices to markups, while providing a simple illustration of the instability and sustained cyclical behaviour that can emanate from such a dynamic model.

The second model is driven by the comparison of sectoral rates of profit to the economy-wide average. This more closely approximates Kalecki's statements in Kalecki 1942.

The third model makes the more realistic assumption that the markup adjustment process will be nonlinear, and its behaviour has some relevance to the debate about the stability of the classical model (Steedman 1984, Flaschel & Semmler 1987, Dumenil & Levy 1987, 1989). The centre of attraction in the classical model is the average rate of profit, not the equilibrium rate. In a linear system, these two necessarily coincide, but in a nonlinear system, this is by no means guaranteed (see Blatt 1982: 211-216 for an exposition of this as regards Goodwin's 1967 model). If the average and the equilibrium rates of profit do not coincide—as in the third model—then the conditions which pertain at equilibrium are doubly irrelevant to the system, since in this circumstance, the equilibrium is not even a centre of attraction of the model.

7.4 A gross profit adjustment mechanism

The markup is modelled as a lagged linear function of previous markups and the direction of change of profits (in what follows, all vectors are column vectors):

$$(109)\quad m_{t+2} = m_{t+1} - a(\Pi_{t+1} - \Pi_t)$$

where a is a constant indicating the degree of responsiveness of markups to changes in profitability[53] ($0<a<1$) and profit Π is costs times the markup:

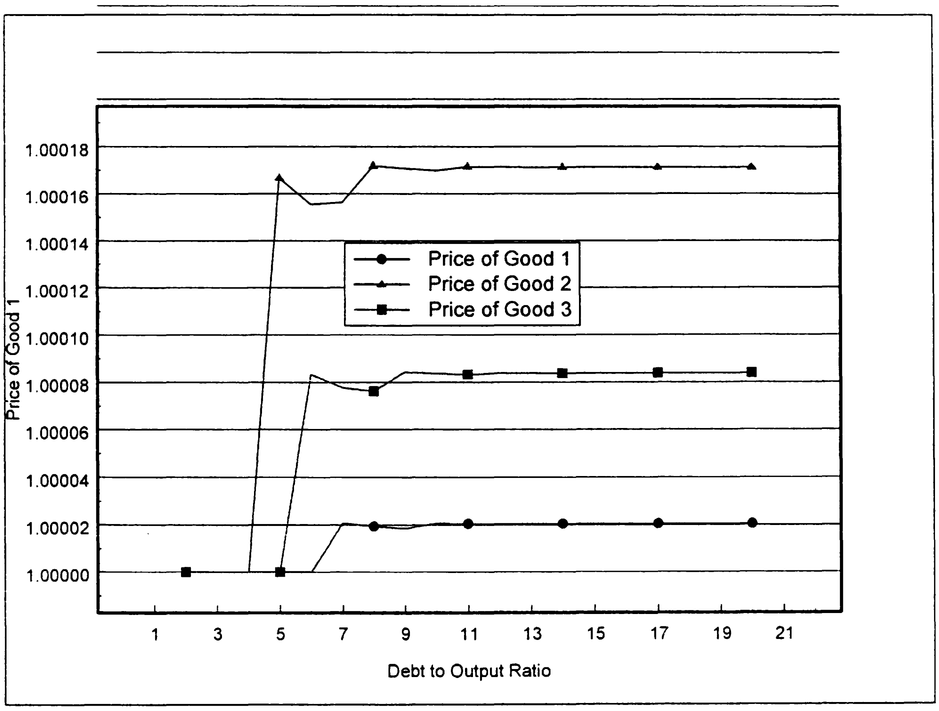
$$(110)\quad \Pi_t = \left(\left(u^T + p_{t-1}^T A \right)^T \hat{m}_t \right)^T$$

This yields a third order relation for markups in terms of prices, and hence a dynamic system of two third order vector difference equations:

$$(111)\quad \begin{aligned} p_{t+3} &= \left((u^T + p_{t+2}^T A)(I + \hat{m}_{t+3}) \right)^T \\ m_{t+3} &= m_{t+2} - a\left(\left(u^T + p_{t+1}^T A \right)\hat{m}_{t+2} - (u^T + u^T +)\hat{m}_{t+1} \right) \end{aligned}$$

The dynamic properties of this system can be quickly indicated with a numerical simulation. If the initial position corresponds to the equilibrium price and markup vectors, then these are maintained indefinitely. However, if the initial position differs from equilibrium, the system does not return to it. Figure 65 shows the time path of prices in a simulation in which a has the value of .1 and the initial value of p_l is 1.01—a 1% divergence from the equilibrium value for one period only. Rather than returning to the equilibrium vector of (1,1,1), prices move to the new vector of (1.0000205336995569, 1.0001713314090339, 1.0000838800373071) after 11 periods, even though the exogenous inputs are the same as for the previous price vector.

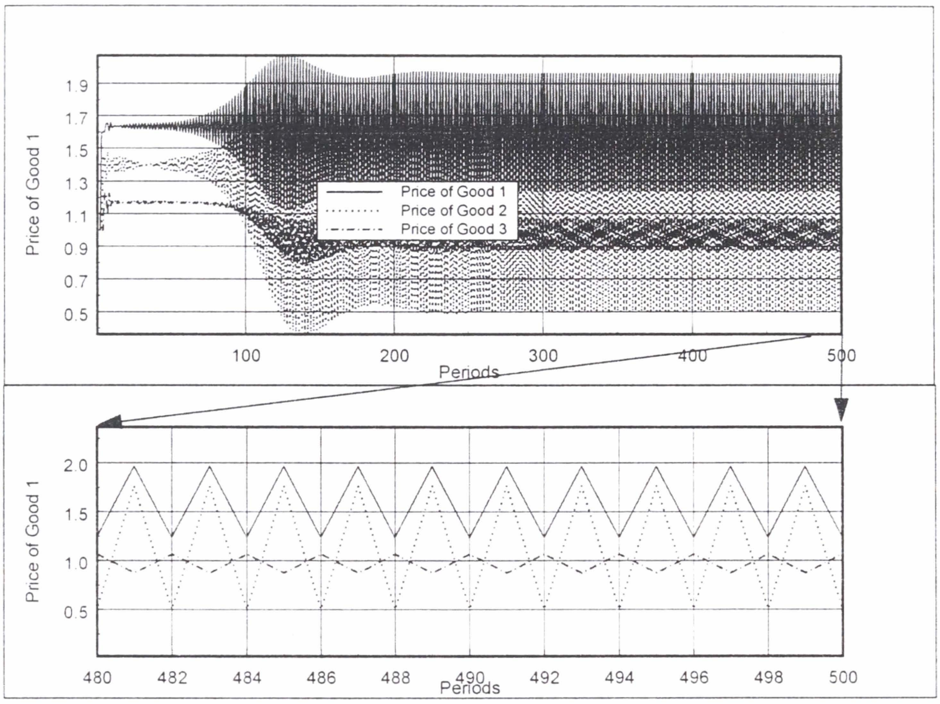
Figure 65 Marginal Instability in Kaleckian System with Gross Profit Markup Adjustment Mechanism



This type of behaviour is evidence of marginal instability, which applies whenever a difference equation has one or more eigenvalues of 1.

Similarly, a change in the u vector, rather than leading to a new equilibrium, can result in sustained instability. Figure 66 plots a system in which a equals .965, prices and markups are as for Steedman's initial u vector, but the first element of u is changed from ½ to 9/10. Rather than settling down to new and different equilibrium price and markup vectors, this model displays sustained oscillations in prices and markups.

Figure 66 Marginal Instability and Sustained Cycles in Kaleckian System with Gross Profit Markup Adjustment Mechanism



These nonequilibrium behaviours occur because the system is marginally unstable: its Jacobian has one or more eigenvalues of one, regardless of the value chosen for a (a sample vector of eigenvalues for $a=.2$, is [0, 0, 0, -0.245+0.41i, -0.245-0.41i, 0.384, 0.222, -0.171+0.025i, -0.171-0.025i, -0.089, -0.085, 1, +4.8e^{-9}i, -4.8e^{-9}i, 6.544e^{-9}, -6.544e^{-9}, 1, 1]). If the system is disturbed from an equilibrium, then it will almost certainly not return to it, and for some parameter values, the non-chaotic cyclical behaviour shown above may result.

The general conclusions which can be derived from this model are that (a) a truly dynamic input-output analysis necessarily results in nonlinear models; (b) these models may have unstable equilibria; (c) this instability is not dependent upon the equilibrium vector being non-feasible; and (d) the conditions which apply in equilibrium can therefore be irrelevant to both the short and long term behaviour of the model. We will now consider a more conventional markup adjustment mechanism.

7.5 A more conventional mechanism

Traditionally, economists of all persuasions have presumed that competition and investment are driven by the search for the highest possible *rate* of profit. Capitalists are presumed to move capital into industries earning a higher than average rate of profit, and out of industries earning a lower than average rate. In this partial, circulating capital model, the rate of profit for each sector reduces to the markup for each sector. The average rate of profit, however, is a weighted sum of sectoral markups. Labelling sectoral costs in period t as c_t, and the average rate of profit as $\overline{\pi}_t$, we have

$$(112)\overline{\pi}_t = \frac{c_t \cdot m_t}{\sum c_t} = \frac{c_t \cdot m_t}{c_t \cdot [1]} = \frac{\left(u^T + p_{t-1}^T \cdot A\right) m_t}{\left(u^T + p_{t-1}^T \cdot A\right) \cdot \hat{m}_t^{-1} \cdot m_t}$$

(where [1] represents a n-vector of 1s).

In this model it is presumed that an individual sector will face increasing competition, and hence downward pressure on its markups, if its rate of return exceeds this average. From the perspective of a single sector j, the markup adjustment process will be of the general form

$$(113)\frac{m_{j_t} - m_{j_{t-1}}}{m_{j_{t-1}}} = \mu\left(\frac{\pi_{j_{t-n}}}{\overline{\pi}_{t-n}}\right)$$

where n represents a time lag, since this is a competitive process, and whereas we can presume markups are known to each industry as soon as they are set, profits of other sectors are known only with a delay. Presuming a single sector lag and a linear form for μ , we get

$$(114)\ \frac{m_{j_t}-m_{j_{t-1}}}{m_{j_{t-1}}} = a\cdot\left(1-\frac{\pi_{j_{t-2}}}{\overline{\pi}_{t-2}}\right) = a\cdot\left(1-\frac{m_{j_{t-2}}}{\overline{\pi}_{t-2}}\right)$$

The matrix form of this equation is

$$(115)\ \begin{aligned} m_t &= m_{t-1} + a\cdot\left(I-\frac{1}{\overline{\pi}_{t-2}}\cdot\hat{m}_{t-2}\right)\cdot m_{t-1} \\ &= m_{t-1} + a\cdot\left(I-\frac{\left(u^T+p_{t-3}^T\cdot A\right)\cdot(\hat{m}_{t-2})^{-1}\cdot m_{t-2}}{(u^T+p_{t-3}\cdot A)m_{t-2}}\cdot\hat{m}_{t-2}\right)m_{t-1}\end{aligned}$$

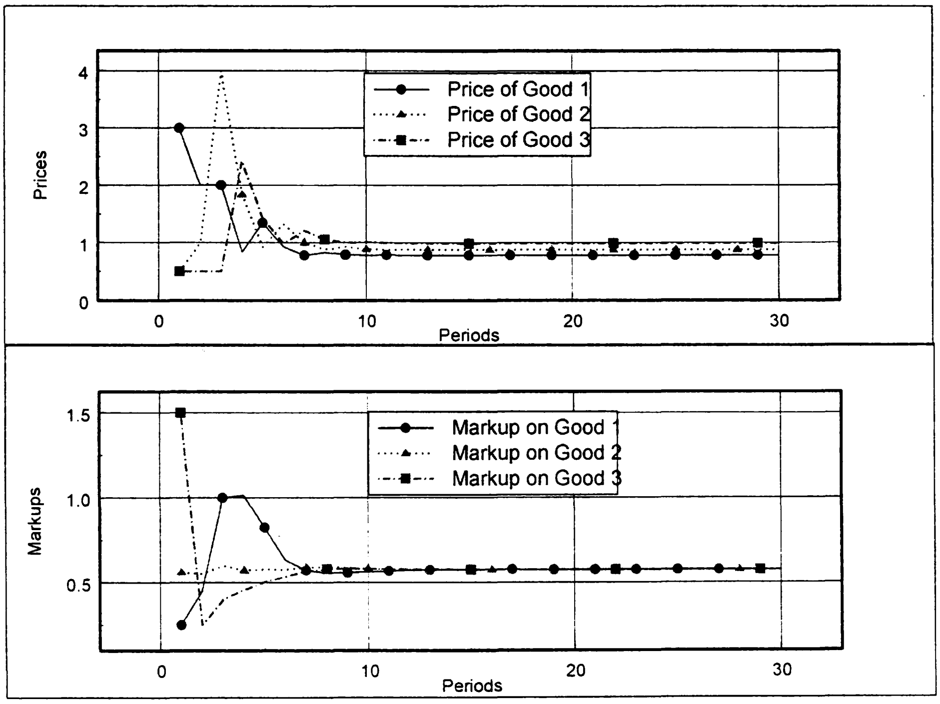
This is a third order equation, so that the resulting Kaleckian system is a pair of coupled third order difference equations:

$$(116)\ p_{t+3} = \left(u^T+p_{t+2}^T\cdot A\right)\cdot(I+\hat{m}_{t+3})$$

$$(117)\ m_{t+3} = \begin{aligned} & m_{t+2}+ \\ & a\cdot\left(I-\frac{\left(u^T+p_t^T\cdot A\right)\cdot(\hat{m}_{t+1})^{-1}\cdot m_{t+1}}{\left(u^T+p_t^T\cdot A\right)\cdot m_{t+1}}\cdot\hat{m}_{t+1}\right)\cdot m_{t+2}\end{aligned}$$

As would be expected, this model is not as unstable as the model of the previous section (markups in particular tend to converge to the average value). However, as with the preceding model, this system is replete with nonlinear terms of the form $\frac{p_t^T\cdot A\cdot(\hat{m}_{t+1})^{-1}\cdot m_{t+1}}{\left(u^T+p_t^T\cdot A\right)\cdot m_{t+1}}\cdot\hat{m}_{t+1}\cdot m_{t+2}$, and a numerical simulation shows that it is also marginally unstable. Symbolic analysis confirms that several of the eigenvalues of its Jacobian have a magnitude of *1* for some values of the markup adjustment parameter *a*.[54] The plots in Figure 67 were generated by a system in which *a* had a value of *.09*, while two of the markups were 2% and 4% respectively below the equilibrium value of *.5* for one of the three initial time periods. Under the influence of the competitive process, markups converged to a equilibrium value of 0.5756; however the price vector of (*0.7846, 0.8807, 0.9878*) clearly differs from the initial equilibrium of (*1,1,1*).

Figure 67 Convergence to Equilibrium and Marginal Instability in Kaleckian System with Linear Average Profit Markup Adjustment Mechanism



7.6 A nonlinear adjustment mechanism

It is a simplification to assume that the response of capitalists to differences in rates of return in different sectors will be a linear one. A linear reaction implies that a tiny difference in rates of return will engender a tiny response, while a thousand fold difference will engender precisely a thousand fold stronger reaction. Two observations can be made against this. Firstly, a tiny divergence from the average is unlikely to generate any response, since the costs of adjustment are likely to exceed any benefit; secondly, there would be a limit to the degree of movement of capital motivated by a large divergence from the average. Both these observations are, of course, a product of the fact that uncertainty is a fundamental aspect of capitalism.

The appropriate model is instead a function which has first derivatives of zero at both zero and maximal divergence from the average rate of profit. However, in order to simplify the presentation, I have chosen to address only the second property, by using a markup function whose first derivatives are zero at plus and minus infinity, the hyperbolic tan. From the point of view of an individual sector, the markup mechanism in this model is

$$(118)\frac{m_{j_t}-m_{j_{t-1}}}{m_{j_{t-1}}} = a \cdot \tanh\left(1 - \frac{m_{j_{t-2}}}{\overline{\pi}_{t-2}}\right)$$

where a is a constant, and the average rate of profit $\overline{\pi}_{t-2}$ is as defined above. This results in the third order system:

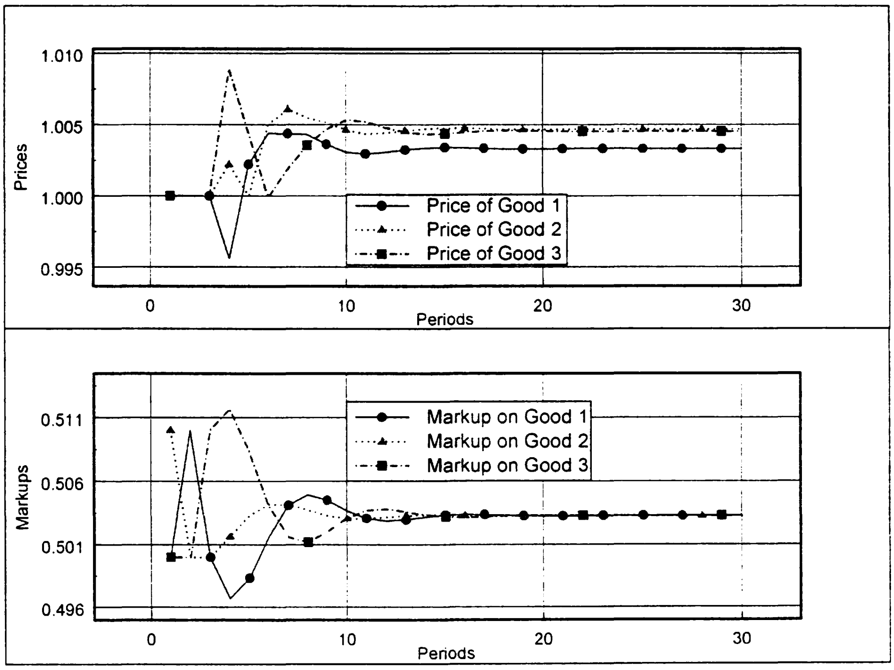
$$(119) p_{t+3} = \left(u^T + p_{t+2}^T \cdot A\right) \cdot (I + \hat{m}_{t+3})$$

$$(120) m_{t+3} = \left(m_{t+2}^T \cdot \left(I + a \cdot ITANH\left(\frac{\left(u^T+p_t^T \cdot A\right)\cdot(\hat{m}_{t-1})^{-1} \cdot m_{t+1}}{\left(u^T+p_t^T \cdot A\right)\cdot m_{t+1}} \cdot \hat{m}_{t+1}\right)\right)\right)^T$$

where *ITANH(x)* takes a vector argument and returns a diagonalised matrix whose *i*th diagonal entry is $\tanh(1 - x_i)$.

This system has the same equilibrium as the preceding models, and if this is the initial state of the system, then it is maintained for all time. However, if the initial conditions diverge from equilibrium, the system cycles away from it towards the average markup, where this is a function of initial conditions and the value of a. For a small divergence and a low value of a (corresponding to an economy with low competitive pressures), the system converges rapidly to an average markup (and therefore to prices of production) which is not too different to the initial equilibrium. Figure 68 shows the time paths of prices and markups with a value of *.5* for *a*, and initial markups differing from the equilibrium by 2% for one sector only in each of the three initial time periods:

Figure 68 Convergence to Equilibrium and Marginal Instability in Kaleckian System with Nonlinear Average Profit Markup Adjustment Mechanism and Low Adjustment Parameter Values



While the system shown in Figure 68 appears similar to the linear system shown in Figure 67, it has fundamental differences which become apparent with larger initial divergences, and larger values for *a*. The larger the initial divergence, the more the final average differs (linearly) from the ex-ante equilibrium, while as *a* approaches *1* (corresponding to an economy with high competitive pressures), cycles maintain longer and the final average markup approaches zero. Figures 69 and 70 respectively show the time paths for prices and markups resulting from a value of *.98* for *a*, and a 20% initial divergence for one sector in each of the initial time periods.

Figure 69 Extensive Price Cycles Prior to Convergence to Equilibrium and Marginal Instability in Kaleckian System with Nonlinear Average Profit Markup Adjustment Mechanism and High Adjustment Parameter Values

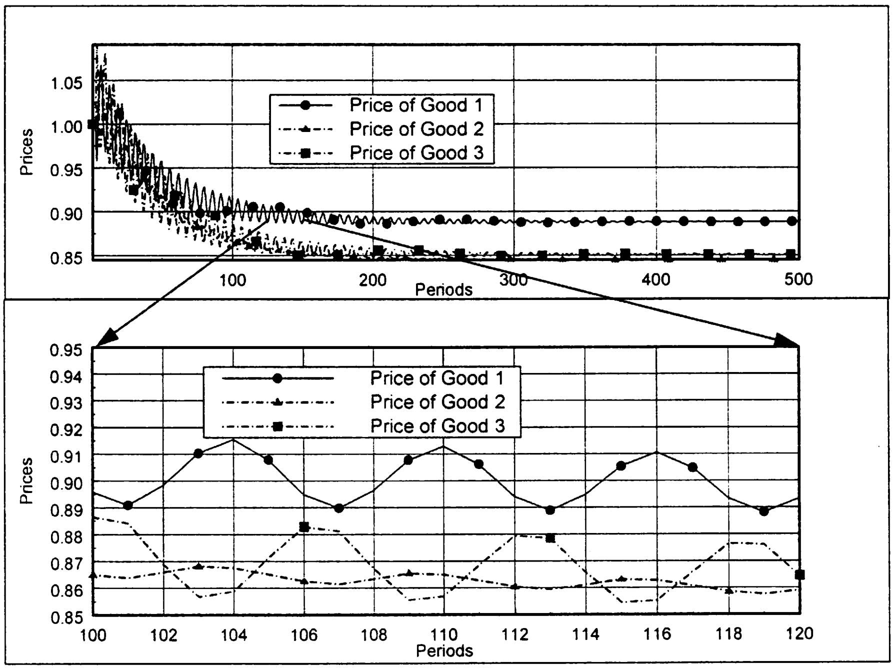
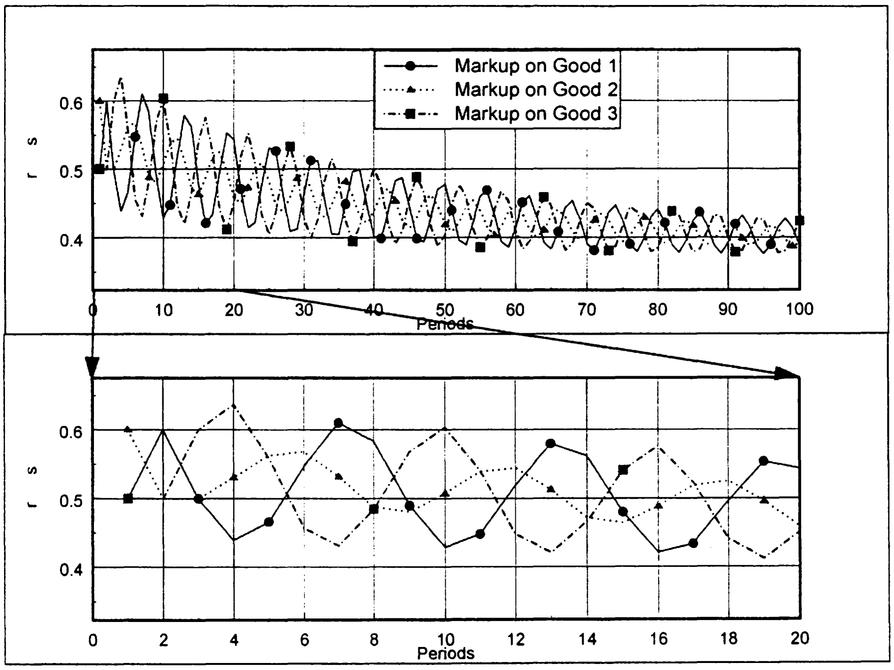


Figure 70 plots the behaviour of markups for the first 100 time periods only; the uniform markup converged to *.3292* after 600 periods.

Figure 70 Extensive Markup Cycles Prior to Convergence to Equilibrium and Marginal Instability in Kaleckian System with Nonlinear Average Profit Markup Adjustment Mechanism and High Adjustment Parameter Values



7.7 Conclusion: general dynamics, nonlinearity and price setting

The models considered above are clearly only partial ones, with quantity dynamics ignored, the wage taken as exogenous, and so on. Yet even when the parameters of this partial system are assumed to be linear, a nonlinear system with a marginally unstable equilibrium results. When nonlinear forms are introduced to increase the realism of the model, the product is a completely unstable equilibrium, though the system does converge to prices of production—at an average rate of profit whose value is dependent upon parameter values and initial conditions.

A general dynamical model, with quantity, wage, finance and technological dynamics added to the price and markup dynamics above would be considerably more nonlinear, firstly because of escalation in the number of product terms and secondly, through the introduction of many more nonlinear relationships (such as a "Phillips curve" for the relationship between wages and employment, as in Goodwin 1967, and nonlinearities from finance as in Keen 1995 and Chapter Five of this thesis). These changes are almost certain to move the model beyond the marginal and equilibrium-average divergence instabilities demonstrated in this paper into full local instability. The equilibria of the system may not be attractors, as in the third model above, and they will almost certainly not be stable, with the result that the conditions which apply at equilibrium will be irrelevant to the system in both the short and long term.

This result justifies Sawyer's distinction, in his reply to Steedman, between a theory of pricing, and a theory of prices (Sawyer 1992: 158). The Kaleckian theory of markup pricing, with markups set according to the degree of competition within each industry, is only irrelevant if the long run outcome of the model corresponds to the static equilibrium position. Since this is not the case, a theory of price-setting must be part of a theory of capitalism. Such a theory must be based upon the setting of prices by disaggregated decision units acting with limited information in an environment of uncertainty. Kaleckians do have such a theory—mark-up pricing, with mark-ups set according to the degree of competition within each industry. Steedman's critique that this does not give rise to a consistent theory of prices can in fact be turned round, by noting that while Steedman proffers a theory of prices, this is not based on any behavioural theory of disaggregated price-setting. Steedman is correct to argue that this theory must be modified to incorporate the impact of input-output effects. However, just as Kaleckians should "learn from Sraffa", Sraffians should "learn from Kalecki", since the results of dynamic input-output analysis are radically different to those of statics.

In the next chapter, an initial attempt is made to achieve such a Kalecki-Sraffa synthesis, as a prelude to developing a multi-sectoral rendition of Minsky's hypothesis. Chapter 8 develops a model of a simplified economy is developed in which prices are set by a competitively determined markup on costs in a multisectoral circulating capital model, and in which finance and debt play an essential role.

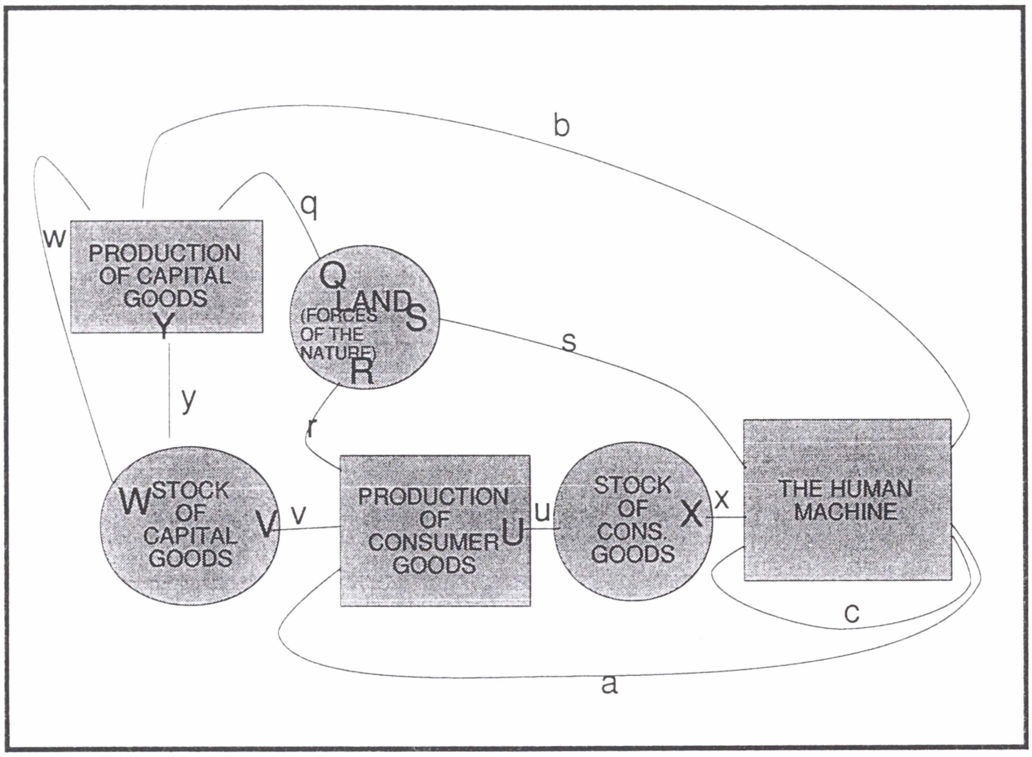
8 A Preliminary Multi-sectoral Model with Finance

8.1 Introduction

This thesis has intertwined a critique of linear economic theory and an exposition of nonlinear analysis with the attempt to model Minsky's Financial Instability Hypothesis, because as Minsky himself came to appreciate (Minsky 1996a), fundamental nonlinearities in a capitalist economy are essential to his analysis. Chapter Three outlined one curious aspect of the economics profession's adoption of linear analysis, and this Chapter begins with another. It is well known that Frisch's "Propagation problems and impulse problems in dynamic economics" (Frisch 1933) played a leading role in establishing the paradigm that the trade cycle was generated by random exogenous shocks disturbing the underlying stable linear propagation system. What is perhaps not so well appreciated is that Frisch expressed a greater ambition in that paper which, if fulfilled, would instead have inaugurated the paradigm of nonlinearity.

Frisch began his paper by expressing the ambition to develop a complete, deterministic dynamic macro-model of the economy, a "Tableau Economique" (see Figure 71) from whose interrelations it would be possible "to explain the movements, cyclical or otherwise, of the system" (Frisch, 1933: 174):

Figure 71 **Frisch's 1933 "Tableau Economique"**



Frisch shrank from this ambition as too difficult, and instead confined himself "to systems that are still more simplified" (Frisch, 1933: 174) than his Tableau, where disaggregation was eschewed, thus condensing "all kinds of production into one variable, all consumption into another" (Frisch, 1933: 173). This was unfortunate, since had Frisch undertaken the full task he set out for himself he would have treated production as a disaggregated process. This would have inaugurated nonlinear analysis in economics, because such a model is necessarily nonlinear, as the previous chapter indicates.

Frisch can be forgiven his decision not to attempt his greater ambition. It is indeed a difficult task, as he surmised, but it is one which has to be undertaken if the full dynamics of Minsky's model of a capitalist economy with finance are to be explored. The approach I have taken is to construct a series of multi-sectoral models, where each new model adds an additional level of realism. This thesis concludes with the first of these models, which has not yet been simulated, and which is still too simple to allow the modelling of Minsky's Hypothesis.

8.2 A preliminary model of a multisectoral economy with finance

This initial model considers an economy in which there is no population growth, no technical change, no fixed capital or stocks, where all commodities last only one period, are produced in one period and either consumed or wasted in the next, and in which there are banks, workers, capitalists, and input-output relations of production. All economic activity is undertaken through the bank accounts of capitalists. The sequence of economic activity is: (a) Interest is paid on outstanding debts/deposits. (b) Production is planned, based on previous periods' output and effective demand. (c) The intermediate inputs needed to produce planned output are purchased from the output of the previous period, at prices set during the previous period. (d) The wage is decided, based on the wage and employment in previous periods. (e) The workers needed to produce planned output are hired. (f) Workers spend all their income on consumption. (g) Bankers consume part of their income, and add the remainder to their "hoards". (h) Capitalists consume part of their income. (i) This period's output is produced.

Conceptually, this can be likened to an agricultural economy with peasants, landlords and moneylenders, where the commodities being produced require each other and labor as inputs, and where the products are perishable.

As simple a society as this is conceptually, it requires a mathematical model of daunting complexity.

The first step in this model is the calculation of the current level of debt, which depends on the previous level of debt and the rate of interest:

$$(121) d_t = d_{t-1} + r(d_{t-1})$$

where d is a vector (all vectors in this model are column vectors) of bank balances (one per sector), and $r(x)$ charges a higher level of interest for a negative x_i (a debt) than for a positive one (deposit).

The second step is the planning of this year's output. Given the condition that all commodities last only one period, planned output cannot involve using all of the previous period's output as input. In general terms:

$$(122) q_t = min0(\theta_{t-n}, q_{max})$$

where $min0(x,y)$ returns a vector where each entry is the minimum of the matching non-zero entries in x and y, θ is a function of previous period output and effective demand (this will be detailed later), and q_{max} is the maximum that could be produced if all of last period's available output (minus a reserve held for workers) were devoted to intermediate consumption (where $i_t = A \times q_t$). Allocating all of last period's output to intermediate demand corresponds to planting all the output from the previous period, thus leaving nothing for workers (or anybody else) to consume. If this were done in this hypothetical economy, then workers would have no incentive to work, since they would be unable to turn their wages into consumption. Hence q_{max} is the amount that could produced from using the entire surplus of last period's output (over the amount consumed by workers)[55] as intermediate input in that period:

$$(123) q_{max} = A^{-1} \times (q_{t-1} - c_{w_{t-1}})$$

Workers are assumed to spend all their wages, employment depends on this period's planned output, they purchase goods produced at the end of the last period at prices set in the last period, and a neoclassical log-linear consumption function is used for demand for all classes. Physical consumption by workers in time t is then:

$$(124) c_{w_t} = w_t \times \sum e_t \times \hat{p}_{t-1}^{-1} \times q_w$$

where p^{-1} is a vector of inverse prices (the ith entry is $\frac{1}{p_i}$) and q_w is the workers consumption vector (whose entries sum to one). Thus q_{max} is:

$$(125) q_{max} = A^{-1} \times \left(q_{t-1} - w_{t-1} \times \sum e_{t-1} \times \hat{p}_{t-2}^{-1} \times q_w \right)$$

Once output is planned, intermediate demand is known:

$$(126) i_t = A \times q_t$$

Expenditure on intermediate inputs is both an income to capitalists, and an expenditure by them, with a net sum of zero. The overall operation results in a further adjustment to bank balances:

$$(127) d_t \rightarrow d_t + \hat{p}_{t-1} \times A \times q_t - \hat{q}_t \times A^T \times p_{t-1}$$

Employment is determined by the (constant) employment coefficient vector E and planned output:

$$(128) e_t = \hat{E} \times q_t$$

where E is the vector of labor input coefficients. Workers are then hired, and the wage at which they are hired is set on the basis of the previous period's employment with a nonlinear wage demand function, Thus the wage in time t is:

$$(129) w_t = w_{t-1} \times \left(1 + W\left(\frac{\sum e_{t-1}}{P}\right)\right)$$

where $W()$ is a nonlinear function, e_t is employment in period t, and P is the fixed population. Employment and the subsequent consumption by workers result in transfers from (to hire the required workforce at the going wage) and to capitalist bank accounts (as consumption by workers):

$$(130)\ d_t \rightarrow d_t - w_t \times (e_t - \Sigma\, e_t \times q_w)$$

Bankers' consumption occurs next, and is the first action in this system which may be quantity constrained: the amount bankers wish to consume may exceed the stocks available after intermediate and worker demand:[56]

$$(131)\ q_{b_t} = min0(mpc_b \times \Sigma{-r}(d_{t-1}) \times \hat{p}_{t-1}^{-1} \times q_b, \\ q_{t-1} - (i_t + q_{w_t}))$$

where mpc_b is the banker's marginal propensity to consume (there is presumed to be one banker only, and $r()$ is negated because it represents interest income to capitalists). This results in a credit adjustment to capitalist bank balances:

$$(132)\ d_t \rightarrow d_t + min0(mpc_b \times \Sigma{-r}(d_{t-1}) \times q_b, \\ (\hat{p}_{t-1} \times (q_{t-1} - A \times q_t) - w_t \times \Sigma\, e_t \times q_w))$$

The remainder of banker's income (if it is positive) is used to augment his/her hoard:

$$(133)\ h_t = h_{t-1} + \Sigma \begin{matrix} -r(d_{t-1}) \\ -min0(mpc_b \times \Sigma{-r}(d_{t-1}) \times q_b, \\ (\hat{p}_{t-1} \times (q_{t-1} - A \times q_t) - w_t \times \Sigma\, e_t \times q_w)) \end{matrix}$$

where h is the banker's hoard. This equation provides the first strong insight from this model: since it is conceivable that banker's income is negative (all the elements of d_{t-1} could be positive), it is feasible that h could be negative. Yet this has no impact on the role of the banker, since hoards are not a determinant part of the system—no other component of the system depends upon h. The banker is a banker merely because he/she is presumed to have a hoard; whether the hoard in fact exists is irrelevant, since alterations in the banker's account entries are then accepted as means of payment for goods, and records of the accumulation of money capital (or of debt). The amount of money in the economy (the sum of all bank accounts, and the sum of transactions on accounts) is thus independent of what is effectively the "monetary base" of the economy. This is a "thought experiment" confirmation of the Minsky/Moore proposition that the money supply is endogenous.

Capitalists are the last to consume in this model (again for reasons of the sustainability of the hypothetical economy). There are four complications for capitalist consumption: firstly, there is one group of capitalists per sector, rather than a class (workers) or a single entity (the banker). Secondly, the consumption of each (other) capitalist sector becomes income for each sector. Thirdly, capitalist consumption is based on income from two sources, profit and interest (both of which can be negative). Finally, as for bankers' capitalist consumption can be quantity constrained; the rationing mechanism used assigns the remaining stocks in proportion to capitalist demand. Capitalist revenue from capitalist consumption is:

$$(134)\hat{p}_{t-1} \times q_{\pi_t} = min0(Q_\pi \times mpc_\pi(\Pi_{t-1}), \hat{p}_{t-1} \times (q_{t-1} - (i_t + c_{w_t} + c_{b_t})))$$

where Q_π is the capitalist consumption matrix, where the *ij*th entry represents the weighting of the *j*th commodity in the consumption vector of the capitalists of sector *i*, $mpc_\pi()$ is the marginal propensity to consume function for capitalists, and Π is total capitalist income.

Capitalist expenditure on consumption is:

$$(135)\quad \widehat{mpc}_\pi(\Pi_{t-1}) \times Q_\pi^T \times dmmin1(\hat{p}_{t-1} \times (\hat{q}_{t-1} - (\hat{i}_t + \hat{c}_{w_t} + \hat{c}_{b_t}) \times \widehat{(Q_\pi \times mpc_\pi(\Pi_{t-1}))}^{-1}\ )$$

where *min1()* returns the minimum of 1 or the diagonals of a matrix. The overall adjustment to bank balances is:

$$(136)\quad d_t \rightarrow d_t + \begin{array}{c} min0\begin{pmatrix} Q_\pi \times mpc_\pi(\Pi_{t-1}), \\ \hat{p}_{t-1} \times (q_{t-1} - (i_t + c_{w_t} + c_{b_t})) \end{pmatrix} \\ -\widehat{mpc}_\pi(\Pi_{t-1}) \times Q_\pi^T \times min1(\hat{p}_{t-1} \times (\hat{q}_{t-1} - \\ (\hat{i}_t + \hat{c}_{w_t} + \hat{c}_{b_t}) \times \widehat{(Q_\pi \times mpc_\pi(\Pi_{t-1}))}^{-1}\ ) \end{array}$$

At this point it is now possible to calculate profits, which in this time-based system are the difference between revenue earned in period t and costs incurred in period t-1:

$$(137)\quad \Pi_t = R_t - C_{t-1}$$

Revenue in period t includes: (a) intermediate demand revenue, where the corresponding expenditure vector is a cost for the previous period; (b) expenditure by workers, where the corresponding costs entry is the wage bill for workers in the previous period; (c) expenditure by bankers; (d) the income aspect of capitalist consumption by all sectors. The expenditure side of capitalist consumption does not appear at all, since that represents "discretionary" disposal of surplus. Thus

$$(138)\quad R_t = \begin{array}{c} \hat{p}_{t-1} \times A \times q_t + w_t \times \Sigma\, e_t \times q_w + \\ min0(mpc_b \times \Sigma\, r(d_{t-1}) \times q_b, \\ (\hat{p}_{t-1} \times (q_{t-1} - A \times q_t) - w_t \times \Sigma\, e_t \times q_w)) + \\ min0(Q_\pi \times mpc_\pi(\Pi_{t-1}), \hat{p}_{t-1} \times (q_{t-1} - (i_t + c_{w_t} + c_{b_t}))) \end{array}$$

Costs are the sum of intermediate inputs and wages:

$$(139)\quad C_{t-1} = \hat{q}_{t-1} \times A^T \times p_{t-2} + w_{t-1} \times e_{t-1}$$

Thus profits are:

$$(140)\quad \left(\begin{array}{c} \hat{p}_{t-1} \times A \times q_t + w_t \times \Sigma\, e_t \times q_w + \\ min0(mpc_b \times \Sigma\, r(d_{t-1}) \times q_b, \\ (\hat{p}_{t-1} \times (q_{t-1} - A \times q_t) - w_t \times \Sigma\, e_t \times q_w)) + \\ min0(Q_\pi \times mpc_\pi(\Pi_{t-1}), \hat{p}_{t-1} \times (q_{t-1} - (i_t + c_{w_t} + c_{b_t}))) \end{array}\right) - \begin{array}{c} \hat{q}_{t-1} \times A^T \times p_{t-2} \\ + w_{t-1} \times e_{t-1} \end{array}$$

Total capitalist income is the sum of profits and interest:

$$(141)\quad \Pi_t = \pi_t + r(d_{t-1})$$

The final components needed to complete the system are the markups applied to the output from this period, the prices for this period's output (which will be sold in the next period), and the quantity to be produced this period. Prices are easily defined:

$$(142)\quad p_t = \left(\left(w_t \times E^T + p_{t-1}^T \times A\right) \times (I + \hat{m}_t)\right)^T$$

The markup reflects the degree of competition in each sector, with a higher rate of profit than average attracting more firms and thus lowering the markup. However in this model it is assumed that there is some minimum level below which markups will not go (reflecting the possibility of earning income from a positive bank balance). The form I have employed here is a simple linear function with a floor:

$$(143)\quad m_t = maxs\left(\hat{m}_{t-1} \times \left(min1(I) - \mu \times \left((\hat{p}_{t-1} \times \hat{q}_{t-1})^{-1} \times meandev(\pi_{t-1})\right)\right), m_{min}\right)$$

where *maxs(x,y)* returns a vector where each entry is the maximum of the entry in vector x and scalar y, *meandev(x)* returns the deviation of each element of x from the average for x, and m_{min} is the minimum markup.

Now that effective demand is known, the quantity equation can be completed. In this simple initial system, the form used presumes that firms attempt to match output to effective demand, using a linear adjustment mechanism:

$$(144)\theta_{t-n} = q_{t-1} - a \times (q_{t-1} - (i_{t-1} + c_{w_{t-1}} + c_{b_{t-1}} + c_{\pi_{t-1}})), 0 < a < 1$$

8.3 Conclusion

This completes the system, yielding a system with five vector and one scalar difference equation, where the number of dimensions is a function of the size of the input-output matrix A. This highly complex and nonlinear model is conceptually far simpler than the one contemplated by Frisch, and it is little wonder that he shrank from constructing his "Tableau Economique", given the absence of computational tools in his day.

It is still a difficult task, even with today's technology, and I have not yet been able to simulate this system. However, its construction has enabled a useful economic insight on the endogeneity of money, and the model gives an indication of the complexity of a more realistic system. The importance of nonlinearity is evident, even though only one explicitly nonlinear function (for wage determination) is employed, since the system is replete with product terms between variables. The degree of nonlinearity would doubtless rise as the model is made more realistic, with the introduction of stocks, fixed capital, population growth and technical change.

As noted in Chapter One, the models in this thesis cannot be solved analytically. This chapter indicates that they can also be difficult to simulate numerically, and yet the simple economy outlined above is many steps removed from a model which would do justice to Minsky's Financial Instability Hypothesis. This indicates that it may be necessary to move towards computer simulation if progress is to be made in modelling Minsky.

9 Conclusion: Balancing Our Way to Instability

Minsky's ambition in constructing the Financial Instability Hypothesis was to build a theory which "makes great depressions one of the possible states in which our type of capitalist economy can find itself" (Minsky, 1982: xi). His purpose was to find "an apt economic theory for our economy" (Minsky 1977, 1982: 68), since it was a manifest fact that capitalist economies periodically find themselves in such a state, and yet neoclassical economics argues that such a state is an aberration.

Minsky's success in developing such an apt theory was mixed. As Chapter One indicates, his verbal model of the Financial Instability Hypothesis successfully blended the insights of Kalecki, Fisher and Keynes into a coherent theory of the financial dynamics of a developed capitalist economy. However, he was unable to derive a meaningful mathematical model of the FIH. This failure was attributable in part to his reliance on the Hansen-Samuelson linear multiplier-accelerator model, which Chapter Two established was economically unsound. The Financial Instability Hypothesis is also fundamentally nonlinear, and could not thus be successfully constructed upon these linear foundations.

In this sense, while Minsky's theory was economically timely—in that it provided a coherent explanation for the Great Depression, and the avoidance of a similar calamity during the era of Big Government—it was ahead of its time mathematically. The appreciation of the importance of nonlinearity in economic analysis is something which was restricted to a few pioneers like

Kaldor, Goodwin and Blatt, until interest in economic dynamics was revived during the 1990s by the general scientific interest in the phenomenon of chaos and complexity theory.

The models of Chapters Four and Five establish that these very modern phenomena are the proper mathematical foundations for Minsky's Hypothesis. These Chapters also integrate the financial analysis of Minsky with the real analysis of Goodwin, producing a set of models which provide an integrated explanation for the 19th century trade cycle, Depressions, and the cyclical but stable economy of the post-War period. These models indicate that a debt deflation is inevitable in a pure capitalist economy, and that Big Government can contain the tendency to deflation.

However the models of Chapters Four and Five cannot capture the rich price dynamics of Minsky's Hypothesis. Chapter Six's defence of the Kaleckian markup pricing theory in the context of input-output dynamics indicates that the introduction of a price system may further enhance the far-from-equilibrium dynamics of the single commodity models. However, given the failure to date to simulate these models, this argument must remain an implication only.

This deficiency aside, I am confident that this thesis has provided a cogent mathematical rendition of the essence of Minsky's Financial Instability Hypothesis. This thesis thus provides an additional reason why the policy implications of Minsky's Hypothesis should be taken seriously.

9.1 Policy Implications of the Financial Instability Hypothesis

There was room for complacency among economic theorists towards Minsky's theory during the long period of economic stability when he first developed the hypothesis, and the system's apparent ability to survive a major downturn in 1973 may have amplified this disposition. But the 1987 Stock Market Crash and the 1987-89 property market boom and bust which followed it gave Australia, Europe, Japan and the United States a taste of debt-deflation, and for a time in the early 1990s all these economies themselves in situations with some of the hallmarks of a depression: a debt-induced deflation in the aftermath to a speculative boom, low or negative rates of money growth, low inflation, a collapse of investment and slow, fragile recoveries in economic activity. Though the crisis itself has now passed in all these economies save Japan, the current speculative bubble on Wall Street—which may at last shortly burst—shows that the behaviour that gave rise to the last crisis is still alive and well, and it is only a matter of time before the next crisis occurs. The possibility remains that the trigger for the next one may yet emanate from the last, if the legacy of debt from Japan's Bubble Economy proves insurmountable.

The policy implications of Minsky's theory should therefore be heeded, but they give little grounds for optimism. Because the hypothesis views capitalism as fundamentally unstable, it does not regard escaping from a collapse as an easy matter. The essential lesson of the Financial Instability

Hypothesis is that we should avoid debt deflations in the first place, by developing and maintaining institutions and policies which enforce "a 'good financial society' in which the tendency by businesses and bankers to engage in speculative finance is constrained" (Minsky 1977, 1982: 69). These institutional arrangements include close and discretionary supervision of financial institutions and financial arrangements, non-discretionary countercyclical fiscal arrangements, and a bias towards income equity rather than inequality.

If a debt deflation does occur, Minsky recommends government "pump priming", but for different reasons than those normally proposed by mainstream Keynesian economists. Large deficits by governments during times of economic crisis generate cash flows which enable businesses to repay the debts incurred during the preceding boom (Minsky 1978, 1982: 112-114). These deficits need to be sustained for long enough to enable businessmen to refinance themselves out of the debt trap induced by the collapse of asset values.

The price system must also be manipulated during a deflation to engender commodity price inflation. This is necessary, not, as Keynes argued, to reduce the real wage (Keynes 1936: 280-291),[57] but so that the real burden of accumulated debt can be rapidly reduced, and the cash flows of corporations made artificially capable of sustaining that debt. Similarly, the rate of interest must be driven to very low levels, not because this will increase the rate of investment, as in IS-LM analysis, but because this low rate of interest will reduce the rate of accumulation of debt. It could be argued that this insight lies behind the current Japanese discount rate of half of one per cent.

However modern-day fiscal policy in America, Europe and Australia is dominated, not by a Minskian vision of capitalism, but by a fetish for zero fiscal deficits which is inspired in equal measure by neoclassical economics and the finance markets. From a Minskian point of view, this fetish is likely to result in a mixed economy which is less able to restrain the tendency towards debt deflation, especially in the current milieu of low commodity price inflation.

9.2 The Zero Budget Deficit Fetish

As of the 24th of January 1997, there were 12 bills before the USA House of Representatives whose objective is to eliminate budget deficits, either via an Act or by amendment to the Constitution.[58] None of these legislative proposals evince any appreciation of the stabilising impact of government deficits during a downturn.[59] One even has a provision that if a deficit does occur one year, it should be repaid entirely in the next, which would therefore ensure a fiscal surplus in the second year of any recession (H.J.RES 11, Section 2; see Appendix C).

The Maastricht Treaty provisions on the reductions of government deficits are less restrictive on the ability of government deficits to ameliorate a downturn only by the fact that a deficit of up to 3% of GDP is allowed, as compared to the US provisions for a zero or positive budget. However, given the comparative mechanics of US and European legislatures, the Maastricht provisions are far more likely to be enacted than the US amendments—though political opposition to both moves should not be underestimated.

The Australian government is pursuing an objective of a balanced budget by 1999, from an initial position of a deficit equivalent to 2 per cent of GDP.[60] This target was set at a time when the unemployment rate was 8.5 per cent, and employment was forecast to grow at an average of less than 2 per cent for the next three years, which would make little inroad into the level of unemployment.[61] The analysis supporting the deficit reduction program argued that its impact on the rate of economic growth would be positive.[62]

Even if the Australian government achieves its objectives, and both Maastricht and the US Balanced Budget amendments become law, governments will still necessarily attenuate the initial downturn, because of the sheer weight of the collapse of government revenues from income tax during a crisis, and the obligatory government function of unemployment payments. But the enforcement of this unsound objective is likely to lead to government action which sustains the ensuing slump, by limiting the extent to which government deficits enable firms to repay their debt commitments. We are thus likely to see lengthier recessions and more anaemic recoveries in future, courtesy of the crusade for smaller government. We will balance our way towards greater economic and financial instability.

On the monetary policy front, the emphasis upon the maintenance of low inflation is likely to see interest rates being used excessively in attempts to restrain "inflationary expectations" during booms. These increased interest rates will instead accelerate the accumulation of debt, thus deepening the debt deflation which follows. While central banks are unlikely to ever reject the lender of last resort function during a crisis, their current behaviour is more likely to precipitate one.

Consequently, from a Minskian point of view, the prospects for capitalist economies as we enter the 21st century are gloomy indeed. Both governments and capitalists have failed to learn the lessons of the past—indeed, as Minsky observed, the system's ability to survive the 1987-89 crisis has built up confidence that such crises can always be survived. However, the initial conditions of the next crisis are likely to be such that the slump will be more severe than that of the 1980s. The low rate of commodity inflation which is the legacy of the 1980s crash means that the next bust is likely to result in actual price deflation, rather than the mere cessation of moderate inflation which occurred in the early '90s. If Fisher's Paradox does return, even Big Government may be unable to stop capitalism entering a genuine Depression—though not a Great one. If American and European governments continue to live up to their current "ideals" in the midst of such a collapse, then the fetish for Small Government may well keep the economy in a Depression for a considerable time.

Ultimately, only a completely different approach to economic theory and policy can save us from a future of financially-driven cycles and the threat of long term stagnation. As Deleplace and Nell emphasise, criticism of the neoclassical orthodoxy is not enough: an alternative edifice must also be offered (Deleplace and Nell 1996: 4). Minsky's hypothesis, and nonlinear models of capitalism akin to those developed in this thesis, must form part of that approach. But it is probable that, as with Keynes's message six decades ago, such theories will only be heeded after capitalism has entered a crisis which is in part the product of its own ideology.

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[1] http://www.nyse.com/public/data/stathtoc.html. The NYSA Index is a comprehensive index of all traded stocks, unlike the Dow Jones Index. For details consult http://www.nyse.com/public/data/histfile/nyadesc.html.

[2] The FIH can be traced further back, to Minsky 1957 (1982: 231-257). However while this paper did deal with the integration of finance with the real economy, many of the key concepts of the FIH—such as the possibility of a debt-deflation, endogenous money (apart from a discussion of an accelerator-multiplier process with an infinitely elastic money supply [1957; 1982: 242-244]), Ponzi financing, the financial interpretation of the impact of fiscal deficits, and the crucial role of the lender of last resort—are absent from this earlier paper, but present in 1963.

[3] These comments of course apply to the Neoclassical School of economics, rather than to its Physiocratic and Classical predecessors, whose analysis was a mixture of both statics and dynamics. As Blatt himself emphasises, Quesnay's "Tableau Economique" was the outstanding instance of dynamical analysis in the pre-Classical period (Blatt 1983: 315-334). To choose but one example from Smith, his analysis of the impact of the division of labour on the level of output, and the feedback impact of the growth of demand on the division of labour (Smith 1776: 748), is clearly a dynamic explanation of the process of growth. The key modern dynamical model used in this thesis is arguably derived from Marx's income distribution dynamical explanation of the trade cycle (Marx 1867: 580-581).

[4] See Miller (1990, 1991: 103), where he basically repeats the Friedman-Schwartz interpretation of the Great Depression.

[5] For simplicity I have used π rather than π_{m}, as in Kalecki's paper, where π_{m} represents the maximum rate of profit, based upon the choice of the most profitable technique of production (Kalecki 1937:)

[6] The derivatives of σ and ρ with respect to k are not in Kalecki, but are implicit in his argument.

[7] See Keen (1997) for a fuller discussion of Frisch's contribution.

[8] It also appears that Minsky was relying on a second-hand account of Hicks's interpretation of Keynes, since his only "Keynesian" reference is to Pigou's *Keynes's General Theory* (Minsky 1957b, 1982: 253, Note 9)

[9] The finance demand for money is absent in the *General Theory*, but becomes an important extension to Keynes's reasoning in the 1937 papers.

[10] Minsky footnotes Friedman M. and Savage L.J., 1948, "The utility analysis of choices involving risk", Journal of Political Economy, Vol. 56 at this point, while later stating that his own ideas about uncertainty "seem to be consistent with those of Keynes" (1969a, 1982: 191, footnote 6), citing Keynes 1937a.

[11] This article also marks the first use of the term "euphoric" (224) to describe capitalist expectations in the accelerating phase of a boom. However the paper also has many neoclassical remnants, including the use of utility and expected utility terminology in the discussion of risk aversion (227-228).

[12] Post-Keynesians in general believe that marginal cost is constant or falling for most firms across their viable range of output; this is of course incompatible with marginal cost pricing. See Reynolds, 1987: 53-62.

[13] This harmony does not imply equilibrium, however. The commodity price-asset price system would be locally unstable, but globally bounded.

[14] Though early in the cycle debt to equity and interest to profit ratios may in fact come in lower than anticipated, due to the multiplier effect of generally increased investment, and the consequently larger profits from each investment.

[15] Ponzi was a 1920s embezzler who peddled both financial and real estate schemes. His "bank" grew rapidly (before its spectacular demise) with its high interest rates financed out of new deposits. During the Florida land boom which preceded the 1929 Wall Street crash, he sold as many as 23 lots to the acre, advertised as being near a city that did not exist (Minsky, 1977, 1982: 70; 1978, 1982: 115; Galbraith 1988: 4-5.)

[16] Hicks's "innovation" vis-a-vis the original Samuelson 1939 paper was to base investment on the increment to output in the preceding two periods. Samuelson achieved the same result, by firstly lagging consumption and then basing investment on the increment to consumption over the preceding period. Hicks's 1949 and Samuelson's 1939 models are thus formally identical.

[17] Minsky's paper showed the results for 5 time periods only. The graph extends the results for 10 time periods to emphasise just how unrealistic are the predictions this model gives for the rate of growth.

[18] I am omitting discussion of Harrod's explanation of cycles since my main objective here is to criticise Hicks's interpretation of Harrod, which focused solely upon the unstable dynamic equilibrium and ignored the nonlinear factors which generated cycles as well as unstable growth.

[19] The term was first coined by Solow (1956: 65) (see Asimakopulos 1985: 631). Harrod protested that he never used the term "knife-edge", or a concept like it (Asimakopulos 1985: 631 n.19), and as Kregel 1980 points out, debate over the knife-edge has led to misinterpretation of Harrod's work, with his interpreters debating the uniqueness of warranted growth whereas Harrod was concerned with its stability (Kregel 1980: 119). However, as Knight observed, Harrod's recollection is not necessarily a good guide to his practice (Knight 1949: 324, footnote 15). Harrod summarised his theory by saying that "A unique warranted line of growth is determined... On either side of this line is a 'field' in which centrifugal forces operate... Departure from the warranted line sets up an inducement to depart further from it. The moving equilibrium of advance is thus a highly unstable one" (Harrod 1939: 23. See also Harrod 1951: 262). The phrase "knife-edge" is a valid shorthand for the concept of a line flanked by fields of centrifugal forces, and in this chapter I am concerned with the instability of dynamic equilibrium, rather than its uniqueness.

[20] It is true only with respect to unstable linear systems, and even in this case there is an exception to the rule, as this Chapter establishes.

[21] Hicks defines c as "the ratio of investment to the increment of output" (1949: 250), which is the definition of C_p, the actual ratio in Harrod's system, rather than C, the desired ratio. For the remainder of the chapter I will continue with Hicks's practice of using c rather than C.

[22] For the remainder of the chapter I will use c for the desired ICOR, and v for the actual accelerator.

[23] The values used were $Y_0=100$, $Y_1=98$, $Y_2=104$, $c=3.3$ and $v=3$.

[24] The derivation of this and all subsequent complex equations can be found in Appendices A and B.

[25] Growth still occurs when negative investment is allowed; non-negativity is imposed simply to add realism.

[26] Goodwin has never explicitly stated the precise section of Capital which inspired this model. This excerpt from Chapter 25 is the closest match to Goodwin's own model.

[27] "It has become abundantly clear that self-maintaining cycles occur because of essential nonlinearities in the structure of the system." Goodwin 1950: 318.

[28] In future work these parameter values could be substituted for those which could be derived by fitting this function to Phillips's original data.

[29] In deriving the "Phillips curve", Phillips reduced his numerous observations of the rate of unemployment and the rate of change in money wages between 1861 and 1913 into seven average pairs of values for the two variables. Though the original data was unavailable, these seven pairs can be roughly derived from his Figure 1. These approximate values were used to perform the generalised regression.These values were not employed in this thesis because Phillips's regression was between the rate of change of nominal wages and the rate of unemployment, whereas this model is in real terms. I therefore felt it preferable to work with the values used by Blatt, so that there would be some benchmark for the simulations in this thesis.

[30] Goodwin's paper omitted depreciation; this is introduced later in the thesis without significantly altering the model's characteristics.

[31] This is a simpler relation than that used by Desai (1973:536:539), since Desai's relation depended upon both the rate of employment and workers' share of output, whereas this depends only upon the rate of employment. The combination of Desai's insight with the models in this paper is a subject for future research.

[32] The data source for capacity utilisation was the Federal Reserve Statistical Release; the data file location was www.bog.frb.fed.us/releases/g17/iphist/utlhist.sa. The data source for the rate of employment was the Bureau of Labor Studies; the data file location was gopher://hopi2.bls.gov:70/00hopiftp.dev/special.requests/lf/unemplr.mon.

[33] As can be seen from Figure 15, the values derived from the regression predict zero capacity utilisation at an unemployment rate of 50 per cent. Logically, this is impossible, since for labor to produce anything, it must have capital to work with. The values obtained from the regression are therefore unduly affected by the (fortunately) relatively small range of unemployment and capacity utilisation data generated in the sample period.

[34] Economists who are unfamiliar with the practice of choosing specific functional forms for economic relations (outside of the traditional monotonic shapes for preference and production functions) should consult Kapur, Kumar & Kumar 1992 to see how prevalent this practice is in physical and scientific research—even when applied to strictly "economic" issues.

[35] This is the only occasion in this thesis in which the functional form for a nonlinear relationship has to be differentiated, so that the functional form itself has a direct bearing on the symbolic form of the system derived.

However the sigmoid relationship used is well supported both by the data, as the preceding regression analysis shows, and by logic.

[36] Unlike neoclassical models which generally employ a Cobb-Douglas production function, productivity in this model is labor productivity, rather than generalised factor productivity. Shai and Desai 1981 illustrate one method in which Goodwin's model can be extended to consider technical change which is directed at the capital to output ratio. A similar extension to this model is a topic for future research.

[37] This should be contrasted with Shah and Desai 1981, where a stable open cycle is generated by the introduction of a technical change function in which capitalists can choose either to increase labor productivity (*a* above) or the capital to output ratio (*v*, which in their model is a variable), with the choice depending upon the wage share of output. This generates a three dimensional model, and therefore has the potential for chaotic behaviour, but the dominant eigenvalues are less than one for economically meaningful parameter values. This approach to the choice of the form of technical change could be used to extend this model in future research.

[38] Any accelerating functional form—such as an exponential—could be used instead of the asymptotic function, since the investment function is not differentiated in the process of deriving the closed form of the model.

[39] One further extension—to include technical change to the *v* coefficient motivated by capitalist reactions to changes in the distribution of income, as in Shah and Desai—is a topic for future research.

[40] Blatt 1983: 215-216. The average rate of growth, however, remains constant at $(\alpha+\beta)$.

[41] Others (see for example Shah and Desai 1981) have introduced endogenous technological change, though with a different mechanism.

[42] Other continuous time extensions of Goodwin's model with unstable equilibria include Semmler (1986) and Sportelli (1995). The vast majority of continuous time extensions have not altered the conservative nature of the underlying model.

[43] The model abstracts from the fact that bankers make their profit on the spread between deposit and loan interest rates.

[44] A more complete model would have bank financial reserves being related to past and present capitalist profits, with a variable money multiplier expanding and contracting the finance these profits can generate. However, the model as specified allows us to focus on the basic antinomy between profits, investment, and long-term debt. Its openness only becomes an issue when the system approaches breakdown—at which time it indicates that capitalists can afford to finance exponentially increasing debt, when in fact they would go bankrupt.

[45] In a single commodity model. In the real world of multiple commodities, prices and inflation, it is quite possible for the ex-post real rate of interest to be negative, as has happened for periods of several months to oine and a half years on five occasions in the USA since 1949.

[46] The "real" rate of interest is highly volatile, due mainly to movements in the rate of inflation rather than to changes in the rate of interest, and given

this variability, an average is a rather meaningless figure. This caveat aside, using the rate paid on AAA corporate bonds as the interest rate, and the unadjusted CPI index for all goods as the rate of inflation, the average real rate of interest in the USA from 1982 till 1996 was 5.85 per cent. See www.bog.frb.fed.us/releases/H15/data/m/aaa.txt for the corporate bonds data, and the Bureau of Labor Statistics series CUUR0000AA0, which can be found at gopher://stats.bls.gov/Time Series.

[47] The derivation of the equivalent relation for the general model is shown in Appendix A.

[48] The percentage rate of change of the debt to output ratio can conceivably be negative, thus reducing the debt to output ratio. This in fact occurs in the final stages of simulations with low rates of interest and initial conditions in which the rate of employment differs substantially from the equilibrium level of 96.6%.

[49] In practice, this near symmetry is disturbed by the discretionary behaviour of governments, such as America's expenditure upon the Vietnam War during the 60s and early 70s.

[50] This aberrant aspect of the model could also be removed by introducing a "social wage" function of government, so that the net gap between taxation revenue and subsidies became a payment to workers. This approach will be explored in future research.

[51] The next pattern involves a slightly different set of actual values, since chaotic processes are aperiodic.

[52] In the vernacular of econometrics, the system has a "unit root".

[53] This should be a vector, indicating that the "barriers to entry" differ between industries, but for the sake of this illustration I will work with a scalar.

[54] The eigenvalues when the parameter *a* equals .09 are (0, 0, 0, 0, 0, 0, 0, 0, 0, -0.227+0.393i, -0.227-0.393i, 0.454, 0.1, 0.9, 0, 0.1, 0.9, 1)

[55] The amount which workers will consume this period depends on the sum of total employment, and since this is a model of capitalism, the capitalists in each sector cannot be presumed to know the sum of employment for the whole economy (since this would presume knowledge of the output plans of capitalists in other sectors). Hence the amount reserved is based on last period's known demand from workers for output from each sector.

[56] It would have been possible to assume a price clearing mechanism at this stage (and also for capitalist consumption) rather than a quantity-constrained process. The arguments against this are: everything else happens once only per time period in this model; if this is done, what is taken to be price for this time period in the future—the initial price, this final one, or some average? This simulation corresponds to a "repeated game", where there is every possibility that the capitalist who wins out of a price adjustment in this period will lose in the next, so a convention from which no-one ever wins but no-one loses is quite feasible.

[57] The simulations of Chapter Four would indicate that reductions in the real wage in fact contribute to the instability which leads to a crisis in the first place, since the increase in income shares going to capitalists results in a greater level of speculative investment, and thus a faster rate of growth of debt.

[58] These are: [H.R.113] Balanced Budget Requirement Act of 1996 (Introduced in the House); [H.R.397]To require that the President transmit to Congress, that the congressional Budget Committees report, and that the Congress consider a balanced budget for each fiscal year. (Introduced in the House); [H.J.RES.11] Proposing an amendment to the Constitution to provide for a balanced budget for the United States Government and for greater accountability in the enactment of tax legislation. (Introduced in the House); [H.J.RES.24] Proposing a balanced budget amendment to the Constitution of the United States. (Introduced in the House); [S.J.RES.7] Proposing an amendment to the Constitution of the United States to require a balanced budget. (Introduced in the Senate); [H.J.RES.7] Proposing a balanced budget amendment to the Constitution of the United States. (Introduced in the House); [H.R.126] Deficit Reduction Lock-box Act of 1997 (Introduced in the House); [S.J.RES.1] Proposing an amendment to the Constitution of the United States to require a balanced budget. (Introduced in the Senate); [H.R.4] Truth in Budgeting Act (Introduced in the House); [H.R.435] Federal Financial Management Improvement Act of 1997 (Introduced in the House); [H.J.RES.1] Proposing an amendment to the Constitution to provide for a balanced budget for the United States Government and for greater accountability in the enactment of tax legislation. (Introduced in the House); [H.J.RES.35] Proposing an amendment to the Constitution to require that congressional resolutions setting forth levels of total budget outlays and Federal revenues must be agreed to by two-thirds... (Introduced in the House). See Appendix C for details of two of these Bills.

[59] The texts of one proposed law and one amendment to the Constitution can be found in Appendix C.

[60] See the 1996/97 Budget Speech, in Appendix C.

[61] The unemployment rate has in fact risen since the budget.

[62] There are aspects of the argument which have some limited merit: specifically, the possibility that the reduction may boost investor confidence, and the possible impact of the deficit reduction on interest rates and, as a flow-on from that, on the exchange rate. However I cannot share the Treasury's confidence that these effects will outweigh "the adverse impact on aggregate demand associated with lower public demand" (Budget Statement One: 18).

Appendix A

This Appendix contains the derivations of the models in this thesis, where they are not trivial and have not already appeared in the text. All derivations have been done by hand, but given the amount of mechanical working involved in some of these (symbolic inversion ofa 3x3 matrix, for example) the result shown is on some occasions output from a symbolic processor (Mathematica or the Maple subset within MathCad).

Chapter Three

This chapter develops two versions of a linear model of cyclical growth. The derivation of the difference equation appears in the body of the chapter; this appendix derives the polynomial form of the model, using the second version, which introduces Samuelson's argument that investment is a function of lagged changes in consumption ($I_t = c \cdot (1 - s) \cdot (Y_{t-1} - Y_{t-2}$). This results in the simple modification of the first equation to $Y_{t+3} = Y_{t+2} + \frac{c \cdot (1-s)}{v} \cdot (Y_{t+1} - Y_t)$. Since the two equations are identical except for the inclusion of a savings function, only the second will be worked with here.

The first step in the analysis of this model is its conversion into a system of first order difference equations. We define

$$
(1)\quad
\begin{aligned}
x_1(t) &= Y_t \\
x_1(t+1) &= Y_{t+1} = x_2(t) \\
x_2(t) &= Y_{t+1} \\
x_2(t+1) &= Y_{t+2} = x_3(t) \\
x_3(t) &= Y_{t+2} \\
x_3(t+1) &= Y_{t+3} = Y_{t+2} + \frac{c \cdot (1-s)}{v} \cdot (Y_{t+1} - Y_t) \\
x_3(t+1) &= x_3(t) + \frac{c \cdot (1-s)}{v} \cdot x_2(t) - \frac{c \cdot (1-s)}{v} \cdot x_1(t)
\end{aligned}
$$

This converts into the following matrix equation:

$$
(2)\quad
\begin{pmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \end{pmatrix}
=
\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{c\cdot(1-s)}{v} & \frac{c\cdot(1-s)}{v} & 1 \end{pmatrix}
\cdot
\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}
$$

In matrix terms, this system is $\overline{x}(t+1) = A \cdot \overline{x}(t)$. We presume a solution of the form $\overline{x}(t) = \lambda^t \cdot \overline{v}$ (where $\overline{x}$ and $\overline{v}$ are vectors). Then

$$\begin{aligned} \overline{x}(t+1) &= \lambda^{t+1} \cdot \overline{v} \\ &= \lambda \cdot \lambda^t \cdot \overline{v} \\ &= \lambda \cdot \overline{x}(t) \end{aligned} \tag{3}$$

Substituting, we get

$$\overline{x}(t+1) = A \cdot \overline{x}(t) = \lambda \cdot \overline{x}(t) \tag{4}$$

This means that

$$\begin{aligned} A \cdot \overline{x}(t) - \lambda \cdot I \cdot \overline{x}(t) &= \overline{0} \\ (A - \lambda \cdot I) \cdot \overline{x}(t) &= \overline{0} \end{aligned} \tag{5}$$

which has a non-trivial solution only if $|A - \lambda \cdot I| = 0$. This is

$$(6)\quad \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -\frac{c\cdot(1-s)}{v} & \frac{c\cdot(1-s)}{v} & 1-\lambda \end{vmatrix} = -\lambda^3 + \lambda^2 + \lambda \cdot \frac{c}{v} - \frac{c}{v}$$

which has roots $\lambda = \left(1, \sqrt{\frac{c\cdot(1-s)}{v}}, -\sqrt{\frac{c\cdot(1-s)}{v}}\right)$. The general solution is thus of the form

$$(7)\quad Y_t = c_1 \cdot 1^t + c_2 \cdot \sqrt{\frac{c\cdot(1-s)}{v}}^{\,t} + c_3 \cdot \left(-\sqrt{\frac{c\cdot(1-s)}{v}}\right)$$

Solving for the constants in terms of initial values Y_0, Y_1 and Y_2 yields the following system of three equations:

$$
\begin{aligned}
&Y_0 = c_1 \cdot 1^0 + c_2 \cdot \sqrt{\frac{c \cdot (1-s)}{v}}^{\,0} + c_3 \cdot \left(-\sqrt{\frac{c \cdot (1-s)}{v}}\right)^0 \\
(8) \quad &Y_1 = c_1 \cdot 1^1 + c_2 \cdot \sqrt{\frac{c \cdot (1-s)}{v}}^{\,1} + c_3 \cdot \left(-\sqrt{\frac{c \cdot (1-s)}{v}}\right)^1 \\
&Y_2 = c_1 \cdot 1^2 + c_2 \cdot \sqrt{\frac{c \cdot (1-s)}{v}}^{\,2} + c_3 \cdot \left(-\sqrt{\frac{c \cdot (1-s)}{v}}\right)^2,
\end{aligned}
$$

which reduces to the matrix equation

$$
(9) \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & \sqrt{\frac{c\cdot(1-s)}{v}} & -\sqrt{\frac{c\cdot(1-s)}{v}} \\ 1 & \frac{c\cdot(1-s)}{v} & \frac{c\cdot(1-s)}{v} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} Y_0 \\ Y_1 \\ Y_2 \end{pmatrix}
$$

This matrix inverts to

(10)
$$\begin{pmatrix} c\frac{-1+s}{-c+cs+v} & 0 & \frac{1}{-c+cs+v}v \\ -\frac{1}{2}\frac{-c+cs-\sqrt{\left(-c\frac{-1+s}{v}\right)}\,v}{\sqrt{\left(-c\frac{-1+s}{v}\right)}\,(-c+cs+v)} & \frac{1}{2\sqrt{\left(-c\frac{-1+s}{v}\right)}} & -\frac{1}{2}\frac{\sqrt{\left(-c\frac{-1+s}{v}\right)}+1}{\sqrt{\left(-c\frac{-1+s}{v}\right)}\,(-c+cs+v)}v \\ \frac{1}{2}\frac{-c+cs+\sqrt{\left(-c\frac{-1+s}{v}\right)}\,v}{\sqrt{\left(-c\frac{-1+s}{v}\right)}\,(-c+cs+v)} & -\frac{1}{2\sqrt{\left(-c\frac{-1+s}{v}\right)}} & -\frac{1}{2}\frac{\sqrt{\left(-c\frac{-1+s}{v}\right)}-1}{\sqrt{\left(-c\frac{-1+s}{v}\right)}\,(-c+cs+v)}v \end{pmatrix}$$

Multiplying this by the Y vector of initial conditions and simplifying results in the expression for Y(t) shown in Chapter Three:

$$Y(t)=\frac{c\cdot(1-s)\cdot Y_0-v\cdot Y_2}{c\cdot(1-s)-v}\ldots$$
$$+\frac{1}{2}\cdot\left[\frac{\sqrt{v\cdot(c\cdot(1-s))}\cdot\left(Y_1-Y_0\right)+v\cdot\left(Y_2-Y_1\right)}{c\cdot(1-s)-\sqrt{v\cdot(c\cdot(1-s))}}\right]\cdot\left[\sqrt{\frac{c\cdot(1-s)}{v}}\right]^t\ldots$$
$$+\frac{1}{2}\cdot\left[\frac{\sqrt{v\cdot(c\cdot(1-s))}\cdot\left(Y_1-Y_0\right)-v\cdot\left(Y_2-Y_1\right)}{c\cdot(1-s)+\sqrt{v\cdot(c\cdot(1-s))}}\right]\cdot\left[-\left[\sqrt{\frac{c\cdot(1-s)}{v}}\right]\right]^t$$

(11)

Chapter Four

The derivation of the basic Goodwin model is shown in the text.

Capacity Utilisation Extension

The capacity utilisation extension employs the following equation for capacity utilisation:

$$(12)\ c(\lambda) = \frac{U}{V+e^{-W(\lambda-X)}}$$

where this affects the relationship between output and capital:

The derivative of this occurs frequently in the model. Working this out:

(13)
$$
\begin{aligned}
\frac{d}{dt}c(\lambda) &= \frac{d}{dt}\frac{U}{V+e^{-W\cdot(\lambda-X)}} \\
&= U\frac{d}{dt}\left(V+e^{-W\cdot(\lambda-X)}\right)^{-1} \\
&= \frac{-U}{(V+e^{-W\cdot(\lambda-X)})^2}\frac{d}{dt}e^{-W\cdot(\lambda-X)} \\
&= \frac{-U}{(V+e^{-W\cdot(\lambda-X)})^2}e^{-W\cdot(\lambda-X)}\frac{d}{dt} - W\cdot(\lambda-X) \\
&= \frac{-U\times -W}{(V+e^{-W\cdot(\lambda-X)})^2}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda \\
&= \frac{U}{V+e^{-W\cdot(\lambda-X)}}\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda \\
&= c(\lambda)\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda
\end{aligned}
$$

The derivation of wages share of output is identical to the basic model, as shown in the text.

Derivation of the rate of change of the rate of employment is of course affected:

$$
\begin{aligned}
\frac{d}{dt}\lambda &= \frac{d}{dt}\frac{L}{N} \\
&= \frac{1}{N}\frac{d}{dt}L - \frac{L}{N^2}\frac{d}{dt}N \\
&= \frac{1}{N}\frac{d}{dt}\frac{Y}{a} - \lambda\frac{1}{N}\cdot\beta\cdot N \\
&= \frac{1}{N}\left(\frac{1}{a}\frac{d}{dt}Y - \frac{Y}{a^2}\frac{d}{dt}a\right) - \lambda\cdot\beta \\
&= \frac{1}{N}\left(\frac{1}{a}\frac{d}{dt}\left(\frac{K}{v}c(\lambda)\right) - \frac{L\cdot a}{a^2}\alpha\cdot a\right) - \lambda\cdot\beta \\
&= \frac{1}{N}\frac{1}{a}\frac{1}{v}\left(c(\lambda)\frac{d}{dt}K + K\frac{d}{dt}c(\lambda)\right) - \lambda\cdot\left(\alpha+\beta\right) \\
&= \frac{1}{N}\frac{1}{a}\frac{1}{v}\left(c(\lambda)\cdot\left((1-\omega)\cdot Y - \gamma\cdot v\cdot Y\right) + Kc(\lambda)\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda\right) - \lambda\cdot\left(\alpha+\beta\right) \\
\frac{d}{dt}\lambda &= \lambda\left(c(\lambda)\left(\frac{(1-\omega)}{v} - \gamma\right) + \frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda - \alpha - \beta\right)
\end{aligned}
\tag{14}
$$

This can be rearranged to yield:

$$\frac{d}{dt}\lambda - \frac{W}{V+e^{-W(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda = \lambda\left(c(\lambda)\left(\frac{(1-\omega)}{v}-\gamma\right)-a-\beta\right)$$

$$(15)\quad \left(1-\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\right)\frac{d}{dt}\lambda = \lambda\left(c(\lambda)\left(\frac{(1-\omega)}{v}-\gamma\right)-a-\beta\right)$$

$$\frac{d}{dt}\lambda = \frac{\lambda\left(c(\lambda)\left(\frac{(1-\omega)}{v}-\gamma\right)-a-\beta\right)}{1-\frac{W}{V+e^{-W(\lambda-X)}}e^{-W\cdot(\lambda-X)}}$$

The complete model is thus

$$\frac{d}{dt}\omega = \omega\cdot(w(\lambda)-a)$$

$$(16)\quad \frac{d}{dt}\lambda = \frac{\lambda\cdot\left(c(\lambda)\left(\frac{(1-\omega)}{v}-\gamma\right)-a-\beta\right)}{1-\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}}$$

Endogenous Technological Change

This introduces a sigmoidal functional relationship between the rate of technological change and the rate of profit:

$$(17)\quad \frac{1}{a}\frac{d}{dt}a = \alpha \cdot a(\pi)$$

This affects both the wage share function:

$$
\begin{aligned}
\frac{d}{dt}\omega &= \frac{d}{dt}\frac{w \cdot L}{L \cdot a} \\
&= \frac{d}{dt}\frac{w}{a} \\
(18)\qquad &= \frac{1}{a}\frac{d}{dt}w - \frac{w}{a^2}\frac{d}{dt}a \\
&= \frac{1}{a}w \cdot w(\lambda) - \omega\frac{1}{a}\frac{d}{dt}a \\
&= \omega \cdot w(\lambda) - \omega \cdot \alpha \cdot a(\pi) \\
&= \omega \cdot (w(\lambda) - \alpha \cdot a(\pi))
\end{aligned}
$$

and the employment function:

$$
\begin{aligned}
\frac{d}{dt}\lambda &= \frac{d}{dt}\frac{L}{N} \\
&= \frac{1}{N}\frac{d}{dt}L - \frac{L}{N^2}\frac{d}{dt}N \\
&= \frac{1}{N}\frac{d}{dt}\frac{Y}{a} - \lambda\frac{1}{N}\beta \cdot N \\
&= \frac{1}{N}\left(\frac{1}{a}\frac{d}{dt}Y - \frac{Y}{a^2}\frac{d}{dt}a\right) - \lambda \cdot \beta \\
&= \frac{1}{N}\left(\frac{1}{a}\frac{d}{dt}\frac{K}{v} - \frac{L \cdot a}{a^2}\frac{d}{dt}a\right) - \lambda \cdot \beta \\
&= \frac{1}{N}\left(\frac{1}{a}\frac{1-\omega}{v}Y - \frac{a \cdot L}{a}\frac{1}{a}\frac{d}{dt}a\right) - \lambda \cdot \beta \\
&= \frac{1}{N}\left(\frac{1}{a}\frac{1-\omega}{v}a \cdot L - L\alpha \cdot a(\pi)\right) - \lambda \cdot \beta \\
&= \lambda\left(\frac{1-\omega}{v} - \alpha \cdot a(\pi) - \beta\right)
\end{aligned}
\tag{19}
$$

The reduced form is thus:

$$(20)\quad \begin{aligned} \frac{d}{dt}\omega &= \omega \cdot (w(\lambda) - \alpha \cdot a(\pi)) \\ \frac{d}{dt}\lambda &= \lambda\left(\frac{1-\omega}{v} - \alpha \cdot a(\pi) - \beta\right) \end{aligned}$$

Combined model

The combined model includes the nonlinear investment function

$$(21)\quad I_g = k(\pi) = \frac{A}{(B-C\cdot\pi)^2} - D$$

and depreciation at γ% p.a. The wages share function is unaltered; the employment function is the same as for variable capacity utilisation, with the alterations that $1-\omega$ is replaced by $k(\pi)$, γ joins α and β as deductions from the numerator, and α is replaced by $\alpha \cdot a(\pi)$ The reduced form is thus:

$$\frac{d}{dt}\omega = \omega \cdot (w(\lambda) - a \cdot a(\pi))$$

(22)

$$\frac{d}{dt}\lambda = \frac{\lambda \cdot \left(\frac{k(\pi)\cdot c(\lambda)}{v} - \gamma - a \cdot a(\pi) - \beta\right)}{1 - \frac{W}{V + e^{-W\cdot(\lambda - X)}} e^{-W\cdot(\lambda - X)}}$$

Chapter Five

The introduction of a banking sector results in a redefinition of the profit share of output:

(23) $\pi_s = 1 - \omega - r \cdot d$

and a relation for the rate of change of debt:

(24) $\frac{dD}{dt} = r \cdot D + I_g - \Pi$

where I_g is gross investment:

(25) $I_g = k(\pi) \cdot Y$

Basic Model

The equations for employment and wages share are unchanged. The new equation is for the rate of change of the debt ratio:

$$\frac{d}{dt}d = \frac{d}{dt}\frac{D}{Y}$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}D - \frac{D}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot\left(r\cdot D + I_g - \Pi\right) - d\cdot\frac{1}{Y}\cdot\frac{d}{dt}Y$$

$$= r\cdot d + k(\pi) - \pi - d\cdot\frac{1}{Y}\cdot\frac{d}{dt}\frac{K}{v} \qquad (26)$$

$$\blacksquare = r\cdot d + k(\pi) - \pi - \frac{d}{v}\cdot\frac{1}{Y}\cdot(k(\pi)\cdot Y - \gamma\cdot K)$$

$$\blacksquare = r\cdot d + k(\pi) - \pi - d\cdot\left(\frac{k(\pi)}{v} - \gamma\right)$$

$$\frac{d}{dt}d = \left[r - \left(\frac{k(\pi)}{v} - \gamma\right)\right]\cdot d + k(\pi) - \pi$$

This equation applies in both the constant and variable interest rate versions, as noted in the text.

General Model

The equations for wages share and the employment rate are unchanged from the model without a banking sector. The derivation ofthe rate of change of the debt ratio is as follows:

$$\frac{d}{dt}d = \frac{d}{dt}\frac{D}{Y}$$

$$\frac{1}{Y}\cdot\frac{d}{dt}D - \frac{D}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot\left(r\cdot D + I_g - \Pi\right) - \frac{D}{Y}\cdot\frac{1}{Y}\cdot\frac{d}{dt}Y$$

(27)

The percentage rate of growth of output requires several steps to derive:

$$\frac{1}{Y}\frac{d}{dt}Y = \frac{1}{Y}\left(\frac{d}{dt}\frac{K\cdot c(\lambda)}{v}\right)$$

(28)

$$= \frac{1}{Y}\frac{1}{v}\left(c(\lambda)\frac{d}{dt}K + K\frac{d}{dt}c(\lambda)\right)$$

$$= \frac{1}{Y}\frac{1}{v}\left(c(\lambda)\cdot\left(k(\pi)\cdot Y - \gamma\frac{v\cdot Y}{c(\lambda)}\right) + \frac{v\cdot Y}{c(\lambda)}c(\lambda)\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{d}{dt}\lambda\right)$$

where $\frac{d\lambda}{dt}$ is already known from the generalised Goodwin model:

$$(29)\quad \frac{d}{dt}\lambda = \frac{\lambda\cdot\left(\frac{k(\pi)\cdot c(\lambda)}{v}-\gamma-a\cdot a(\pi)-\beta\right)}{1-\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}}$$

Substituting, this yields

$$(30)\quad \frac{1}{Y}\frac{d}{dt}Y = \frac{1}{Y}\frac{1}{v}\left(c(\lambda)\cdot\left(k(\pi)\cdot Y-\gamma\frac{v\cdot Y}{c(\lambda)}\right)+\frac{v\cdot Y}{c(\lambda)}c(\lambda)\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{\lambda\cdot\left(\frac{k(\pi)\cdot c(\lambda)}{v}-\gamma-a\cdot a(\pi)-\beta\right)}{1-\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}}\right)$$

which simplies to

$$(31)\quad \frac{1}{Y}\frac{d}{dt}Y = \left(\frac{c(\lambda)\cdot k(\pi)}{v}-\gamma\right)+\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}\frac{\lambda\cdot\left(\frac{k(\pi)\cdot c(\lambda)}{v}-\gamma-a\cdot a(\pi)-\beta\right)}{1-\frac{W}{V+e^{-W\cdot(\lambda-X)}}e^{-W\cdot(\lambda-X)}}$$

This can be simplified further by factoring out the exponential to arrive at:

$$(32)\quad \frac{1}{Y}\frac{d}{dt}Y = \left(\frac{c(\lambda)\cdot k(\pi)}{v}-\gamma\right)+W\cdot e^{-W\cdot(\lambda-X)}\frac{\lambda\cdot\left(\frac{k(\pi)\cdot c(\lambda)}{v}-\gamma-a\cdot a(\pi)-\beta\right)}{V+(1-W\cdot\lambda)\cdot e^{-W\cdot(\lambda-X)}}$$

Substituting this back into the equation for the debt ratio yields:

(33) $$\frac{d}{dt}d = r \cdot d + k(\pi) - \pi - d \cdot \left(\left(\frac{c(\lambda)\cdot k(\pi)}{v} - \gamma \right) + W \cdot e^{-W\cdot(\lambda - X)} \frac{\lambda \cdot \left(\frac{k(\pi)\cdot c(\lambda)}{v} - \gamma - a \cdot a(\pi) - \beta \right)}{V + (1 - W\cdot\lambda)\cdot e^{-W\cdot(\lambda - X)}} \right)$$

(34) $$\frac{d}{dt}d = \left(r - \left(\left(\frac{c(\lambda)\cdot k(\pi)}{v} - \gamma \right) + W \cdot e^{-W\cdot(\lambda - X)} \frac{\lambda \cdot \left(\frac{k(\pi)\cdot c(\lambda)}{v} - \gamma - a \cdot a(\pi) - \beta \right)}{V + (1 - W\cdot\lambda)\cdot e^{-W\cdot(\lambda - X)}} \right) \right) \cdot d + k(\pi) - \pi$$

Chapter Six

This chapter adds government taxation and subsidy functions, and also requires the division of debt into private and public components. The equations for wages share and employment remain unaltered from the respective preceding models. Gross profit reverts to the pre-banking form:

(35) $$\begin{aligned} \Pi &= Y - W \\ \pi &= 1 - \omega \end{aligned}$$

while net profit must also be defined:

$$\Pi_n = Y - W - T + G - r \cdot D_k$$

(36)

$$\pi_n = 1 - \omega - t + g - r \cdot d_k$$

Basic Model

The first new equation is for the rate of change of the capitalist debt to output ratio:

$$\frac{d}{dt} d_k = \frac{d}{dt} \frac{D_k}{Y}$$

$$= \frac{1}{Y} \cdot \frac{d}{dt} D_k - \frac{D_k}{Y^2} \cdot \frac{d}{dt} Y$$

$$= \frac{1}{Y} \cdot \frac{d}{dt} D_k - d_k \cdot \frac{1}{Y} \cdot \frac{d}{dt} Y$$

(37)

The rate of change of capitalist debt is:

$$(38)\quad \frac{d}{dt}D_k = r \cdot D_k + I_g - \Pi + T - G$$

The percentage rate of growth is a common term in several of the equations of this model. Deriving it separately:

$$(39)\quad \begin{aligned} \frac{1}{Y}\frac{d}{dt}Y &= \frac{1}{Y}\frac{d}{dt}\frac{K}{v} \\ &= \frac{1}{Y}\frac{1}{v}\frac{d}{dt}K \\ &= \frac{1}{Y}\frac{1}{v}\left(k(\pi_n) \cdot Y - \gamma \cdot v \cdot Y\right) \\ &= \left(\frac{k(\pi_n)}{v} - \gamma\right) \end{aligned}$$

Substituting this back into the equation for the rate of change of the capitalist debt ratio:

(40)
$$
\begin{aligned}
\frac{d}{dt}d_k &= \frac{1}{Y}\left(r \cdot D_k + I_g - \Pi + T - G\right) - d_k \cdot \left(\frac{k(\pi_n)}{v} - \gamma\right) \\
&= r \cdot d_k + \frac{I_g}{Y} - \pi + t - g - d_k \cdot \left(\frac{k(\pi_n)}{v} - \gamma\right) \\
&= r \cdot d_k + \frac{k(\pi_n) \cdot Y}{Y} - \pi + t - g - d_k \cdot \left(\frac{k(\pi_n)}{v} - \gamma\right) \\
&= \left(r - \left(\frac{k(\pi_n)}{v} - \gamma\right)\right) \cdot d_k + k(\pi_n) - \pi + t - g
\end{aligned}
$$

The rate of change of government debt is:

(41) $\frac{d}{dt}D_g = r \cdot D_g - T + G$

The rate of change of the government debt ratio is:

$$\frac{d}{dt}d_g=\frac{d}{dt}\frac{D_g}{Y}$$

$$=\frac{1}{Y}\cdot\frac{d}{dt}D_g-d_g\cdot\frac{1}{Y}\cdot\frac{d}{dt}Y$$

$$=\frac{1}{Y}\cdot\left[r\cdot D_g-(T-G)\right]-d_g\cdot\left(\frac{k(\pi_n)}{v}-\gamma\right)$$

$$=r\cdot d_g-t+g-d_g\cdot\left(\frac{k(\pi_n)}{v}-\gamma\right)$$

$$=\left[r-\left(\frac{k(\pi_n)}{v}-\gamma\right)\right]\cdot d_g-t+g$$

(42)

The rate of change of government taxation is:

$$\frac{d}{dt}t = \frac{d}{dt}\frac{T}{Y}$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}T - \frac{T}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot\left(t\left(\pi_n\right)\cdot Y\right) - t\cdot\left(\frac{k\left(\pi_n\right)}{v} - \gamma\right)$$

$$= t\left(\pi_n\right) - t\cdot\left(\frac{k\left(\pi_n\right)}{v} - \gamma\right)$$

(43)

The rate of change of government subsidies:

$$\frac{d}{dt}g = \frac{d}{dt}\frac{G}{Y}$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}G - \frac{G}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot g(\lambda)\cdot Y - g\cdot\left(\frac{k(\pi_n)}{v} - \gamma\right)$$

$$= g(\lambda) - g\cdot\left(\frac{k(\pi_n)}{v} - \gamma\right)$$

(44)

General Model

The rate of change of the capitalist debt ratio commences as above:

$$\frac{d}{dt} d_k = \frac{d}{dt} \frac{D_k}{Y}$$

$$= \frac{1}{Y} \cdot \frac{d}{dt} D_k - \frac{D_k}{Y^2} \cdot \frac{d}{dt} Y$$

(45)
$$= \frac{1}{Y} \cdot \frac{d}{dt} D_k - d_k \cdot \frac{1}{Y} \cdot \frac{d}{dt} Y$$

The percentage rate of growth of output for the general model has been worked out earlier. Substituting this yields:

(46)
$$\frac{d}{dt} d_k = \frac{1}{Y}\left(r \cdot D_k + k(\pi_n) \cdot Y - (\Pi - T + G)\right) - d_k \cdot \left(\left(\frac{c(\lambda) \cdot k(\pi_n)}{v} - \gamma \right) + W \cdot e^{-W \cdot (\lambda - X)} \frac{\lambda \cdot \left(\frac{k(\pi_n) \cdot c(\lambda)}{v} - \gamma - \alpha \cdot a(\pi_n) - \beta \right)}{V + (1 - W \cdot \lambda) \cdot e^{-W \cdot (\lambda - X)}} \right)$$

This can be rearranged to yield:

(47) $$\frac{d}{dt}d_k = \left(r - \left(\left(\frac{c(\lambda)\cdot k(\pi_n)}{v} - \gamma \right) + W\cdot e^{-W\cdot(\lambda - X)} \frac{\lambda\cdot\left(\frac{k(\pi_n)\cdot c(\lambda)}{v} - \gamma - a\cdot a(\pi_n) - \beta\right)}{V + (1 - W\cdot\lambda)\cdot e^{-W\cdot(\lambda - X)}} \right) + k(\pi_n) + t - \pi - g \right)\cdot d_k$$

The rate of change of government debt is:

$$\frac{d}{dt}d_g = \frac{d}{dt}\frac{D_g}{Y}$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}D_g \quad \frac{D_g}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot\left(r\cdot D_g + G - T\right) - d_g\cdot\left[\frac{1}{Y}\cdot\left(\frac{d}{dt}Y\right)\right]$$

(48)

Substituting in the term for the percentage rate of growth yields:

$$(49)\quad \frac{d}{dt}d_g = \left(r - \left(\left(\frac{c(\lambda)\cdot k(\pi_n)}{v} - \gamma \right) + W \cdot e^{-W\cdot(\lambda - X)} \frac{\lambda\cdot\left(\frac{k(\pi_n)\cdot c(\lambda)}{v} - \gamma - a\cdot a(\pi_n) - \beta\right)}{V + (1 - W\cdot\lambda)\cdot e^{-W\cdot(\lambda - X)}} \right) \right) \cdot d_g + g - t$$

The rate of change of the tax share of output is:

$$\frac{d}{dt}t = \frac{d}{dt}\frac{T}{Y}$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}T - \frac{T}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}T - t\cdot\left[\frac{1}{Y}\cdot\left(\frac{d}{dt}Y\right)\right]$$

$$(50)\quad = \frac{1}{Y}\cdot(t(\pi)\cdot Y) - t\cdot\left[\frac{1}{Y}\cdot\left(\frac{d}{dt}Y\right)\right]$$

Substituting for the rate of growth:

$$(51)\quad \frac{d}{dt}t = t(\pi) - t\cdot\left(\left(\frac{c(\lambda)\cdot k(\pi_n)}{v} - \gamma\right) + W\cdot e^{-W\cdot(\lambda - X)}\frac{\lambda\cdot\left(\frac{k(\pi_n)\cdot c(\lambda)}{v} - \gamma - a\cdot a(\pi_n) - \beta\right)}{V + (1 - W\cdot\lambda)\cdot e^{-W\cdot(\lambda - X)}}\right)$$

The rate of change of the subsidies share of output:

$$\frac{d}{dt}g = \frac{d}{dt}\frac{G}{Y}$$

$$= \frac{1}{Y}\cdot\frac{d}{dt}G - \frac{G}{Y^2}\cdot\frac{d}{dt}Y$$

$$= \frac{1}{Y}\cdot(g(1-\lambda)\cdot Y) - g\cdot\left[\frac{1}{Y}\cdot\left(\frac{d}{dt}Y\right)\right]$$

(52)

Substituting the rate of growth:

$$(53)\quad \frac{d}{dt}g = g(\lambda) - g\cdot\left(\left(\frac{c(\lambda)\cdot k(\pi_n)}{v} - \gamma\right) + W\cdot e^{-W\cdot(\lambda - X)}\frac{\lambda\cdot\left(\frac{k(\pi_n)\cdot c(\lambda)}{v} - \gamma - a\cdot a(\pi_n) - \beta\right)}{V + (1 - W\cdot\lambda)\cdot e^{-W\cdot(\lambda - X)}}\right)$$

Chapter Seven

The conditions for price instability

Steedman's dynamic equation:

(54) $p_{t+1} = (u + p_t A)(I + \hat{m})$

has to be rearranged to put it in a form suitable for dynamical analysis:

(55) $p_{t+1} = p_t A(I + \hat{m}) + u(I + \hat{m})$

Solving the homogenous problem initially, we presume a solution of the form

(56) $p_t = v \cdot X^t$

Expanding this:

(57)
$$\begin{aligned} p_{t+1} &= v \cdot X^{t+1} = p_t A(I+\hat{m}) = v \cdot X^t \cdot A(I+\hat{m}) \\ v \cdot X^{t+1} &= v \cdot X^t \cdot A(I+\hat{m}) \end{aligned}$$

Rearranging this yields:

(58)
$$\begin{aligned} v \cdot X^{t+1} - v \cdot X^t \cdot A(I+\hat{m}) &= 0 \\ v \cdot X^{t+1}\left(X - A(I+\hat{m})\right) &= 0 \end{aligned}$$

which has the solution

(59) $X = A(I+\hat{m})$

Hence the solution to the homogenous problem is:

(60) $p_t = v \cdot \left(A(I+\hat{m})\right)^t$

For the particular problem, we presume a solution of the form $p_{t+1} = b$ where b is a vector of constants. Then we have

$$(61)\quad \begin{aligned} p_{t+1} &= p_t A(I+\widehat{m}) + u(I+\widehat{m}) \\ b &= b \cdot A(I+\widehat{m}) + u(I+\widehat{m}) \end{aligned}$$

Rearranging this yields:

$$(62)\quad \begin{aligned} b - b \cdot A(I+\widehat{m}) &= u(I+\widehat{m}) \\ b \cdot \left(I - A(I+\widehat{m})\right) &= u(I+\widehat{m}) \end{aligned}$$

which has the solution

$$(63)\quad b = u(I+\widehat{m}) \cdot \left(I - A(I+\widehat{m})\right)^{-1}$$

This is the solution which Steedman put forward as the general solution to his dynamic process. Instead, the general solution is

$$(64)\quad p_t = v \cdot \left(A(I+\widehat{m})\right)^{t} + u(I+\widehat{m}) \cdot \left(I - A(I+\widehat{m})\right)^{-1}$$

which will converge to b only if the dominant eigenvalue of $A(I+\hat{m})$ is less than one. This depends upon the values of A and m. It is easily shown that feasible values of A and m lead to eigenvalues greater than one, so that convergence to the equilibrium of b is conditional only.

A gross profit adjustment mechanism

See the appended Mathematica document (page 260).

An average rate of profit adjustment mechanism for markups

See the appended Mathematica document (page 267).

Chapter Eight

This derivation of this model is explained in the Chapter. This appendix will concentrate on details which were not elaborated upon there. The functions are written in MathCad's programming language.

The interest rate function *r(x)* in the debt equation:

(65) $d_t = d_{t-1} + r(d_{t-1})$

is a differential rate of interest on a vector of deposits x:

$$
r(x) = \left|
\begin{array}{l}
r_d \leftarrow l \\
r_l \leftarrow h \\
s \leftarrow length(x) \\
for \;\; i \in 1 .. s \\
\quad \left| \begin{array}{l} r_i \leftarrow r_d x_i \;\; if \;\; x_i \geq 0 \\ r_i \leftarrow r_l x_i \;\; otherwise \end{array} \right. \\
r
\end{array}
\right.
$$

(66)

The *Min0(x,y)* function which is used several times in the model, including in the determination output on the basis of previous effective demand and a reserved quantity of output:

(67) $q_t = min0(\theta_{t-n}, q_{max})$

is a function which selects the non-negative minimum entries of two vectors.:

$$min0(x,y) \equiv \left| \begin{array}{l} s \leftarrow length(x) \\ for\ \ i \in 1..s \\ \quad \left| \begin{array}{l} m_i \leftarrow x_i \ \ if\ \left(x_i < y_i\right) \\ m_i \leftarrow y_i \ \ otherwise \end{array} \right. \\ for\ \ i \in 1..s \\ \quad \left| \begin{array}{l} n_i \leftarrow m_i \ \ if\ \ m_i > 0 \\ n_i \leftarrow 0 \ \ otherwise \end{array} \right. \\ n \end{array} \right. \tag{68}$$

The output decision q_t determines the demand for intermediate inputs i_t. The intersectoral transfers generated by demand for intemediate inputs sum to zero for the capitalist class as a whole, but of course involve net gains for some sectors and net outgoings for others. The revenue side of intersectoral demand is

(69) $$diag\left(p_{t-1}\right)\cdot A\cdot q_t$$

while the expenditure side is

(70) $$diag\left(q_t\right)\cdot A^T\cdot p_{t-1}$$

Such matrix transposes occur at several points in the model. The function *diag(x)* takes a vector and returns a diagonalised matrix:

(71)
```
diag(x) ≡ | s ← length(x)
          | for i ∈ 1..s
          |   for j ∈ 1..s
          |     | M[i,j] ← 0  if i≠j
          |     | M[i,j] ← x[i]  otherwise
          | M
```

In this model workers are assumed to spend all their wages on consumption, while bankers and capitalists spend a proportion based upon their marginal propensity to consume. q_w is the vector of workers' consumption preferences, which is derived from a log linear utility function. The same function is used for bankers and capitalists: all are assumed to attempt to maximise a utility function of the form

$$U(q_1, q_2, \ldots) = \alpha_1 \cdot ln(q_1) + \alpha_2 \cdot ln(q_2) + \ldots + \alpha_n \cdot ln(q_n) \quad (72)$$

where the α_i's are constants, subject to the expenditure constraint that

$$p(q_1, q_2, \ldots, q_n, y) = p_1 \cdot q_1 + p_2 \cdot q_2 + \ldots + p_n \cdot q_n - y = 0 \quad (73)$$

where in the case of workers, y equals the entire wage. This can be solved via a Lagrangian, where the general solution is of the form

$$\nabla U_q = \lambda \cdot \nabla_{p,y} \quad (74)$$

Expressing this in terms of commodity *1*, we are looking for solutions of the form

$$\frac{d}{dq_1}\left(\alpha_1 \cdot ln(q_1) + \alpha_2 \cdot ln(q_2) + \alpha_3 \cdot ln(q_3)\right) = \lambda \cdot \left|\frac{d}{dq_1}\left[\left(p_1 \cdot q_1 + p_2 \cdot q_2 + p_3 \cdot q_3\right) - y\right]\right| \quad (75)$$

and so on. This leads to n equations of the form $\frac{\alpha_i}{q_i} = \lambda \cdot p_i$ or $q_i = \frac{\alpha_i}{\lambda \cdot p_i}$. Substituting this back into the constraint yields

$$p_1 \cdot q_1 + p_2 \cdot q_2 + \ldots + p_n \cdot q_n - y = \left(p_1 \cdot \frac{\alpha_1}{p_1 \cdot \lambda} + p_2 \cdot \frac{\alpha_2}{p_2 \cdot \lambda} + \ldots + p_n \cdot \frac{\alpha_n}{p_n \cdot \lambda} \right) - y = 0$$

(76)

so that in general terms

$$\lambda = \frac{\sum \alpha_i}{y}$$

(77)

This in turn implies that the *i*th entry in the consumption vector is

$$q_i = \frac{\alpha_i}{p_i \cdot \lambda} = \frac{\alpha_i}{p_i \cdot \frac{\sum \alpha_i}{y}} = \frac{\alpha_i \cdot y}{p_i \cdot \sum \alpha_i}$$

(78)

so that the consumption vector in quantity terms is

$$q = y \cdot \begin{bmatrix} \dfrac{\alpha_1}{\left(p_1 \cdot \sum \alpha_i\right)} \\ \dfrac{\alpha_2}{\left(p_2 \cdot \sum \alpha_i\right)} \\ \cdots \\ \dfrac{\alpha_n}{\left(p_3 \cdot \sum \alpha_i\right)} \end{bmatrix} \tag{79}$$

(where it is possible to normalise the α's so that $\sum a_i = 1$). This can be expressed as

$$q = w \cdot \sum e \cdot \begin{bmatrix} \frac{1}{p_1} & 0 & \dots & 0 \\ 0 & \frac{1}{p_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{p_n} \end{bmatrix} \cdot \begin{bmatrix} \frac{\omega_1}{\sum \omega_i} \\ \frac{\omega_2}{\sum \omega_i} \\ \dots \\ \frac{\omega_n}{\sum \omega_n} \end{bmatrix}$$

(80)

where workers total income is the wage times the sum of employment, and ω replaces α to signify that these are workers preferences. Making the substitution that

$$\begin{bmatrix} \dfrac{\omega_1}{\sum \omega_i} \\ \dfrac{\omega_2}{\sum \omega_i} \\ \cdots \\ \dfrac{\omega_n}{\sum \omega_n} \end{bmatrix} = q_w \tag{81}$$

and dating consumption, this means that physical consumption by workers at time t is

$$q_{w_t} = w_t \cdot \sum e_t \cdot diag\left(p_{t-1}\right)^{(-1)} \cdot q_w \tag{82}$$

and consumption expenditure by workers at time t is

$$c_{w_t} = diag(p_{t-1}) \cdot q_{w_t} = diag(p_{t-1}) \cdot \left[w_t \cdot \sum e_t \cdot diag(p_{t-1})^{(-1)} \cdot q_w \right] = w_t \cdot \sum e_t \cdot q_w$$
(83)

Bankers' consumption is similarly derived; capitalists' consumption is complicated by the fact that there is one group of capitalists per sector, rather than a class (workers) or a single entity (the banker). Thus banker's consumption preferences are indicated by a vector q_b, as for workers, whereas capitalist consumption preferences are represented by a matrix Q_π . Physical consumption by bankers and capitalists can also be quantity constrained to the difference between the previous year's output and the sum of intermediate demand and workers' demand, thus the function *min0(x,y)* recurs here

Capitalist expenditure on consumption and revenue from consumption need not be reciprocal, since it is possible that one sector's expenditures are not quantity constrained, but its revenues are. The function *dmmin1(x,y)* is used in this case, where it returns a vector whose entries are the minimum of 1 and the diagonal entries of a diagonal matrix:

$$
dmmin1(x) \equiv \left|
\begin{array}{l}
s \leftarrow rows(x) \\
for\ \ i \in 1..s \\
\quad for\ \ j \in 1..s \\
\quad\quad m_i \leftarrow x_{i,j}\ \ if\ \ i=j \\
for\ \ i \in 1..s \\
\quad \left| \begin{array}{l} n_i \leftarrow m_i\ \ if\ \ m_i<1 \\ n_i \leftarrow 1\ \ otherwise \end{array} \right. \\
for\ \ i \in 1..s \\
\quad \left| \begin{array}{l} o_i \leftarrow n_i\ \ if\ \ m_i>0 \\ o_i \leftarrow 0\ \ otherwise \end{array} \right. \\
o
\end{array}
\right.
$$

(84)

The function *maxs(x,y)* is used to provide a minimum level for markups (which could be related to the rate of interest on loans). It returns a vector where each entry is the maximum of the entry in vector x and scalar y:

$$maxs(v,s) \equiv \left| \begin{array}{l} l \leftarrow length(v) \\ for\ \ i \in 1..l \\ \quad \left| \begin{array}{l} m_i \leftarrow v_i\ \ if\ \ v_i > s \\ m_i \leftarrow s\ \ otherwise \end{array} \right. \\ m \end{array} \right. \tag{85}$$

meandev(x) returns the deviation of each element of *x* from the average for *x*:

$$meandev(x) \equiv \left| \begin{array}{l} for\ \ i \in 1..rows(x) \\ \quad m_i \leftarrow x_i - mean(x) \\ m \end{array} \right. \tag{86}$$

Appendix B
Chapter Seven

Section 4: A gross profit adjustment mechanism for markups

This document calculates the eigenvalues for the Jacobian of this system, and shows that for all values of the markup adjustment parameter (shown as α in this document), there is at least one eigenvalue of 1. Consequently the equilibrium of the system is marginally unstable. The calculations are done only for the example system used by Steedman; however it would be possible to modify the document to calculate the equilibrium of any given system and calculate its eigenvalues.

This document works with a 3x3 input - output matrix :

```
A = Array[a, {3, 3}];
```

```
TraditionalForm[A]
```

$$\begin{pmatrix} a(1,1) & a(1,2) & a(1,3) \\ a(2,1) & a(2,2) & a(2,3) \\ a(3,1) & a(3,2) & a(3,3) \end{pmatrix}$$

```
I3 = IdentityMatrix[3];
```

The markup adjustment process is as shown in Chapter Seven :

$$\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix} = \begin{pmatrix} m_{1_{t+2}} \\ m_{2_{t+2}} \\ m_{3_{t+2}} \end{pmatrix} - \alpha \times \text{Transpose}\left[(u_1 \quad u_2 \quad u_3 + p_{1_{t+1}} \quad p_{2_{t+1}} \quad p_{3_{t+1}} \;.\; A) \;.\; \begin{pmatrix} m_{1_{t+2}} & 0 & 0 \\ 0 & m_{2_{t+2}} & 0 \\ 0 & 0 & m_{3_{t+2}} \end{pmatrix} - (u_1 \quad u_2 \quad u_3 + p_{1_t} \quad p_{2_t} \quad p_{3_t} \;.\; A) \;.\; \begin{pmatrix} m_{1_{t+1}} & 0 & 0 \\ 0 & m_{2_{t+1}} & 0 \\ 0 & 0 & m_{3_{t+1}} \end{pmatrix} \right];$$

$$\text{TraditionalForm}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}\right]$$

$$\begin{pmatrix} m_{1_{t+2}} - \alpha (m_{1_{t+2}} (a(1,1)\, p_{1_{t+1}} + a(2,1)\, p_{2_{t+1}} + a(3,1)\, p_{3_{t+1}} + u_1) - m_{1_{t+1}} (a(1,1)\, p_{1_t} + a(2,1)\, p_{2_t} + a(3,1)\, p_{3_t} + u_1)) \\ m_{2_{t+2}} - \alpha (m_{2_{t+2}} (a(1,2)\, p_{1_{t+1}} + a(2,2)\, p_{2_{t+1}} + a(3,2)\, p_{3_{t+1}} + u_2) - m_{2_{t+1}} (a(1,2)\, p_{1_t} + a(2,2)\, p_{2_t} + a(3,2)\, p_{3_t} + u_2)) \\ m_{3_{t+2}} - \alpha (m_{3_{t+2}} (a(1,3)\, p_{1_{t+1}} + a(2,3)\, p_{2_{t+1}} + a(3,3)\, p_{3_{t+1}} + u_3) - m_{3_{t+1}} (a(1,3)\, p_{1_t} + a(2,3)\, p_{2_t} + a(3,3)\, p_{3_t} + u_3)) \end{pmatrix}$$

Prices are determined by a markup on exogenous inputs and prices times the input - output matrix :

$$\begin{pmatrix} p_{1_{t+3}} \\ p_{2_{t+3}} \\ p_{3_{t+3}} \end{pmatrix} = \text{Transpose}\left[(u_1 \quad u_2 \quad u_3 + (p_{1_{t+2}} \quad p_{2_{t+2}} \quad p_{3_{t+2}}) \;.\; A) \;.\; \left(\text{I3} + \text{DiagonalMatrix}\left[\left\{\text{Extract}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}, \{1, 1\}\right], \text{Extract}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}, \{2, 1\}\right], \text{Extract}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}, \{3, 1\}\right]\right\}\right]\right)\right];$$

$$\text{TraditionalForm}\left[\begin{pmatrix} p_{1_{t+3}} \\ p_{2_{t+3}} \\ p_{3_{t+3}} \end{pmatrix}\right]$$

$$\begin{pmatrix} (a(1,1)\, p_{1_{t+2}} + a(2,1)\, p_{2_{t+2}} + a(3,1)\, p_{3_{t+2}} + u_1)(m_{1_{t+2}} - \alpha (m_{1_{t+2}} (a(1,1)\, p_{1_{t+1}} + a(2,1)\, p_{2_{t+1}} + a(3,1)\, p_{3_{t+1}} + u_1) - m_{1_{t+1}} (a(1,1)\, p_{1_t} + a(2,1)\, p_{2_t} + a(3,1)\, p_{3_t} + u_1)) + 1) \\ (a(1,2)\, p_{1_{t+2}} + a(2,2)\, p_{2_{t+2}} + a(3,2)\, p_{3_{t+2}} + u_2)(m_{2_{t+2}} - \alpha (m_{2_{t+2}} (a(1,2)\, p_{1_{t+1}} + a(2,2)\, p_{2_{t+1}} + a(3,2)\, p_{3_{t+1}} + u_2) - m_{2_{t+1}} (a(1,2)\, p_{1_t} + a(2,2)\, p_{2_t} + a(3,2)\, p_{3_t} + u_2)) + 1) \\ (a(1,3)\, p_{1_{t+2}} + a(2,3)\, p_{2_{t+2}} + a(3,3)\, p_{3_{t+2}} + u_3)(m_{3_{t+2}} - \alpha (m_{3_{t+2}} (a(1,3)\, p_{1_{t+1}} + a(2,3)\, p_{2_{t+1}} + a(3,3)\, p_{3_{t+1}} + u_3) - m_{3_{t+1}} (a(1,3)\, p_{1_t} + a(2,3)\, p_{2_t} + a(3,3)\, p_{3_t} + u_3)) + 1) \end{pmatrix}$$

The stability of the linearised version of the system is calculated about its equilibrium point . Mapping all price and markup variables to a standard variable :

$\mathbf{p}_{1_t} = \mathbf{x}_1;$
$\mathbf{p}_{1_{t+1}} = \mathbf{x}_2;$
$\mathbf{p}_{1_{t+2}} = \mathbf{x}_3;$
$\mathbf{p}_{2_t} = \mathbf{x}_4;$
$\mathbf{p}_{2_{t+1}} = \mathbf{x}_5;$
$\mathbf{p}_{2_{t+2}} = \mathbf{x}_6;$
$\mathbf{p}_{3_t} = \mathbf{x}_7;$
$\mathbf{p}_{3_{t+1}} = \mathbf{x}_8;$
$\mathbf{p}_{3_{t+2}} = \mathbf{x}_9;$

$\mathbf{m}_{1_t} = \mathbf{x}_{10};$
$\mathbf{m}_{1_{t+1}} = \mathbf{x}_{11};$
$\mathbf{m}_{1_{t+2}} = \mathbf{x}_{12};$
$\mathbf{m}_{2_t} = \mathbf{x}_{13};$
$\mathbf{m}_{2_{t+1}} = \mathbf{x}_{14};$
$\mathbf{m}_{2_{t+2}} = \mathbf{x}_{15};$
$\mathbf{m}_{3_t} = \mathbf{x}_{16};$
$\mathbf{m}_{3_{t+1}} = \mathbf{x}_{17};$
$\mathbf{m}_{3_{t+2}} = \mathbf{x}_{18};$

TraditionalForm[$\mathbf{p}_{1_{t+3}}$]

$$(u_1 + a(1, 1)\, x_3 + a(2, 1)\, x_6 + a(3, 1)\, x_9)\,(x_{12} - \alpha\,((u_1 + a(1, 1)\, x_2 + a(2, 1)\, x_5 + a(3, 1)\, x_8)\, x_{12} - (u_1 + a(1, 1)\, x_1 + a(2, 1)\, x_4 + a(3, 1)\, x_7)\, x_{11}) + 1)$$

TraditionalForm[$\mathbf{p}_{2_{t+3}}$]

$$(u_2 + a(1, 2)\, x_3 + a(2, 2)\, x_6 + a(3, 2)\, x_9)\,(x_{15} - \alpha\,((u_2 + a(1, 2)\, x_2 + a(2, 2)\, x_5 + a(3, 2)\, x_8)\, x_{15} - (u_2 + a(1, 2)\, x_1 + a(2, 2)\, x_4 + a(3, 2)\, x_7)\, x_{14}) + 1)$$

TraditionalForm[$\mathbf{p}_{3_{t+3}}$]

$$(u_3 + a(1, 3)\, x_3 + a(2, 3)\, x_6 + a(3, 3)\, x_9)\,(x_{18} - \alpha\,((u_3 + a(1, 3)\, x_2 + a(2, 3)\, x_5 + a(3, 3)\, x_8)\, x_{18} - (u_3 + a(1, 3)\, x_1 + a(2, 3)\, x_4 + a(3, 3)\, x_7)\, x_{17}) + 1)$$

```
TraditionalForm[m1t+3]
```

$$x_{12} - \alpha\,((u_1 + a(1,\,1)\,x_2 + a(2,\,1)\,x_5 + a(3,\,1)\,x_8)\,x_{12} - (u_1 + a(1,\,1)\,x_1 + a(2,\,1)\,x_4 + a(3,\,1)\,x_7)\,x_{11})$$

```
TraditionalForm[m2t+3]
```

$$x_{15} - \alpha\,((u_2 + a(1,\,2)\,x_2 + a(2,\,2)\,x_5 + a(3,\,2)\,x_8)\,x_{15} - (u_2 + a(1,\,2)\,x_1 + a(2,\,2)\,x_4 + a(3,\,2)\,x_7)\,x_{14})$$

```
TraditionalForm[m3t+3]
```

$$x_{18} - \alpha\,((u_3 + a(1,\,3)\,x_2 + a(2,\,3)\,x_5 + a(3,\,3)\,x_8)\,x_{18} - (u_3 + a(1,\,3)\,x_1 + a(2,\,3)\,x_4 + a(3,\,3)\,x_7)\,x_{17})$$

The linearised model in the vicinity of the equilibrium point is thus :

$$f = \{x_2,\ x_3,\ p1_{t+3},\ x_5,\ x_6,\ p2_{t+3},\ x_8,\ x_9,\ p3_{t+3},\ x_{11},\ x_{12},\ m1_{t+3},\ x_{14},\ x_{15},\ m2_{t+3},\ x_{17},\ x_{18},\ m3_{t+3}\};$$

```
MatrixForm[f]
```

$$\begin{pmatrix}
x_2 \\
x_3 \\
(u_1 + a[1,\ 1]\, x_3 + a[2,\ 1]\, x_6 + a[3,\ 1]\, x_9)\,(1 + x_{12} - \alpha\,(-(u_1 + a[1,\ 1]\, x_1 + a[2,\ 1]\, x_4 + a[3,\ 1]\, x_7)\, x_{11} + (u_1 + a[1,\ 1]\, x_2 + a[2,\ 1]\, x_5 + a[3,\ 1]\, x_8)\, x_{12})) \\
x_5 \\
x_6 \\
(u_2 + a[1,\ 2]\, x_3 + a[2,\ 2]\, x_6 + a[3,\ 2]\, x_9)\,(1 + x_{15} - \alpha\,(-(u_2 + a[1,\ 2]\, x_1 + a[2,\ 2]\, x_4 + a[3,\ 2]\, x_7)\, x_{14} + (u_2 + a[1,\ 2]\, x_2 + a[2,\ 2]\, x_5 + a[3,\ 2]\, x_8)\, x_{15})) \\
x_8 \\
x_9 \\
(u_3 + a[1,\ 3]\, x_3 + a[2,\ 3]\, x_6 + a[3,\ 3]\, x_9)\,(1 + x_{18} - \alpha\,(-(u_3 + a[1,\ 3]\, x_1 + a[2,\ 3]\, x_4 + a[3,\ 3]\, x_7)\, x_{17} + (u_3 + a[1,\ 3]\, x_2 + a[2,\ 3]\, x_5 + a[3,\ 3]\, x_8)\, x_{18})) \\
x_{11} \\
x_{12} \\
x_{12} - \alpha\,(-(u_1 + a[1,\ 1]\, x_1 + a[2,\ 1]\, x_4 + a[3,\ 1]\, x_7)\, x_{11} + (u_1 + a[1,\ 1]\, x_2 + a[2,\ 1]\, x_5 + a[3,\ 1]\, x_8)\, x_{12}) \\
x_{14} \\
x_{15} \\
x_{15} - \alpha\,(-(u_2 + a[1,\ 2]\, x_1 + a[2,\ 2]\, x_4 + a[3,\ 2]\, x_7)\, x_{14} + (u_2 + a[1,\ 2]\, x_2 + a[2,\ 2]\, x_5 + a[3,\ 2]\, x_8)\, x_{15}) \\
x_{17} \\
x_{18} \\
x_{18} - \alpha\,(-(u_3 + a[1,\ 3]\, x_1 + a[2,\ 3]\, x_4 + a[3,\ 3]\, x_7)\, x_{17} + (u_3 + a[1,\ 3]\, x_2 + a[2,\ 3]\, x_5 + a[3,\ 3]\, x_8)\, x_{18})
\end{pmatrix}$$

The Jacobian of this system determines its stability properties :

```
Jac = Table[D[f[[i]], x_j], {i, 1, 18}, {j, 1, 18}];

TraditionalForm[Jac]
```

$$
\left(
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
\alpha\, a(1,1)\,(u_1 + a(1,1)\,x_3 + a(2,1)\,x_6 + a(3,1)\,x_9)\,x_{11} & -\alpha\, a(1,1)\,(u_1 + a(1,1)\,x_3 + a(2,1)\,x_6 + a(3,1)\,x_9)\,x_{12} & a(1,1)\,(x_{12} - \alpha\,((u_1 + a(1,1)\,x_2 + a(2,1)\,x_5 + a(3,1)\,x_8)\,x_{12} - (u_1 + a(1,1)\,x_1 + a(\\
0 & 0 & 0 \\
0 & 0 & 0 \\
\alpha\, a(1,2)\,(u_2 + a(1,2)\,x_3 + a(2,2)\,x_6 + a(3,2)\,x_9)\,x_{14} & -\alpha\, a(1,2)\,(u_2 + a(1,2)\,x_3 + a(2,2)\,x_6 + a(3,2)\,x_9)\,x_{15} & a(1,2)\,(x_{15} - \alpha\,((u_2 + a(1,2)\,x_2 + a(2,2)\,x_5 + a(3,2)\,x_8)\,x_{15} - (u_2 + a(1,2)\,x_1 + a(\\
0 & 0 & 0 \\
0 & 0 & 0 \\
\alpha\, a(1,3)\,(u_3 + a(1,3)\,x_3 + a(2,3)\,x_6 + a(3,3)\,x_9)\,x_{17} & -\alpha\, a(1,3)\,(u_3 + a(1,3)\,x_3 + a(2,3)\,x_6 + a(3,3)\,x_9)\,x_{18} & a(1,3)\,(x_{18} - \alpha\,((u_3 + a(1,3)\,x_2 + a(2,3)\,x_5 + a(3,3)\,x_8)\,x_{18} - (u_3 + a(1,3)\,x_1 + a(\\
0 & 0 & 0 \\
0 & 0 & 0 \\
\alpha\, a(1,1)\,x_{11} & -\alpha\, a(1,1)\,x_{12} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\alpha\, a(1,2)\,x_{14} & -\alpha\, a(1,2)\,x_{15} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\alpha\, a(1,3)\,x_{17} & -\alpha\, a(1,3)\,x_{18} & 0
\end{array}
\right.
$$

The parameter values at equilibrium :

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \\ x_{18} \end{pmatrix} = \begin{pmatrix} 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{pmatrix}; \begin{pmatrix} a(1,1) & a(1,2) & a(1,3) \\ a(2,1) & a(2,2) & a(2,3) \\ a(3,1) & a(3,2) & a(3,3) \end{pmatrix} = \begin{pmatrix} 0. & .5 & 0 \\ 0. & 0 & .3333 \\ .1666 & 0 & 0 \end{pmatrix}; \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} .5 \\ .1666 \\ .3333 \end{pmatrix};$$

Calculating the eigenvalues for an arbitrary value of the markup adjustment parameter :

```
TraditionalForm[Eigenvalues[Jac]] /. α -> .2
```

Eigenvalues::eival : Unable to find all roots of the characteristic polynomial.

$\{1., 1., 1., -0.275569 + 0.385047\,I, -0.275569 - 0.385047\,I, 0.417564, -0.0966791, -0.0848531 + 0.00830558\,I, 0.0848531 - 0.00830558\,I, -6.95538 \times 10^{9}, 6.95538 \times 10^{9}, 6.46093 \times 10^{9} + 7.98417 \times 10^{10}\,I, 6.46093 \times 10^{9} - 7.98417 \times 10^{10}\,I, -6.46093 \times 10^{9} + 7.98419 \times 10^{10}\,I, -6.46093 \times 10^{9} - 7.98419 \times 10^{10}\,I, 0., 0., 0.\}$

The value $\alpha = .2$ generates 3 eigenvalues of 1. The same result applies for any value of α in the relevant range $0 < \alpha < 1$.

Section 5: An average rate of profit adjustment mechanism for markups

This document calculates the eigenvalues for the Jacobian of this system, and shows that for all values of the markup adjustment parameter (shown as α in this document), there is at least one eigenvalue of 1. Consequently the equilibrium of the system is marginally unstable. The calculations are done only for the example system used by Steedman; however it would be possible to modify the document to calculate the equilibrium of any given system and calculate its eigenvalues.

This document works with a 3 x3 input - output matrix :

```
A = Array[a, {3, 3}];
```

```
TraditionalForm[A]
```

$$\begin{pmatrix} a(1,1) & a(1,2) & a(1,3) \\ a(2,1) & a(2,2) & a(2,3) \\ a(3,1) & a(3,2) & a(3,3) \end{pmatrix}$$

```
I3 = IdentityMatrix[3];
```

The markup adjustment process is as shown in Chapter Seven :

$$\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix} = \begin{pmatrix} m_{1_{t+2}} \\ m_{2_{t+2}} \\ m_{3_{t+2}} \end{pmatrix} + \alpha \times \left(\text{I3} - \text{Extract}\left[(\begin{matrix} u_1 & u_2 & u_3 \end{matrix} + (\begin{matrix} p_{1_t} & p_{2_t} & p_{3_t} \end{matrix}) . \text{A}) . \text{Inverse}\left[\begin{pmatrix} m_{1_{t+1}} & 0 & 0 \\ 0 & m_{2_{t+1}} & 0 \\ 0 & 0 & m_{3_{t+1}} \end{pmatrix} \right] . \begin{pmatrix} m_{1_{t+1}} \\ m_{2_{t+1}} \\ m_{3_{t+1}} \end{pmatrix}, \{1, 1\} \right] \times \right.$$

$$\left. \left(\text{Extract}\left[(\begin{matrix} u_1 & u_2 & u_3 \end{matrix} + (\begin{matrix} p_{1_t} & p_{2_t} & p_{3_t} \end{matrix}) . \text{A}) . \begin{pmatrix} m_{1_{t+1}} \\ m_{2_{t+1}} \\ m_{3_{t+1}} \end{pmatrix}, \{1, 1\} \right] \right)^{-1} \times \begin{pmatrix} m_{1_{t+1}} & 0 & 0 \\ 0 & m_{2_{t+1}} & 0 \\ 0 & 0 & m_{3_{t+1}} \end{pmatrix} \right) .$$

$$\begin{pmatrix} m_{1_{t+2}} \\ m_{2_{t+2}} \\ m_{3_{t+2}} \end{pmatrix};$$

$$\text{TraditionalForm}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix} \right]$$

$$\begin{pmatrix}
\alpha \left(1 - \frac{m_{1_{t+1}} (a(1,1)\, p_{1_t} + a(1,2)\, p_{1_t} + a(1,3)\, p_{1_t} + a(2,1)\, p_{2_t} + a(2,2)\, p_{2_t} + a(2,3)\, p_{2_t} + a(3,1)\, p_{3_t} + a(3,2)\, p_{3_t} + a(3,3)\, p_{3_t} + u_1 + u_2 + u_3)}{m_{1_{t+1}} (a(1,1)\, p_{1_t} + a(2,1)\, p_{2_t} + a(3,1)\, p_{3_t} + u_1) + m_{2_{t+1}} (a(1,2)\, p_{1_t} + a(2,2)\, p_{2_t} + a(3,2)\, p_{3_t} + u_2) + m_{3_{t+1}} (a(1,3)\, p_{1_t} + a(2,3)\, p_{2_t} + a(3,3)\, p_{3_t} + u_3)} \right) m_{1_{t+2}} + m_{1_{t+2}} \\
\alpha \left(1 - \frac{m_{2_{t+1}} (a(1,1)\, p_{1_t} + a(1,2)\, p_{1_t} + a(1,3)\, p_{1_t} + a(2,1)\, p_{2_t} + a(2,2)\, p_{2_t} + a(2,3)\, p_{2_t} + a(3,1)\, p_{3_t} + a(3,2)\, p_{3_t} + a(3,3)\, p_{3_t} + u_1 + u_2 + u_3)}{m_{1_{t+1}} (a(1,1)\, p_{1_t} + a(2,1)\, p_{2_t} + a(3,1)\, p_{3_t} + u_1) + m_{2_{t+1}} (a(1,2)\, p_{1_t} + a(2,2)\, p_{2_t} + a(3,2)\, p_{3_t} + u_2) + m_{3_{t+1}} (a(1,3)\, p_{1_t} + a(2,3)\, p_{2_t} + a(3,3)\, p_{3_t} + u_3)} \right) m_{2_{t+2}} + m_{2_{t+2}} \\
\alpha \left(1 - \frac{m_{3_{t+1}} (a(1,1)\, p_{1_t} + a(1,2)\, p_{1_t} + a(1,3)\, p_{1_t} + a(2,1)\, p_{2_t} + a(2,2)\, p_{2_t} + a(2,3)\, p_{2_t} + a(3,1)\, p_{3_t} + a(3,2)\, p_{3_t} + a(3,3)\, p_{3_t} + u_1 + u_2 + u_3)}{m_{1_{t+1}} (a(1,1)\, p_{1_t} + a(2,1)\, p_{2_t} + a(3,1)\, p_{3_t} + u_1) + m_{2_{t+1}} (a(1,2)\, p_{1_t} + a(2,2)\, p_{2_t} + a(3,2)\, p_{3_t} + u_2) + m_{3_{t+1}} (a(1,3)\, p_{1_t} + a(2,3)\, p_{2_t} + a(3,3)\, p_{3_t} + u_3)} \right) m_{3_{t+2}} + m_{3_{t+2}}
\end{pmatrix}$$

Prices are determined by a markup on exogenous inputs and prices times the input - output matrix :

$$\begin{pmatrix} p_{1_{t+3}} \\ p_{2_{t+3}} \\ p_{3_{t+3}} \end{pmatrix} = \text{Transpose}\left[(\begin{matrix} u_1 & u_2 & u_3 \end{matrix} + (\begin{matrix} p_{1_{t+2}} & p_{2_{t+2}} & p_{3_{t+2}} \end{matrix}) . \text{A}) . \right.$$

$$\left. \left(\text{I3} + \text{DiagonalMatrix}\left[\left\{ \text{Extract}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}, \{1, 1\} \right], \text{Extract}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}, \{2, 1\} \right], \text{Extract}\left[\begin{pmatrix} m_{1_{t+3}} \\ m_{2_{t+3}} \\ m_{3_{t+3}} \end{pmatrix}, \{3, 1\} \right] \right\} \right] \right) \right];$$

```
TraditionalForm[ ( p1_{t+3} ; p2_{t+3} ; p3_{t+3} ) ]
```

$$\text{TraditionalForm}\left[\begin{pmatrix} p1_{t+3} \\ p2_{t+3} \\ p3_{t+3} \end{pmatrix}\right]$$

$$\begin{pmatrix}
(a(1,1)\,p_{1_{t+2}} + a(2,1)\,p_{2_{t+2}} + a(3,1)\,p_{3_{t+2}} + u_1)\left(\alpha\left(1 - \frac{m_{1_{t+1}}\,(a(1,1)\,p_{1_t} + a(1,2)\,p_{1_t} + a(1,3)\,p_{1_t} + a(2,1)\,p_{2_t} + a(2,2)\,p_{2_t} + a(2,3)\,p_{2_t} + a(3,1)\,p_{3_t} + a(3,2)\,p_{3_t} + a(3,3)\,p_{3_t} + u_1 + u_2 + u_3)}{m_{1_{t+1}}\,(a(1,1)\,p_{1_t} + a(2,1)\,p_{2_t} + a(3,1)\,p_{3_t} + u_1) + m_{2_{t+1}}\,(a(1,2)\,p_{1_t} + a(2,2)\,p_{2_t} + a(3,2)\,p_{3_t} + u_2) + m_{3_{t+1}}\,(a(1,3)\,p_{1_t} + a(2,3)\,p_{2_t} + a(3,3)\,p_{3_t} + u_3)}\right) m_{1_{t+2}} + m_{1_{t+2}} + 1\right) \\
(a(1,2)\,p_{1_{t+2}} + a(2,2)\,p_{2_{t+2}} + a(3,2)\,p_{3_{t+2}} + u_2)\left(\alpha\left(1 - \frac{m_{2_{t+1}}\,(a(1,1)\,p_{1_t} + a(1,2)\,p_{1_t} + a(1,3)\,p_{1_t} + a(2,1)\,p_{2_t} + a(2,2)\,p_{2_t} + a(2,3)\,p_{2_t} + a(3,1)\,p_{3_t} + a(3,2)\,p_{3_t} + a(3,3)\,p_{3_t} + u_1 + u_2 + u_3)}{m_{1_{t+1}}\,(a(1,1)\,p_{1_t} + a(2,1)\,p_{2_t} + a(3,1)\,p_{3_t} + u_1) + m_{2_{t+1}}\,(a(1,2)\,p_{1_t} + a(2,2)\,p_{2_t} + a(3,2)\,p_{3_t} + u_2) + m_{3_{t+1}}\,(a(1,3)\,p_{1_t} + a(2,3)\,p_{2_t} + a(3,3)\,p_{3_t} + u_3)}\right) m_{2_{t+2}} + m_{2_{t+2}} + 1\right) \\
(a(1,3)\,p_{1_{t+2}} + a(2,3)\,p_{2_{t+2}} + a(3,3)\,p_{3_{t+2}} + u_3)\left(\alpha\left(1 - \frac{m_{3_{t+1}}\,(a(1,1)\,p_{1_t} + a(1,2)\,p_{1_t} + a(1,3)\,p_{1_t} + a(2,1)\,p_{2_t} + a(2,2)\,p_{2_t} + a(2,3)\,p_{2_t} + a(3,1)\,p_{3_t} + a(3,2)\,p_{3_t} + a(3,3)\,p_{3_t} + u_1 + u_2 + u_3)}{m_{1_{t+1}}\,(a(1,1)\,p_{1_t} + a(2,1)\,p_{2_t} + a(3,1)\,p_{3_t} + u_1) + m_{2_{t+1}}\,(a(1,2)\,p_{1_t} + a(2,2)\,p_{2_t} + a(3,2)\,p_{3_t} + u_2) + m_{3_{t+1}}\,(a(1,3)\,p_{1_t} + a(2,3)\,p_{2_t} + a(3,3)\,p_{3_t} + u_3)}\right) m_{3_{t+2}} + m_{3_{t+2}} + 1\right)
\end{pmatrix}$$

The stability of the linearised version of the system is calculated about its equilibrium point. Mapping all price and markup variables to a standard variable :

$p1_t = x_1$;
$p1_{t+1} = x_2$;
$p1_{t+2} = x_3$;
$p2_t = x_4$;
$p2_{t+1} = x_5$;
$p2_{t+2} = x_6$;
$p3_t = x_7$;
$p3_{t+1} = x_8$;
$p3_{t+2} = x_9$;

```
m1t = x10;
m1t+1 = x11;
m1t+2 = x12;
m2t = x13;
m2t+1 = x14;
m2t+2 = x15;
m3t = x16;
m3t+1 = x17;
m3t+2 = x18;
```

TraditionalForm[$p_{1_{t+3}}$]

$$(u_1 + a(1, 1)\, x_3 + a(2, 1)\, x_6 + a(3, 1)\, x_9) \left(\alpha \left(1 - \frac{(u_1 + u_2 + u_3 + a(1, 1)\, x_1 + a(1, 2)\, x_1 + a(1, 3)\, x_1 + a(2, 1)\, x_4 + a(2, 2)\, x_4 + a(2, 3)\, x_4 + a(3, 1)\, x_7 + a(3, 2)\, x_7 + a(3, 3)\, x_7)\, x_{11}}{(u_1 + a(1, 1)\, x_1 + a(2, 1)\, x_4 + a(3, 1)\, x_7)\, x_{11} + (u_2 + a(1, 2)\, x_1 + a(2, 2)\, x_4 + a(3, 2)\, x_7)\, x_{14} + (u_3 + a(1, 3)\, x_1 + a(2, 3)\, x_4 + a(3, 3)\, x_7)\, x_{17}}\right) x_{12} + x_{12} + 1\right)$$

TraditionalForm[$p_{2_{t+3}}$]

$$(u_2 + a(1, 2)\, x_3 + a(2, 2)\, x_6 + a(3, 2)\, x_9) \left(\alpha \left(1 - \frac{(u_1 + u_2 + u_3 + a(1, 1)\, x_1 + a(1, 2)\, x_1 + a(1, 3)\, x_1 + a(2, 1)\, x_4 + a(2, 2)\, x_4 + a(2, 3)\, x_4 + a(3, 1)\, x_7 + a(3, 2)\, x_7 + a(3, 3)\, x_7)\, x_{14}}{(u_1 + a(1, 1)\, x_1 + a(2, 1)\, x_4 + a(3, 1)\, x_7)\, x_{11} + (u_2 + a(1, 2)\, x_1 + a(2, 2)\, x_4 + a(3, 2)\, x_7)\, x_{14} + (u_3 + a(1, 3)\, x_1 + a(2, 3)\, x_4 + a(3, 3)\, x_7)\, x_{17}}\right) x_{15} + x_{15} + 1\right)$$

TraditionalForm[$p_{3_{t+3}}$]

$$(u_3 + a(1, 3)\, x_3 + a(2, 3)\, x_6 + a(3, 3)\, x_9) \left(\alpha \left(1 - \frac{(u_1 + u_2 + u_3 + a(1, 1)\, x_1 + a(1, 2)\, x_1 + a(1, 3)\, x_1 + a(2, 1)\, x_4 + a(2, 2)\, x_4 + a(2, 3)\, x_4 + a(3, 1)\, x_7 + a(3, 2)\, x_7 + a(3, 3)\, x_7)\, x_{17}}{(u_1 + a(1, 1)\, x_1 + a(2, 1)\, x_4 + a(3, 1)\, x_7)\, x_{11} + (u_2 + a(1, 2)\, x_1 + a(2, 2)\, x_4 + a(3, 2)\, x_7)\, x_{14} + (u_3 + a(1, 3)\, x_1 + a(2, 3)\, x_4 + a(3, 3)\, x_7)\, x_{17}}\right) x_{18} + x_{18} + 1\right)$$

TraditionalForm[$m_{1_{t+3}}$]

$$\alpha\left(1-\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)x_{11}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17}}\right)x_{12}+x_{12}$$

TraditionalForm[$m_{2_{t+3}}$]

$$\alpha\left(1-\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)x_{14}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17}}\right)x_{15}+x_{15}$$

TraditionalForm[$m_{3_{t+3}}$]

$$\alpha\left(1-\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)x_{17}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17}}\right)x_{18}+x_{18}$$

The linearised model in the vicinity of the equilibrium point is thus :

f = {x_2, x_3, $p_{1_{t+3}}$, x_5, x_6, $p_{2_{t+3}}$, x_8, x_9, $p_{3_{t+3}}$, x_{11}, x_{12}, $m_{1_{t+3}}$, x_{14}, x_{15}, $m_{2_{t+3}}$, x_{17}, x_{18}, $m_{3_{t+3}}$};

MatrixForm[f]

$$\begin{pmatrix} x_2 \\ x_3 \\ (u_1 + a[1, 1]\, x_3 + a[2, 1]\, x_6 + a[3, 1]\, x_9)\,\left(1 + x_{12} + \alpha\, x_{12}\, \left(1 - \frac{(u_1+u_2+u_3+a[1,1]\, x_1+a[1,2]\, x_1+a[1,3]\, x_1+a[2,1]\, x_4+a[2,2]\, x_4+a[2,3]\, x_4+a[3,1]\, x_7+a[3,2]\, x_7+a[3,3]\, x_7)\, x_{11}}{(u_1+a[1,1]\, x_1+a[2,1]\, x_4+a[3,1]\, x_7)\, x_{11}+(u_2+a[1,2]\, x_1+a[2,2]\, x_4+a[3,2]\, x_7)\, x_{14}+(u_3+a[1,3]\, x_1+a[2,3]\, x_4+a[3,3]\, x_7)\, x_{17}}\right)\right) \\ x_5 \\ x_6 \\ (u_2 + a[1, 2]\, x_3 + a[2, 2]\, x_6 + a[3, 2]\, x_9)\,\left(1 + x_{15} + \alpha\, x_{15}\, \left(1 - \frac{(u_1+u_2+u_3+a[1,1]\, x_1+a[1,2]\, x_1+a[1,3]\, x_1+a[2,1]\, x_4+a[2,2]\, x_4+a[2,3]\, x_4+a[3,1]\, x_7+a[3,2]\, x_7+a[3,3]\, x_7)\, x_{14}}{(u_1+a[1,1]\, x_1+a[2,1]\, x_4+a[3,1]\, x_7)\, x_{11}+(u_2+a[1,2]\, x_1+a[2,2]\, x_4+a[3,2]\, x_7)\, x_{14}+(u_3+a[1,3]\, x_1+a[2,3]\, x_4+a[3,3]\, x_7)\, x_{17}}\right)\right) \\ x_8 \\ x_9 \\ (u_3 + a[1, 3]\, x_3 + a[2, 3]\, x_6 + a[3, 3]\, x_9)\,\left(1 + x_{18} + \alpha\, \left(1 - \frac{(u_1+u_2+u_3+a[1,1]\, x_1+a[1,2]\, x_1+a[1,3]\, x_1+a[2,1]\, x_4+a[2,2]\, x_4+a[2,3]\, x_4+a[3,1]\, x_7+a[3,2]\, x_7+a[3,3]\, x_7)\, x_{17}}{(u_1+a[1,1]\, x_1+a[2,1]\, x_4+a[3,1]\, x_7)\, x_{11}+(u_2+a[1,2]\, x_1+a[2,2]\, x_4+a[3,2]\, x_7)\, x_{14}+(u_3+a[1,3]\, x_1+a[2,3]\, x_4+a[3,3]\, x_7)\, x_{17}}\right)\, x_{18}\right) \\ x_{11} \\ x_{12} \\ x_{12} + \alpha\, x_{12}\, \left(1 - \frac{(u_1+u_2+u_3+a[1,1]\, x_1+a[1,2]\, x_1+a[1,3]\, x_1+a[2,1]\, x_4+a[2,2]\, x_4+a[2,3]\, x_4+a[3,1]\, x_7+a[3,2]\, x_7+a[3,3]\, x_7)\, x_{11}}{(u_1+a[1,1]\, x_1+a[2,1]\, x_4+a[3,1]\, x_7)\, x_{11}+(u_2+a[1,2]\, x_1+a[2,2]\, x_4+a[3,2]\, x_7)\, x_{14}+(u_3+a[1,3]\, x_1+a[2,3]\, x_4+a[3,3]\, x_7)\, x_{17}}\right) \\ x_{14} \\ x_{15} \\ x_{15} + \alpha\, x_{15}\, \left(1 - \frac{(u_1+u_2+u_3+a[1,1]\, x_1+a[1,2]\, x_1+a[1,3]\, x_1+a[2,1]\, x_4+a[2,2]\, x_4+a[2,3]\, x_4+a[3,1]\, x_7+a[3,2]\, x_7+a[3,3]\, x_7)\, x_{14}}{(u_1+a[1,1]\, x_1+a[2,1]\, x_4+a[3,1]\, x_7)\, x_{11}+(u_2+a[1,2]\, x_1+a[2,2]\, x_4+a[3,2]\, x_7)\, x_{14}+(u_3+a[1,3]\, x_1+a[2,3]\, x_4+a[3,3]\, x_7)\, x_{17}}\right) \\ x_{17} \\ x_{18} \\ x_{18} + \alpha\, \left(1 - \frac{(u_1+u_2+u_3+a[1,1]\, x_1+a[1,2]\, x_1+a[1,3]\, x_1+a[2,1]\, x_4+a[2,2]\, x_4+a[2,3]\, x_4+a[3,1]\, x_7+a[3,2]\, x_7+a[3,3]\, x_7)\, x_{17}}{(u_1+a[1,1]\, x_1+a[2,1]\, x_4+a[3,1]\, x_7)\, x_{11}+(u_2+a[1,2]\, x_1+a[2,2]\, x_4+a[3,2]\, x_7)\, x_{14}+(u_3+a[1,3]\, x_1+a[2,3]\, x_4+a[3,3]\, x_7)\, x_{17}}\right)\, x_{18} \end{pmatrix}$$

The Jacobian of this system determines its stability properties :

Jac = Table[D[f[[i]], x_j], {i, 1, 18}, {j, 1, 18}];

TraditionalForm[Jac]

$$0$$
$$0$$
$$\alpha\,(u_1 + a(1,1)\,x_3 + a(2,1)\,x_6 + a(3,1)\,x_9)\,x_{12}\,\Big(\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)\,x_{11}\,(a(1,1)x_{11}+a(1,2)x_{14}+a(1,3)x_{17})}{((u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17})^2} - \frac{(a(1,1)+a(1,2)+a(1,3}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a}$$
$$0$$
$$0$$
$$\alpha\,(u_2 + a(1,2)\,x_3 + a(2,2)\,x_6 + a(3,2)\,x_9)\,x_{15}\,\Big(\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)\,x_{14}\,(a(1,1)x_{11}+a(1,2)x_{14}+a(1,3)x_{17})}{((u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17})^2} - \frac{(a(1,1)+a(1,2)+a(1,3}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a}$$
$$0$$
$$0$$
$$\alpha\,(u_3 + a(1,3)\,x_3 + a(2,3)\,x_6 + a(3,3)\,x_9)\,\Big(\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)\,x_{17}\,(a(1,1)x_{11}+a(1,2)x_{14}+a(1,3)x_{17})}{((u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17})^2} - \frac{(a(1,1)+a(1,2)+a(1,3))\,x_{17}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)}$$
$$0$$
$$0$$
$$\alpha\,x_{12}\,\Big(\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)\,x_{11}\,(a(1,1)x_{11}+a(1,2)x_{14}+a(1,3)x_{17})}{((u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17})^2} - \frac{(a(1,1)+a(1,2)+a(1,3))\,x_{11}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a}$$
$$0$$
$$0$$
$$\alpha\,x_{15}\,\Big(\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)\,x_{14}\,(a(1,1)x_{11}+a(1,2)x_{14}+a(1,3)x_{17})}{((u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17})^2} - \frac{(a(1,1)+a(1,2)+a(1,3))\,x_{14}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a}$$
$$0$$
$$0$$
$$\alpha\,\Big(\frac{(u_1+u_2+u_3+a(1,1)x_1+a(1,2)x_1+a(1,3)x_1+a(2,1)x_4+a(2,2)x_4+a(2,3)x_4+a(3,1)x_7+a(3,2)x_7+a(3,3)x_7)\,x_{17}\,(a(1,1)x_{11}+a(1,2)x_{14}+a(1,3)x_{17})}{((u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)x_4+a(3,3)x_7)x_{17})^2} - \frac{(a(1,1)+a(1,2)+a(1,3))\,x_{17}}{(u_1+a(1,1)x_1+a(2,1)x_4+a(3,1)x_7)x_{11}+(u_2+a(1,2)x_1+a(2,2)x_4+a(3,2)x_7)x_{14}+(u_3+a(1,3)x_1+a(2,3)}$$

The parameter values at equilibrium :

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \\ x_{18} \end{pmatrix} = \begin{pmatrix} 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{pmatrix}; \begin{pmatrix} a(1,1) & a(1,2) & a(1,3) \\ a(2,1) & a(2,2) & a(2,3) \\ a(3,1) & a(3,2) & a(3,3) \end{pmatrix} = \begin{pmatrix} 0. & .5 & 0 \\ 0. & 0 & .3333 \\ .1666 & 0 & 0 \end{pmatrix}; \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} .5 \\ .1666 \\ .3333 \end{pmatrix};$$

Calculating the eigenvalues for an arbitrary value of the markup adjustment parameter :

Eigenvalues[Jac] /. α -> .2

{0, 0, 0, 0, 0., 0., 0., 0., 0., 2.592×10^{-17}, 0.276393, 0.276393, 0.454204, 0.723607, 0.723607, 1., -0.227102 - 0.393353 I,
-0.227102 + 0.393353 I}

The value α = .2 generates one eigenvalue of 1. The same result applies for any value of α in the relevant range $0 < \alpha < 1$.

Appendix C

This Appendix provides extracts from government and legislative documents in the United States of America, Europe, and Australia, which are all motivated by the supposed desirability of zero or near zero government deficits. They give a strong indication of the extent to which the view that the role of the government budget is to stabilise the economy has been superceded by the view that any budget deficit greater than zero is necessarily bad. From the perspective of Minsky's Financial Instability Hypothesis, such naivety is likely to amplify economic instability, and reduce the effectiveness of the government sector as a bulkward against debt deflation.

1 United States of America

These three bills are indicative of the content of the twelve bills which, as noted in the Conclusion, were before the United States House of Representatives which were attempting to enshrine, either in legislation or in an amendment to the Constitution, the principle that the Budget should be in balance at all times. The current Bills before the House of Representatives can be found at http://thomas.loc.gov/home/c105query.html.

1.1 H.J.RES.11

> Proposing an amendment to the Constitution to provide for a balanced budget for the United States Government and for greater accountability in the enactment of tax legislation.

Resolved by the Senate and House of Representatives of the United States of America in Congress assembled (two-thirds of each House concurring therein), That the following article is proposed as an amendment to the Constitution of the United States, which shall be valid to all intents and purposes as part of the Constitution if ratified by the legislatures of three-fourths of the several States within seven years after its submission to the States for ratification:

`Article-

`SECTION 1. Prior to each fiscal year, the Congress and the President shall agree on an estimate of total receipts for that fiscal year by enactment into law of a joint resolution devoted solely to that subject. Total outlays for that year shall not exceed the level of estimated receipts set forth in such joint resolution, unless three-fifths of the total membership of each House of Congress shall provide, by a rollcall vote, for a specific excess of outlays over estimated receipts.

SECTION 2. Whenever actual outlays exceed actual receipts for any fiscal year, the Congress shall, in the ensuing fiscal year, provide by law for the repayment of such excess. The public debt of the United States shall not be increased unless three-fifths of the total membership of each House shall provide by law for such an increase by a rollcall vote.

SECTION 3. Prior to each fiscal year, the President shall transmit to the Congress a proposed budget for the United States Government for that fiscal year in which total outlays do not exceed total receipts.

SECTION 4. No bill to increase revenue shall become law unless approved by a majority of the total membership of each House by a rollcall vote.

SECTION 5. The provisions of this article are waived for any fiscal year in which a declaration of war is in effect.

SECTION 6. Total receipts shall include all receipts of the United States except those derived from borrowing. Total outlays shall include all outlays of the United States except for those for repayment of debt principal.

SECTION 7. This article shall take effect beginning with the second fiscal year after its ratification.'.

1.2 H.R.113

A BILL To require that the President transmit to Congress, that the congressional Budget Committees report, and that the Congress consider a balanced budget for each fiscal year.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,

TITLE I--AMENDMENT TO TITLE 31, UNITED STATES CODE

SEC. 101. SUBMISSION OF BALANCED BUDGET BY THE PRESIDENT.

Section 1105 of title 31, United States code, is amended by inserting at the end the following new subsection:

`(g)(1) Except as provided by paragraph (2), any budget submitted to Congress pursuant to subsection (a) for the ensuing fiscal year shall not be in deficit.

`(2) For any fiscal year with respect to which the President determines that it is infeasible to submit a budget in compliance with paragraph (1), the

President shall submit on the same day two budgets, one of which shall be in compliance with paragraph (1), together with written reasons in support of that determination.'.

TITLE II--AMENDMENT TO CONGRESSIONAL BUDGET ACT OF 1974

SEC. 201. REPORTING OF BALANCED BUDGETS BY COMMITTEES ON THE BUDGET OF THE HOUSE OF REPRESENTATIVES AND SENATE.

Section 301 of the Congressional Budget Act of 1974 is amended by inserting at the end the following new subsection:

`(j) Reporting of Balanced Budgets

`(1) Except as provided by paragraph (2), the concurrent resolution on the budget for a fiscal year referred to in subsection (a) as reported by the Committee on the Budget of each House shall not be in deficit.

`(2) For any fiscal year with respect to which the Committee on the Budget of either House determines that it is infeasible to report a concurrent resolution on the budget in compliance with paragraph (1) and includes written reasons in support of that determination in its report accompanying a concurrent resolution on the budget, the committee shall report two concurrent resolutions on the budget, one of which shall be in compliance with paragraph (1).

`(3) Each concurrent resolution on the budget reported by the Committee on the Budget of either House shall contain reconciliation directives described in section 310 necessary to effectuate the provisions and requirements of such resolution.'.

SEC. 202. PROCEDURE IN THE HOUSE OF REPRESENTATIVES.

Section 305(a) of the Congressional Budget Act of 1974 is amended by inserting at the end the following:

`(8)(A) If the Committee on Rules of the House of Representatives reports any rule or order providing for the consideration of any concurrent resolution on the budget for a fiscal year, then it shall also, within the same rule or order, provide for-

`(i) the consideration of the text of any concurrent resolution on the budget for that fiscal year reported by the Committee on the Budget of the House of Representatives pursuant to section 301(j); and

`(ii) the consideration of the text of each concurrent resolution on the budget as introduced by the Majority Leader pursuant to subparagraph (B);

and such rule or order shall assure that a separate vote occurs on each such budget .`(B) The Majority Leader of the House of Representatives shall introduce a concurrent resolution on the budget reflecting, without substantive revision, each budget submitted by the President pursuant to section 1105(g) of title 31, United States Code, as soon as practicable after its submission.'.

SEC. 203. PROCEDURE IN THE SENATE.

Section 305(b) of the Congressional Budget Act of 1974 is amended by inserting at the end the following:

`(7) Notwithstanding any other rule, it shall always be in order in the Senate to consider an amendment to a concurrent resolution on the budget for a fiscal year comprising the text of any budget submitted by the President for that fiscal year as described in section 1105(g)(1) of title 31, United States

Code, and, whenever applicable, an amendment comprising the text of any other budget submitted by the President for that fiscal year as described in section 1105(g)(2) of title 31, United States Code.'.

TITLE III--EFFECTIVE DATE

SEC. 301. EFFECTIVE DATE.

This Act and the amendments made by it shall become effective for fiscal year 1999 and shall be fully reflected in the fiscal year 1999 budget submitted by the President as required by section 1105(a) of title 31, United States Code.

2 Europe

The Maastricht Treaty, while not as inflexible as the US Bills in that it requires budget deficits of member nations to be no more than 3 per cent of GDP, is far more likely to be enacted. The relevant Articles concerning limitations on government deficits are extracted below. The full document can be found at http://europa.eu.int/en/record/mt/top.html.

2.1 ARTICLE 104c

1. Member States shall avoid excessive governmental deficits.

2. The Commission shall monitor the development of the budgetary situation and of the stock of government debt in the Member States with a view to identifying gross errors. In particular it shall examine compliance with budgetary discipline on the basis of the following two criteria:

(a) whether the ratio of the planned or actual government deficit to gross domestic product exceeds a reference value, unless either the ratio has declined substantially and continuously and reached a level that comes close to the reference valu; or, alternatively, the excess over the reference value is only exceptional and temporary and the ratio remains close to the reference value;

(b) whether the ratio of government debt to gross domestic product exceeds a reference value, unless the ratio is sufficiently diminishing and approaching the reference value at a satisfactory pace.

The reference values are specified in the Protocol on the excessive deficit procedure annexed to this Treaty.

3. If a Member State does not fulfil the requirements under one or both of these criteria, the Commission shall prepare a report. The report of the Commission shall also take into account whether the government deficit exceeds government investment expenditure and take into account all other relevant factors, including the medium term economic and budgetary position of the Member State.

The Commission may also prepare a report if, notwithstanding the fulfillment of the requirement under the criteria, it is of the opinion that there is a risk of an excessive deficit in a Member State.

4. The Committee provided for in Article 109c shall formulate an opinion on the report of the Commission.

5. If the Commission considers that an excessive deficit in a Member State exists or may occur, the Commission shall address an opinion to the Council.

6. The Council shall, acting by a qualified majority on a recommendation from the Commission, and having considered any observations which the Member State concerned may wish to make, decide after an overall assessment whether an excessive deficit exists.

7. Where the existence of an excessive deficit is decided according to paragraph 6, the Council shall make recommendations to the Member State concerned with a view to bringing that situation to an end within a given period. Subject to the provisions of paragraph 8, these recommendations shall not be made public.

8. Where it establishes that there has been no effective action in response to its recommendations within the period laid down, the Council may make its recommendations public.

9. If a Member State persists in failing to put into practice the recommendations of the Council, the Council may decide to give notice to the Member State to take, within a specified time limit, measures for the deficit reduction which is judged necessary by the Council in order to remedy the situation. In such a case, the Council may request the Member State concerned to submit reports in accordance with a specific timetable in order to examine the adjustment efforts of that Member State.

10. The right to bring actions provided for in Articles 169 and 170 may not be exercised within the framework of paragraphs 1 to 9 of this Article.

11. As long as a Member State fails to comply with a decision taken in accordance with paragraph 9, the Council may decide to apply the following measures: to require the Member State concerned to publish additional information, to be specified by the Council, before issuing bonds and securities; to invite the European Investment Bank to reconsider its lending policy towards the Member State concerned; to require the Member State concerned to make a non-interest-bearing deposit of an appropriate size with the Community until

the excessive deficit has, in the view of the Council, been corrected; to impose fines of an appropriate size.

The President of the Council shall inform the European Parliament of the decisions taken.

12. The Council shall abrogate some or all of its decisions referred to in paragraphs 6 to 9 and 11 to the extent that the excessive deficit in the Member State concerned has, in the view of the Council, been corrected. If the Council has previously made public recommendations, it shall, as soon as the decision under paragraph 8 has been abrogated, make a public statement that an excessive deficit in the Member State concerned no longer exists.

13. When taking the decisions referred to in paragraphs 7 to 9, 11 and 12, the Council shall act on a recommendation from the Commission by a majority of two thirds of the votes of its members weighted in accordance with Article 148(2), excluding the votes of the representative of the Member State concerned.

14. Further provisions relating to the implementation of the procedure described in this Article are set out in the Protocol on the excessive deficit procedure annexed to this Treaty.

The Council shall, acting unanimously on a proposal from the Commission and after consulting the European Parliament and the ECB, adopt the appropriate provisions which shall then replace the said Protocol.

Subject to the other provisions of this paragraph the Council shall, before 1 January 1994, acting by a qualified majority on a proposal from the Commission and after consulting the European Parliament, lay down detailed rules and definitions for the application of the provisions of the said Protocol.

2.2 ARTICLE 105

1. The primary objective of the ESCB shall be to maintain price stability. Without prejudice to the objective of price stabilty, the ESCB shall support the general economic policies in the Community with a view to contributing to the achievement of the objectives of the Community as laid down in Article 2. The ESCB shall act in accordance with the principle of an open market economy with free competition, favouring an efficient allocation of resources, and in compliance with the principles set out in Article 3a.

2. The basic tasks to be carried out through the ESCB shall be: to define and implement the monetary policy of the Community; to conduct foreign exchange operations consistent with the provisions of Article 109; to hold and manage the official foreign reserves of the Member States; to promote the smooth operation of payment systems.

3. The third indent of paragraph 2 shall be without prejudice to the holding and management by the government of Member States of foreign exchange working balances.

4. The ECB shall be consulted: on any proposed Community act in its fields of competence; by national authorities regarding any draft legislative provision in its fields of competence, but within the limits and under the conditions set out by the Council in accordances with the procedure laid down in Article 106(6). The ECB may submit opinions to the appropriate Commu-

nity institutions or bodies or to national authorities on matters in its fields of competence.

5. The ESCB shall contribute to the smooth conduct of policies pursued by the competent authorities relating to the prudential supervision of credit institutions and the stability of the financial system.

6. The Council may, acting unanimously on a proposal from the Commission and after consulting the ECB and after receiving the assent of the European Parliament, confer upon the ECB specific tasks concerning policies relating to the prudential supervision of credit institutions and other financial institutions with the exception of insurance undertakings.

2.3 PROTOCOL ON THE EXCESSIVE DEFICIT PROCEDURE

THE HIGH CONTRACTING PARTIES

DESIRING to lay down the details of the excessive deficit procedure referred to in Article 104c of the treaty establishing the European Community,

HAVE AGREED upon the following provisions, which shall be annexed to the Treaty establishing the European Community:

ARTICLE 1

The reference values referred to in Article 104c(2) of this Treaty are:

3% for the ratio of the planned or actual government deficit to gross domestic product at market prices; 60% for the ratio of government debt to gross domestic product at market prices.

ARTICLE 2

In Article 104c of this Treaty and in this Protocol:

government means general government, that is central government, regional or local government and social security funds, to the exclusion of commercial operations, as defined in the European System of Integrated Economic Accounts;

deficit means net borrowing as defined in the European System of Integrated Economic Accounts;

investment means gross fixed capital formation as defined in the European System of Integrated Economic Accounts;

debt means total gross debt at nominal value outstanding at the end of the year and consolidated between and within the sectors of general government as defined in the first indent.

ARTICLE 3

In order to ensure the effectiveness of the excessive deficit procedure, the governments of the Member States shall be responsible under this procedure for the deficits of general government as defined in the first indent of Article 2. The Member States shall ensure that national procedures in the Budgetary area enable them to meet their obligations in this area deriving from this Treaty. The Member States shall report their planned and actual deficits and the levels of their debt promptly and regularly to the Commission.

ARTICLE 4.

The statistical data to be used for the application of this Protocol shall be provided by the Commission.

3 Australia

While the United States Billsand Europe's Maastricht Treaty are still at present proposals, the Australian Government has already commenced its attempt to achieve a balanced Budget. The following sections provide an excerpt from the Treasurer's Budget Speech, and an extract from the Supporting Documents to the Budget. The documents are available at http://www.treasury.gov.au/publications/budget96/budget96.htm.

3.1 BUDGET SPEECH 1996/97

DELIVERED ON 20 AUGUST 1996 ON THE SECOND READING

OF THE APPROPRIATION BILL (NO. 1) 199697

BY

THE HONOURABLE PETER COSTELLO, MP

TREASURER OF THE COMMONWEALTH OF AUSTRALIA

Mr Speaker, I move that the Bill now be read a second time.

Tonight I announce a programme

- for families
- for small business
- for older Australians
- for major improvements in health care and, importantly,
- a programme to repair the nation's finances and secure our future.

This Budget implements the key election commitments of the Coalition and it does so as part of a responsible economic strategy.

The focus is savings — savings for investment, sustainable growth and real jobs. This Budget turns around failure and sets us on a winning course.

BUDGET REPAIR

The budget outcome for the past year was a deficit of $10.3 billion. If we took no corrective measures it would be $9.6 billion this year. We would still have an underlying deficit of 2 per cent of GDP even though the economy has had five years of growth. Debt would be increasing and Australia would be dangerously exposed to shifts in the international outlook or sentiment.

In periods of growth we must put away savings for the downturns. But far from saving, the previous Government kept racheting up our debts — spending money it didn't have.

Our predecessors had Australia on a path of deficit and debt to the next century.

Make no mistake, this path would only make future choices harder, future possibilities bleaker and rob Australians of the future opportunities they deserve.

Our Government could not stand back and ignore the problem. Although we did not create it, we will take the responsibility to fix it.

The measures I announce tonight will reduce the underlying deficit by around $4 billion this year and $7.2 billion over two years. These measures will balance the budget over the term of this Parliament. These are the net effect on the budget bottom line after the introduction of new policy to meet the Coalition's election commitments. These measures represent a historic turnaround in Commonwealth finances. In 199798, outlays will have fallen around 1.8 per cent of GDP, while revenues will have risen only 0.2 per cent of GDP....

3.1 Budget Supporting Papers

The Economic Outlook

Australia's economic outlook is for another year of strong growth. The international economy remains supportive, the underlying economic fundamentals continue to be favourable and policy is directed at raising Australia's longterm growth potential. Of particular importance for the 199697 outlook, inflation is subdued, asset prices are stable and business and consumer confidence remain positive, while corporate profits and the net rate of return on investment are both healthy. In addition, the constraining influences of the housing and stock cycles felt through 199596 appear to be waning. It is against this favourable economic backdrop that the longerterm benefits of fiscal consolidation are being sought. Those longerterm benefits of structural improvements in the budget balance are widely accepted. There is less certainty about the net shortterm impacts. In the short term, the direct effects of changes in real Commonwealth expenditure and net transfer payments to individuals and other governments are reasonably clear, but the indirect effects on confidence in the business and household sectors and on financial markets are more difficult to assess. To the extent that private sector activity is boosted because confidence is improved, or market interest rates or the exchange rate are lower than they otherwise would be, the indirect effects provide offsetting influences to the direct contractionary impact of the consolidation. The forecasts presented here incorporate an assumption that possible confidence effects, while significant, do not fully offset the direct shortterm effects of the measures, resulting in a small net contractionary impact on activity in 1996/97...

Glossary

Term	Definition	Formula
Common Terms		
Y	Output	
Y_t	Output at time t	
Y_{t-1}	Output at time t-1	
Y_{t-2}	Output at time t-2	
Y_{t+1}	Output at time t+1	
Y_{t+2}	Output at time t+2	
Y_{t+3}	Output at time t+3	
Chapter Two		
α	Average propensity to consume	$Y_t = \left(a+\beta\right) \cdot Y_{t-1} - \beta \cdot Y_{t-2}$
β	Desired incremental capital to output ratio	
π	Prospective gross profit	$\pi = k \cdot \varepsilon$
k	Capital available to invest for a given capitalist	
ε	Expected rate of return on investment	
g	Prospective net gain	$g = \pi - \left(\rho + \sigma\right) \cdot k$
ρ	Market rate of return	
σ	Perceived risk of project	
S	Ex-post savings	$(S-I)+(T-G) \equiv 0$
I	Ex-post investment	
T	Ex-post taxation	
G	Ex-post government spending	
M	Exogenous money supply	$M = L(i)$
L()	Liquidity function demand for money	
I_x	Investment goods output	$I_x = C(i)$
C()	Investment function	
S()	Savings function	$I_x = S(I)$
i	Interest rate	
I	Income	

Chapter Three		
α	Average propensity to consume	$Y_t = (\alpha + \beta) \cdot Y_{t-1} - \beta \cdot Y_{t-2}$
β	Desired incremental capital to output ratio	
S	Ex-post savings	$S = S(Y)$
S()	Savings function	
I	Ex-post investment	$I = I\left(\frac{d}{dt}Y\right)$
I()	Investment function	
C_p	Actual incremental capital to output ratio	$C_p = \frac{I}{\delta Y}$
G	Actual rate of growth	$G = \frac{\delta Y}{Y}$
s	Savings to output ratio	$s = \frac{S}{Y}$
G_w	Warranted rate of growth	
C	Desired incremental capital to output ratio	
G_m	"Natural" or maximum rate of growth	
δY	Change in output	$\delta Y = Y_n - Y_{n-1}$
c	Desired incremental capital to output ratio	$I_t = c \cdot (Y_{t-1} - Y_{t-2})$
I_t	Actual Investment in period t	$I_t \equiv K_t - K_{t-1}$
K_t	Capital stock at the end of period t	
K_{t-1}	Capital stock at the end of period t-1	
v	The accelerator, or capital to output ratio	$K_t = v \cdot Y_t$
C_t	Consumption in period t	$C_t = (1-s) \cdot Y_{t-1}$ or $= (1-s) \cdot Y_t$
S_t	Savings in period t	$S_t = Y_t - (1-s) \cdot Y_{t-1}$ or $= s \cdot Y_t$
g	Actual rate of growth	$g = \frac{s}{v-1}$ or $g = \frac{s}{v-s}$
$x_1(t)$	Output in period t	See Appendix A for derivation
$x_2(t)$	Output in period t+1	
$x_3(t)$	Output in period t+2	
Norm(x,y)	A function which returns a random variable with mean x and standard deviation y	

if(x,y,z)	A function which returns y if x is true, and z otherwise	
	Chapter Four	
E	The state of expectations	$\frac{1}{E}\frac{dE}{dt} = \frac{dY}{dt}$ or $= \pi$
Y	The level of output	
π	The rate of profit	
p	The number of prey	$\frac{1}{p}\frac{dp}{dt} = a - b \cdot P$
P	The number of predators	$\frac{1}{P}\frac{dP}{dt} = -c + d \cdot p$
a,b,c,d	Constants in the predator-prey equations	
L	The level of employment	$L = \frac{Y}{a}$
Y	The level of output	
a	The level of labour productivity	
α	The rate of growth of labour productivity	$a = a_0 \cdot e^{\alpha \cdot t}$
a_0	The initial level of labour productivity	
N	The level of population	$N = N_0 \cdot e^{\beta \cdot t}$
β	The rate of population growth	
N_0	The initial level of population	
w	The wage rate	$\frac{1}{w}\frac{dw}{dt} = w(\lambda)$
w(λ)	The "Phillips curve" relation between the rate of employment and the rate of change of wages	$w(\lambda) = \frac{A}{(B - C \cdot \lambda)^2} - D$
A,B,C,D	Constants in the Phillips curve relation	
λ	The rate of employment	$\lambda = \frac{L}{N}$
π_s	The profit share of output	$\pi_s = \frac{\Pi}{Y}$
ω	The wages share of output	$\omega = \frac{W}{Y} = \frac{w \cdot L}{L \cdot a} = \frac{w}{a}$
γ	The rate of depreciation	
c(λ)	The capacity utilisation relation	$c(\lambda) = \frac{U}{v + e^{-W \cdot (\lambda - X)}}$
U,V,W,X	Constants used in the capacity utilisation relation	

$a(\pi)$	The profit rate to technical change relation	$a(\pi) = \frac{Q}{R+e^{-S\cdot\pi}} - T$
Q,R,S,T	Constants used in the profit rate to technical change relation	
$k(\pi)$	The investment to profit relation	$k(\pi) = \frac{E}{(F-G\cdot\pi)^2} - H$
E,F,G,H	Constants used in the investment to profit relation	
	Chapter Five	
r(d)	The debt ratio to interest rate relation	$r(d) = \rho \cdot \frac{A}{B+e^{-\Psi\cdot(d-\Delta)}} + E$
ρ	The exogenous component of the rate of interest	
A,B,X,Δ,E	Constants used in the debt ratio to interest rate relation	
D	The level of debt	$\frac{dD}{dt} = r \cdot D + I - \Pi$
d	The debt to output ratio	$d = \frac{D}{Y}$
g_y	The percentage rate of growth of output	$g_y = \frac{1}{Y}\frac{dY}{dt}$
π_s	The profit share of output	$\pi_s = 1 - \omega - r \cdot d$
	Chapter Six	
$g(\lambda)$	The rate of change of government subsidies function	$g(\lambda) = \frac{I}{(J-K\cdot\lambda)^2} - L$
I,J,K,L	Constants used in the rate of change of government subsidies function	
G	The level of government subsidies	$g = \frac{G}{Y}$
g	The government subsidies to output ratio	
$t(\pi)$	The rate of change of taxation function	$t(\pi) = \frac{M}{(N-O\cdot\pi)^2} - P$
M,N,O.P	Constants used in the rate of change of taxation function	
T	The level of taxes	$t = \frac{T}{Y}$
t	The taxation to output ratio	
D_k	The level of capitalist debt	$d_k = \frac{D_k}{Y}$

d_k	The capitalist debt to output ratio	
D_g	The level of government debt	$d_g = \frac{D_g}{Y}$
d_g	The government debt to output ratio	
π_s	Gross profit share of output	$\pi_s = 1 - \omega$
π_{ns}	Net profit share of output	$\pi_{ns} = 1 - \omega - t + g - r \cdot d_k$
	Chapter Seven	
p	The row vector of equilibrium prices	$p = \left(u + p \cdot A\right) \cdot (I + \hat{m})$
u	The row vector of exogenous inputs	
A	The input output coefficients matrix	
I	The identity matrix	
m	The row vector of sectoral markups	
p_1,p_2	Price vectors in periods 1 and 2	$p_2 = \left(u + du + p_1 \cdot A\right) \cdot (I + \hat{m})$
$p_t,p_{t+1},$	Price vectors in period t and t+1	$p_{t+1} = \left(u + p_t \cdot A\right) \cdot (I + \hat{m})$
		$p_{t+1} = \left(u + p_t \cdot A\right) \cdot (I + \hat{m}_{t+1})$
Π_t	Profit at time t	$\pi_t = \left(\Pi_t^T \cdot \left(\hat{C}_{t-1}\right)^{-1}\right)^T$
π_t	The rate of profit at time t	
C_t,c_t	Costs at time t	$u^T + p_{t-1}^T \cdot A$
$\mu\left(\frac{\pi_{j_{t-n}}}{\overline{\pi}_{t-n}}\right)$	An average markup adjustment mechanism	
$\overline{\pi}_{t-n}$	The average rate of profit	$\frac{\left(u^T + p_{t-1}^T \cdot A\right) m_t}{\left(u^T + p_{t-1}^T \cdot A\right) \cdot \hat{m}_t^{-1} \cdot m_t}$
ITANH(x)	A function which takes the diagonal matrix x and returns a diagonal matrix with entries $\tanh(1-x_i)$	$ITANH\left(\frac{\left(u^T + p_t^T \cdot A\right) \cdot \left(\hat{m}_{t+1}\right)^{-1} \cdot m_{t+1}}{\left(u^T + p_t^T \cdot A\right) \cdot m_{t+1}} \cdot \hat{m}_{t+1}\right)$
	Chapter Seven	
d_t	The vector of capitalist debts at time t	$d_t = d_{t-1} + r(d_{t-1})$

min0(a,b)	A vector function which returns the non-zero minimum of each element in the vectors a and b	$min0(x,y) \equiv$ $s \leftarrow length(x)$; for $i \in 1..s$: $m_i \leftarrow x_i$ if $(x_i < y_i)$, $m_i \leftarrow y_i$ otherwise; for $i \in 1..s$: $n_i \leftarrow m_i$ if $m_i > 0$, $n_i \leftarrow 0$ otherwise; n
Θ(t-n)	A lagged function of previous output and effective demand	
q_{max}	The maximum amount which could be produced if all of last period's output was used as intermediate output	$q_{max} = A^{-1} \times \left(\begin{array}{c} q_{t-1} - w_{t-1} \times \\ \sum e_{t-1} \times \hat{p}_{t-2}^{-1} \times q_w \end{array} \right)$
c_{w_t}	Consumption by workers in period t	$c_{w_t} = w_t \times \sum e_t \times \hat{p}_{t-1}^{-1} \times q_w$
e_t	Employment at time t	$e_t = \hat{E} \times q_t$
q_w	The workers consumption bundle	
i_t	Intermediate consumption in period t	$i_t = A \times q_{ti}$
w_t	Wages at time t	$w_t = w_{t-1} \times (1 + W(\frac{\sum e_{t-1}}{P}))$
$W(\frac{\sum e_{t-1}}{P})$	The "Phillips curve" relation	
q_{b_t}	Consumption by bankers in period t	$q_{b_t} = min0(mpc_b \times \sum -r(d_{t-1}) \times \hat{p}_{t-1}^{-1} \times q_b, q_{t-1} - (i_t + q_{w_t}))$
q_{π_t}	Consumption by capitalists in period t	$\widehat{mpc}_\pi(\Pi_{t-1}) \times Q_\pi^T \times dmmin1(\hat{p}_{t-1} \times (\hat{q}_{t-1} -$
Π_t	Gross profits in time t	$\Pi_t = R_t - C_{t-1}$
m_t	Markups in time t	$maxs\left(\hat{m}_{t-1} \times \left(\begin{array}{c} min1(I) - \mu \times \\ \left(\left(\hat{p}_{t-1} \times \hat{q}_{t-1}\right)^{-1} \times \right. \\ meandev(\pi_{t-1} \end{array} \right. \right., m_{min}$